Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/1.2.2.6-P-x-d-x-^m-a+b-x^2+c-x^4-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [145]. This is test number [43].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (145)	% 0. (0)
Mathematica	% 100. (145)	% 0. (0)
Maple	% 97.93 (142)	% 2.07 (3)
Maxima	% 42.07 (61)	% 57.93 (84)
Fricas	% 79.31 (115)	% 20.69 (30)
Sympy	% 61.38 (89)	% 38.62 (56)
Giac	% 60. (87)	% 40. (58)

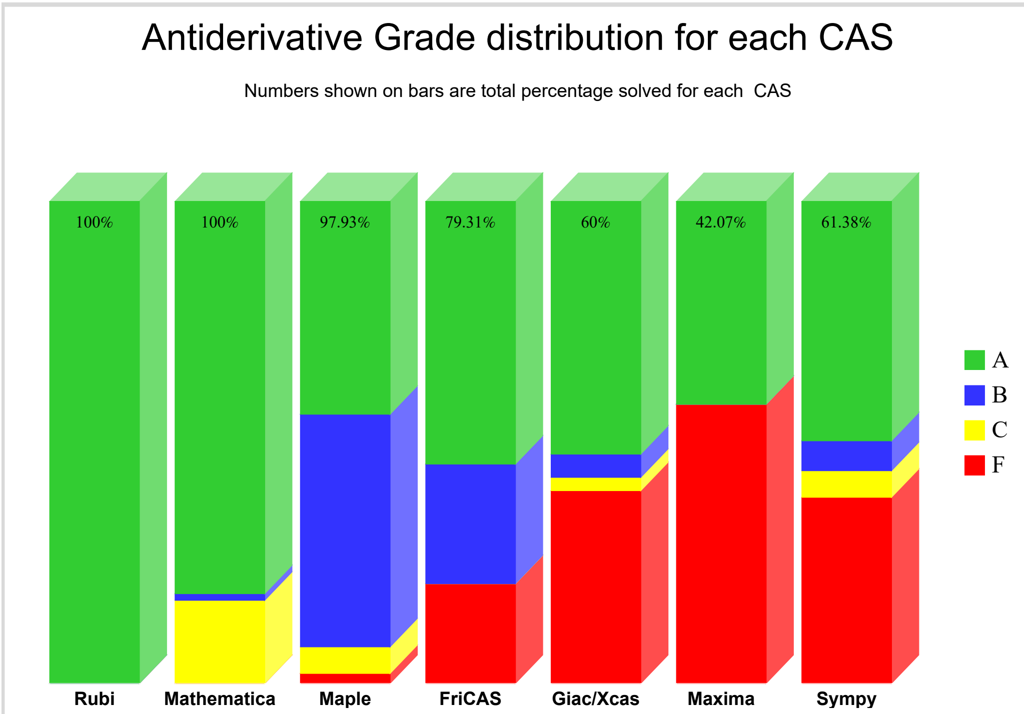
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

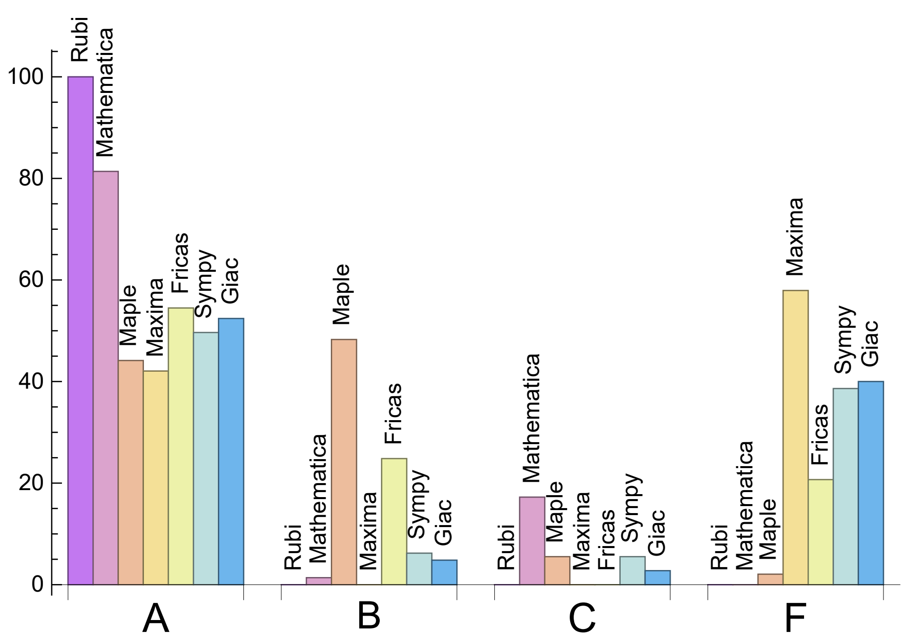
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	81.38	1.38	17.24	0.
Maple	44.14	48.28	5.52	2.07
Maxima	42.07	0.	0.	57.93
Fricas	54.48	24.83	0.	20.69
Sympy	49.66	6.21	5.52	38.62
Giac	52.41	4.83	2.76	40.

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.84	207.48	1.03	179.	1.
Mathematica	0.46	207.06	1.02	144.	1.
Maple	0.02	584.	2.09	224.5	1.67
Maxima	1.25	111.54	1.23	81.	1.18
Fricas	6.52	2775.24	10.67	327.	3.67
Sympy	13.52	223.51	1.69	70.	0.95
Giac	2.76	717.02	3.52	103.	1.3

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {40, 41}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

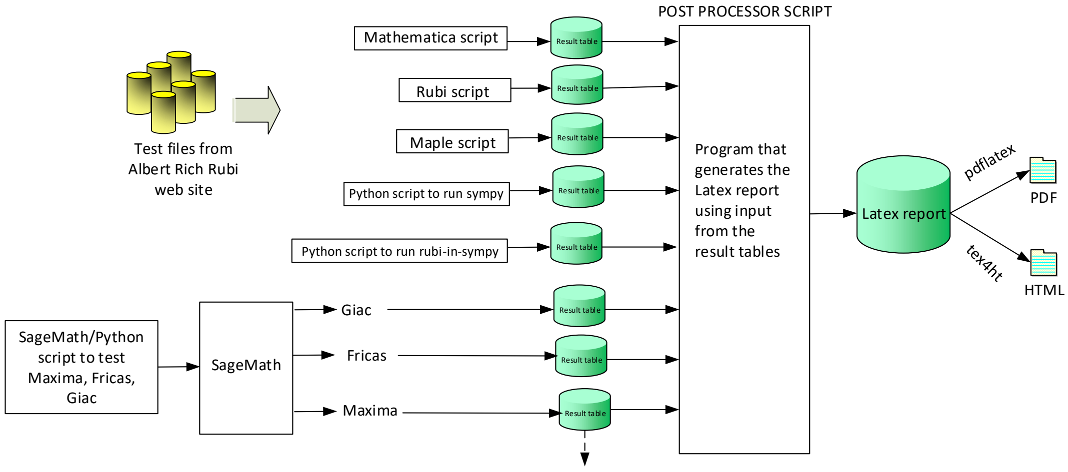
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 125, 126, 127, 128, 129, 130, 131, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade: { 135, 136 }

C grade: { 40, 41, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 132, 133, 134 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 51, 64, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 131, 132, 133, 134, 143, 144, 145 }

B grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130 }

C grade: { 135, 136, 137, 138, 139, 140, 141, 142 }

F grade: { 30, 40, 41 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 131, 132, 133, 134, 139, 140

B grade: { }

C grade: { }

F grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 135, 136, 137, 138, 141, 142, 143, 144, 145 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 47, 48, 49, 50, 51, 52, 53, 54, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 125, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade: { 37, 38, 39, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124 }

C grade: { }

F grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 42, 43, 44, 45, 46, 68, 73, 126, 127, 128, 129, 130 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 39, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124 }

B grade: { 47, 48, 49, 50, 57, 58, 63, 64, 125 }

C grade: { 134, 135, 136, 137, 139, 140, 141, 142 }

F grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 72, 73, 126, 127, 128, 129, 130, 131, 132, 133, 138, 143, 144, 145 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 125, 132, 133, 134, 139, 140 }

B grade: { 37, 38, 39, 131, 143, 144, 145 }

C grade: { 24, 25, 26, 27 }

F grade: { 21, 22, 23, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 42, 43, 44, 45, 46, 55, 56, 57, 58, 59, 60, 68, 69, 70, 71, 72, 73, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 135, 136, 137, 138, 141, 142 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	61	81	169	68	86
normalized size	1	1.	1.	0.82	1.09	2.28	0.92	1.16
time (sec)	N/A	0.082	0.015	0.001	0.932	1.052	0.087	1.093

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	61	81	169	68	86
normalized size	1	1.	1.	0.82	1.09	2.28	0.92	1.16
time (sec)	N/A	0.057	0.012	0.001	0.961	1.124	0.074	1.085

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	161	65	82
normalized size	1	1.	1.	0.84	1.12	2.33	0.94	1.19
time (sec)	N/A	0.036	0.013	0.	0.946	1.122	0.11	1.093

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	60	74	143	63	81
normalized size	1	1.	1.	0.92	1.14	2.2	0.97	1.25
time (sec)	N/A	0.04	0.015	0.003	0.941	1.271	0.45	1.095

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	74	157	58	77
normalized size	1	1.	1.	0.9	1.17	2.49	0.92	1.22
time (sec)	N/A	0.051	0.023	0.007	0.965	1.289	0.492	1.095

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	58	58	74	154	60	78
normalized size	1	1.	0.92	0.92	1.17	2.44	0.95	1.24
time (sec)	N/A	0.048	0.04	0.006	0.945	1.211	0.67	1.083

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	57	76	149	61	76
normalized size	1	1.	0.95	0.9	1.21	2.37	0.97	1.21
time (sec)	N/A	0.051	0.047	0.006	0.959	1.263	1.013	1.106

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	62	58	76	154	61	77
normalized size	1	1.	0.98	0.92	1.21	2.44	0.97	1.22
time (sec)	N/A	0.051	0.03	0.007	0.969	1.291	2.982	1.097

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	60	76	159	63	77
normalized size	1	1.	1.	0.95	1.21	2.52	1.	1.22
time (sec)	N/A	0.052	0.058	0.007	0.952	1.208	8.345	1.091

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	63	80	159	66	81
normalized size	1	1.	1.	0.93	1.18	2.34	0.97	1.19
time (sec)	N/A	0.048	0.048	0.006	0.968	1.272	16.138	1.096

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	159	142	193	400	168	208
normalized size	1	1.	1.	0.89	1.21	2.52	1.06	1.31
time (sec)	N/A	0.214	0.047	0.002	0.943	1.113	0.094	1.115

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	159	142	193	397	163	208
normalized size	1	1.	1.	0.89	1.21	2.5	1.03	1.31
time (sec)	N/A	0.143	0.036	0.001	0.949	1.13	0.095	1.1

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	154	139	189	385	165	204
normalized size	1	1.	1.	0.9	1.23	2.5	1.07	1.32
time (sec)	N/A	0.111	0.031	0.001	0.973	1.103	0.095	1.091

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	150	149	186	335	156	201
normalized size	1	1.	1.	0.99	1.24	2.23	1.04	1.34
time (sec)	N/A	0.107	0.04	0.003	0.955	1.247	0.5	1.095

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	145	147	185	365	156	198
normalized size	1	1.	1.	1.01	1.28	2.52	1.08	1.37
time (sec)	N/A	0.121	0.097	0.007	0.931	1.251	0.503	1.088

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	139	148	188	362	151	200
normalized size	1	1.	0.93	0.99	1.26	2.43	1.01	1.34
time (sec)	N/A	0.123	0.109	0.008	0.944	1.3	0.641	1.136

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	151	146	189	354	158	197
normalized size	1	1.	1.01	0.98	1.27	2.38	1.06	1.32
time (sec)	N/A	0.137	0.082	0.006	0.956	1.288	0.886	1.088

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	130	144	188	346	151	192
normalized size	1	1.	0.88	0.97	1.27	2.34	1.02	1.3
time (sec)	N/A	0.142	0.083	0.008	0.984	1.237	2.622	1.114

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	142	144	186	344	151	189
normalized size	1	1.	0.99	1.01	1.3	2.41	1.06	1.32
time (sec)	N/A	0.147	0.082	0.007	0.97	1.204	8.278	1.121

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	144	148	189	346	153	190
normalized size	1	1.	0.97	0.99	1.27	2.32	1.03	1.28
time (sec)	N/A	0.143	0.101	0.009	0.953	1.217	29.718	1.129

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	339	339	460	1622	0	0	0	0
normalized size	1	1.	1.36	4.78	0.	0.	0.	0.
time (sec)	N/A	1.856	0.644	0.046	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	377	1171	0	0	0	0
normalized size	1	1.	1.36	4.21	0.	0.	0.	0.
time (sec)	N/A	0.466	0.451	0.033	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	360	1327	0	0	0	0
normalized size	1	1.	1.33	4.91	0.	0.	0.	0.
time (sec)	N/A	0.835	0.396	0.036	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	240	728	0	0	0	7461
normalized size	1	1.	1.08	3.26	0.	0.	0.	33.46
time (sec)	N/A	0.213	0.396	0.023	0.	0.	0.	3.237

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	234	616	0	0	0	9080
normalized size	1	1.	1.11	2.92	0.	0.	0.	43.03
time (sec)	N/A	0.266	0.223	0.017	0.	0.	0.	2.946

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	285	488	0	0	0	4740
normalized size	1	1.	1.24	2.13	0.	0.	0.	20.7
time (sec)	N/A	0.259	0.468	0.027	0.	0.	0.	2.447

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	315	811	0	0	0	7089
normalized size	1	1.	1.21	3.12	0.	0.	0.	27.27
time (sec)	N/A	0.471	1.135	0.027	0.	0.	0.	3.037

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	377	1054	0	0	0	0
normalized size	1	1.	1.31	3.66	0.	0.	0.	0.
time (sec)	N/A	0.474	0.984	0.037	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	444	1429	0	0	0	0
normalized size	1	1.	1.08	3.47	0.	0.	0.	0.
time (sec)	N/A	1.334	1.561	0.042	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	358	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.618	0.995	180.	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	378	1119	0	0	0	0
normalized size	1	1.	1.06	3.14	0.	0.	0.	0.
time (sec)	N/A	0.902	1.161	0.036	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	335	1344	0	0	0	0
normalized size	1	1.	1.06	4.24	0.	0.	0.	0.
time (sec)	N/A	0.415	1.411	0.117	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	393	1813	0	0	0	0
normalized size	1	1.	1.07	4.93	0.	0.	0.	0.
time (sec)	N/A	0.867	1.456	0.105	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	458	1603	0	0	0	0
normalized size	1	1.	1.14	3.98	0.	0.	0.	0.
time (sec)	N/A	0.932	1.676	0.042	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	514	514	559	2398	0	0	0	0
normalized size	1	1.	1.09	4.67	0.	0.	0.	0.
time (sec)	N/A	1.486	2.278	0.057	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	534	534	655	2512	0	0	0	0
normalized size	1	1.	1.23	4.7	0.	0.	0.	0.
time (sec)	N/A	1.992	2.761	0.062	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	296	5520	0	11187	0	10541
normalized size	1	1.	0.74	13.83	0.	28.04	0.	26.42
time (sec)	N/A	0.425	1.271	0.016	0.	2.672	0.	1.338

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	185	2187	0	4469	0	4324
normalized size	1	1.	0.71	8.41	0.	17.19	0.	16.63
time (sec)	N/A	0.223	0.383	0.01	0.	1.825	0.	1.195

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	90	585	0	1181	3735	1234
normalized size	1	1.	0.66	4.27	0.	8.62	27.26	9.01
time (sec)	N/A	0.088	0.122	0.004	0.	1.577	2.79	1.123

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	368	368	168	0	0	0	0	0
normalized size	1	1.	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.622	0.22	0.043	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	685	670	242	0	0	0	0	0
normalized size	1	0.98	0.35	0.	0.	0.	0.	0.
time (sec)	N/A	2.378	0.345	0.029	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	378	1119	0	0	0	0
normalized size	1	1.	1.06	3.14	0.	0.	0.	0.
time (sec)	N/A	0.924	1.187	0.	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	378	1119	0	0	0	0
normalized size	1	1.	1.06	3.14	0.	0.	0.	0.
time (sec)	N/A	0.371	0.793	0.025	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	378	1119	0	0	0	0
normalized size	1	1.	1.06	3.14	0.	0.	0.	0.
time (sec)	N/A	0.37	0.171	0.022	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	378	1119	0	0	0	0
normalized size	1	1.	1.06	3.14	0.	0.	0.	0.
time (sec)	N/A	0.356	0.165	0.021	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	378	1119	0	0	0	0
normalized size	1	1.	1.06	3.14	0.	0.	0.	0.
time (sec)	N/A	0.359	0.16	0.024	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	260	622	0	1854	1392	413
normalized size	1	1.	0.95	2.28	0.	6.79	5.1	1.51
time (sec)	N/A	0.854	0.208	0.007	0.	4.168	49.741	1.15

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	193	474	0	1404	1044	289
normalized size	1	1.	0.95	2.33	0.	6.92	5.14	1.42
time (sec)	N/A	0.424	0.143	0.006	0.	2.823	35.705	1.162

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	136	321	0	994	721	190
normalized size	1	1.	0.94	2.23	0.	6.9	5.01	1.32
time (sec)	N/A	0.272	0.107	0.005	0.	1.966	19.719	1.15

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	100	211	0	691	498	134
normalized size	1	1.	0.97	2.05	0.	6.71	4.83	1.3
time (sec)	N/A	0.179	0.069	0.005	0.	1.747	11.093	1.147

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	178	165	0	683	0	131
normalized size	1	1.	1.84	1.7	0.	7.04	0.	1.35
time (sec)	N/A	0.2	0.143	0.008	0.	2.633	0.	1.174

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	203	227	0	873	0	182
normalized size	1	1.	1.72	1.92	0.	7.4	0.	1.54
time (sec)	N/A	0.285	0.162	0.009	0.	2.991	0.	1.129

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	314	356	0	1283	0	286
normalized size	1	1.	1.8	2.05	0.	7.37	0.	1.64
time (sec)	N/A	0.407	0.353	0.011	0.	5.267	0.	1.137

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	416	523	0	1747	0	423
normalized size	1	1.	1.7	2.14	0.	7.16	0.	1.73
time (sec)	N/A	0.573	0.352	0.014	0.	13.701	0.	1.146

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	456	1450	0	31190	0	0
normalized size	1	1.	1.24	3.93	0.	84.53	0.	0.
time (sec)	N/A	4.577	0.546	0.036	0.	100.239	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	365	1035	0	18515	0	0
normalized size	1	1.	1.29	3.67	0.	65.66	0.	0.
time (sec)	N/A	3.59	0.538	0.031	0.	22.088	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	258	676	0	11135	1151	0
normalized size	1	1.	1.18	3.09	0.	50.84	5.26	0.
time (sec)	N/A	0.637	0.347	0.025	0.	11.18	90.156	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	253	563	0	11429	1192	0
normalized size	1	1.	1.19	2.64	0.	53.66	5.6	0.
time (sec)	N/A	0.839	0.328	0.025	0.	5.209	96.593	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	284	727	0	19478	0	0
normalized size	1	1.	1.06	2.72	0.	72.95	0.	0.
time (sec)	N/A	1.065	0.373	0.029	0.	27.674	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	394	1121	0	31905	0	0
normalized size	1	1.	1.2	3.41	0.	96.98	0.	0.
time (sec)	N/A	1.942	0.604	0.036	0.	102.526	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	309	1167	0	4393	0	572
normalized size	1	1.	0.97	3.65	0.	13.73	0.	1.79
time (sec)	N/A	1.233	0.551	0.018	0.	3.462	0.	19.825

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	236	832	0	3043	0	377
normalized size	1	1.	1.	3.53	0.	12.89	0.	1.6
time (sec)	N/A	0.44	0.385	0.017	0.	2.305	0.	19.206

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	175	336	0	2033	1030	263
normalized size	1	1.	1.06	2.04	0.	12.32	6.24	1.59
time (sec)	N/A	0.287	0.288	0.013	0.	1.7	145.522	19.602

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	130	205	0	1374	474	189
normalized size	1	1.	1.06	1.67	0.	11.17	3.85	1.54
time (sec)	N/A	0.184	0.122	0.011	0.	1.446	40.153	19.251

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	268	462	0	2333	0	306
normalized size	1	1.	1.61	2.78	0.	14.05	0.	1.84
time (sec)	N/A	0.394	0.469	0.018	0.	7.603	0.	19.512

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	403	722	0	3637	0	387
normalized size	1	1.	1.72	3.09	0.	15.54	0.	1.65
time (sec)	N/A	0.725	0.692	0.023	0.	16.641	0.	21.974

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	592	1078	0	5341	0	722
normalized size	1	1.	1.8	3.28	0.	16.23	0.	2.19
time (sec)	N/A	1.157	1.219	0.028	0.	39.111	0.	19.648

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	550	550	648	2558	0	0	0	0
normalized size	1	1.	1.18	4.65	0.	0.	0.	0.
time (sec)	N/A	13.227	2.313	0.053	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	511	1977	0	26999	0	0
normalized size	1	1.	1.17	4.53	0.	61.92	0.	0.
time (sec)	N/A	5.541	1.74	0.042	0.	75.811	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	414	1300	0	18090	0	0
normalized size	1	1.	1.14	3.59	0.	49.97	0.	0.
time (sec)	N/A	2.498	1.235	0.035	0.	37.949	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	382	1182	0	18375	0	0
normalized size	1	1.	1.1	3.42	0.	53.11	0.	0.
time (sec)	N/A	1.896	1.196	0.033	0.	35.917	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	444	1575	0	28044	0	0
normalized size	1	1.	1.11	3.95	0.	70.29	0.	0.
time (sec)	N/A	2.203	1.473	0.045	0.	79.06	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	575	575	548	2180	0	0	0	0
normalized size	1	1.	0.95	3.79	0.	0.	0.	0.
time (sec)	N/A	9.906	1.96	0.048	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	56	78	220	61	85
normalized size	1	1.	0.91	0.82	1.15	3.24	0.9	1.25
time (sec)	N/A	0.126	0.035	0.015	0.972	1.962	0.163	1.119

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	51	72	208	54	78
normalized size	1	1.	1.	0.84	1.18	3.41	0.89	1.28
time (sec)	N/A	0.118	0.027	0.016	1.016	1.949	0.156	1.111

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	46	65	186	48	72
normalized size	1	1.	1.	0.85	1.2	3.44	0.89	1.33
time (sec)	N/A	0.108	0.024	0.014	0.996	1.964	0.155	1.131

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	41	58	169	42	61
normalized size	1	1.	1.	0.84	1.18	3.45	0.86	1.24
time (sec)	N/A	0.086	0.022	0.013	1.027	1.975	0.156	1.093

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	51	144	36	54
normalized size	1	1.	1.	0.86	1.21	3.43	0.86	1.29
time (sec)	N/A	0.049	0.017	0.015	0.965	2.003	0.15	1.121

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	38	59	185	39	63
normalized size	1	1.	1.	0.86	1.34	4.2	0.89	1.43
time (sec)	N/A	0.077	0.022	0.017	1.019	1.749	0.164	1.084

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	50	45	72	219	51	72
normalized size	1	1.	0.91	0.82	1.31	3.98	0.93	1.31
time (sec)	N/A	0.104	0.025	0.017	1.001	1.859	0.19	1.11

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	56	50	76	231	56	89
normalized size	1	1.	0.88	0.78	1.19	3.61	0.88	1.39
time (sec)	N/A	0.111	0.029	0.018	0.953	1.727	0.204	1.094

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	71	56	78	247	66	78
normalized size	1	1.	1.01	0.8	1.11	3.53	0.94	1.11
time (sec)	N/A	0.085	0.046	0.013	1.468	1.843	0.186	1.083

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	58	49	69	209	54	69
normalized size	1	1.	1.02	0.86	1.21	3.67	0.95	1.21
time (sec)	N/A	0.082	0.048	0.013	1.486	1.756	0.185	1.084

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	46	65	198	53	65
normalized size	1	1.	1.02	0.82	1.16	3.54	0.95	1.16
time (sec)	N/A	0.073	0.043	0.01	1.488	1.903	0.19	1.147

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	50	41	58	180	48	58
normalized size	1	1.	1.02	0.84	1.18	3.67	0.98	1.18
time (sec)	N/A	0.066	0.039	0.01	1.487	2.14	0.186	1.101

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	46	38	54	170	44	54
normalized size	1	1.	0.96	0.79	1.12	3.54	0.92	1.12
time (sec)	N/A	0.028	0.041	0.011	1.499	2.06	0.182	1.081

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	43	61	185	49	61
normalized size	1	1.	0.96	0.81	1.15	3.49	0.92	1.15
time (sec)	N/A	0.073	0.049	0.012	1.458	2.18	0.201	1.123

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	56	48	70	213	56	70
normalized size	1	1.	0.9	0.77	1.13	3.44	0.9	1.13
time (sec)	N/A	0.084	0.053	0.014	1.471	1.587	0.225	1.115

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	61	53	77	239	61	77
normalized size	1	1.	0.88	0.77	1.12	3.46	0.88	1.12
time (sec)	N/A	0.09	0.06	0.016	1.466	1.53	0.243	1.142

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	77	58	84	271	66	84
normalized size	1	1.	1.01	0.76	1.11	3.57	0.87	1.11
time (sec)	N/A	0.1	0.057	0.013	1.531	1.565	0.263	1.077

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	71	64	96	325	75	82
normalized size	1	1.	0.88	0.79	1.19	4.01	0.93	1.01
time (sec)	N/A	0.112	0.06	0.013	1.494	1.585	0.241	1.096

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	66	62	92	313	75	78
normalized size	1	1.	0.82	0.78	1.15	3.91	0.94	0.98
time (sec)	N/A	0.1	0.055	0.011	1.495	1.538	0.238	1.113

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	60	56	85	285	68	72
normalized size	1	1.	0.8	0.75	1.13	3.8	0.91	0.96
time (sec)	N/A	0.091	0.061	0.011	1.496	1.626	0.241	1.085

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	55	54	81	274	65	68
normalized size	1	1.	0.76	0.75	1.12	3.81	0.9	0.94
time (sec)	N/A	0.068	0.061	0.012	1.49	1.64	0.237	1.1

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	56	53	81	279	65	68
normalized size	1	1.	0.78	0.74	1.12	3.88	0.9	0.94
time (sec)	N/A	0.066	0.063	0.012	1.497	1.521	0.236	1.105

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	56	53	81	281	65	68
normalized size	1	1.	0.78	0.74	1.12	3.9	0.9	0.94
time (sec)	N/A	0.037	0.06	0.013	1.515	1.573	0.231	1.106

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	63	58	88	302	70	74
normalized size	1	1.	0.8	0.73	1.11	3.82	0.89	0.94
time (sec)	N/A	0.103	0.068	0.014	1.486	1.567	0.26	1.127

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	78	64	97	336	76	84
normalized size	1	1.	0.91	0.74	1.13	3.91	0.88	0.98
time (sec)	N/A	0.119	0.06	0.016	1.548	1.544	0.278	1.11

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	73	68	104	370	82	90
normalized size	1	1.	0.78	0.73	1.12	3.98	0.88	0.97
time (sec)	N/A	0.134	0.079	0.015	1.474	1.569	0.301	1.121

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	78	74	96	270	85	103
normalized size	1	1.	0.91	0.86	1.12	3.14	0.99	1.2
time (sec)	N/A	0.135	0.047	0.011	1.472	1.5	0.172	1.131

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	73	69	89	255	80	96
normalized size	1	1.	0.9	0.85	1.1	3.15	0.99	1.19
time (sec)	N/A	0.127	0.031	0.01	1.499	1.545	0.168	1.121

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	66	64	80	235	73	89
normalized size	1	1.	0.89	0.86	1.08	3.18	0.99	1.2
time (sec)	N/A	0.121	0.03	0.008	1.469	1.557	0.164	1.08

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	61	59	73	219	66	73
normalized size	1	1.	0.94	0.91	1.12	3.37	1.02	1.12
time (sec)	N/A	0.105	0.027	0.008	1.471	1.503	0.168	1.112

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	54	66	194	60	66
normalized size	1	1.	1.	0.93	1.14	3.34	1.03	1.14
time (sec)	N/A	0.067	0.022	0.01	1.47	1.527	0.162	1.109

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	93	58	74	238	65	84
normalized size	1	1.	1.41	0.88	1.12	3.61	0.98	1.27
time (sec)	N/A	0.108	0.06	0.01	1.479	1.481	0.174	1.08

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	97	63	89	275	75	89
normalized size	1	1.	1.37	0.89	1.25	3.87	1.06	1.25
time (sec)	N/A	0.134	0.051	0.014	1.458	1.615	0.201	1.098

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	105	68	96	289	80	107
normalized size	1	1.	1.31	0.85	1.2	3.61	1.	1.34
time (sec)	N/A	0.137	0.061	0.012	1.475	1.49	0.214	1.083

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	110	73	103	317	85	113
normalized size	1	1.	1.26	0.84	1.18	3.64	0.98	1.3
time (sec)	N/A	0.149	0.069	0.013	1.499	1.527	0.237	1.094

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	145	427	0	2484	71	0
normalized size	1	1.	0.58	1.72	0.	10.02	0.29	0.
time (sec)	N/A	0.345	0.178	0.105	0.	1.747	0.537	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	132	419	0	2055	63	0
normalized size	1	1.	0.56	1.77	0.	8.67	0.27	0.
time (sec)	N/A	0.293	0.165	0.026	0.	1.665	0.541	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	129	416	0	2102	58	0
normalized size	1	1.	0.56	1.79	0.	9.06	0.25	0.
time (sec)	N/A	0.292	0.162	0.022	0.	1.775	0.533	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	121	412	0	2072	51	0
normalized size	1	1.	0.54	1.83	0.	9.21	0.23	0.
time (sec)	N/A	0.297	0.17	0.018	0.	1.756	0.535	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	115	408	0	1993	48	0
normalized size	1	1.	0.51	1.82	0.	8.9	0.21	0.
time (sec)	N/A	0.215	0.274	0.02	0.	1.724	0.522	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	126	414	0	1871	53	0
normalized size	1	1.	0.55	1.81	0.	8.17	0.23	0.
time (sec)	N/A	0.31	0.185	0.023	0.	1.706	0.551	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	131	419	0	2205	60	0
normalized size	1	1.	0.55	1.76	0.	9.26	0.25	0.
time (sec)	N/A	0.335	0.31	0.023	0.	1.723	0.575	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	140	424	0	2392	65	0
normalized size	1	1.	0.57	1.73	0.	9.76	0.27	0.
time (sec)	N/A	0.329	0.303	0.023	0.	1.741	0.59	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	156	429	0	2853	82	0
normalized size	1	1.	0.64	1.77	0.	11.74	0.34	0.
time (sec)	N/A	0.36	0.222	0.02	0.	1.794	0.591	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	155	426	0	2421	80	0
normalized size	1	1.	0.64	1.76	0.	10.	0.33	0.
time (sec)	N/A	0.31	0.211	0.022	0.	1.81	0.607	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	138	422	0	2670	71	0
normalized size	1	1.	0.59	1.8	0.	11.36	0.3	0.
time (sec)	N/A	0.3	0.33	0.022	0.	1.773	0.587	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	129	418	0	2402	68	0
normalized size	1	1.	0.54	1.76	0.	10.09	0.29	0.
time (sec)	N/A	0.29	0.304	0.019	0.	1.835	0.592	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	133	418	0	2074	68	0
normalized size	1	1.	0.54	1.7	0.	8.43	0.28	0.
time (sec)	N/A	0.284	0.299	0.022	0.	1.734	0.592	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	129	418	0	2261	68	0
normalized size	1	1.	0.52	1.69	0.	9.12	0.27	0.
time (sec)	N/A	0.254	0.302	0.022	0.	1.68	0.571	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	140	424	0	2529	73	0
normalized size	1	1.	0.55	1.68	0.	10.	0.29	0.
time (sec)	N/A	0.343	0.379	0.023	0.	1.733	0.607	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	139	429	0	2952	80	0
normalized size	1	1.	0.53	1.64	0.	11.27	0.31	0.
time (sec)	N/A	0.366	0.332	0.024	0.	1.75	0.627	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	142	357	0	1021	789	197
normalized size	1	1.	0.95	2.4	0.	6.85	5.3	1.32
time (sec)	N/A	0.295	0.127	0.005	0.	2.351	49.275	1.175

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	594	594	721	3028	0	0	0	0
normalized size	1	1.	1.21	5.1	0.	0.	0.	0.
time (sec)	N/A	14.113	2.847	0.059	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	471	471	575	2300	0	0	0	0
normalized size	1	1.	1.22	4.88	0.	0.	0.	0.
time (sec)	N/A	6.662	2.116	0.05	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	512	1760	0	0	0	0
normalized size	1	1.	1.14	3.92	0.	0.	0.	0.
time (sec)	N/A	2.867	1.833	0.042	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	460	529	2045	0	0	0	0
normalized size	1	1.	1.15	4.45	0.	0.	0.	0.
time (sec)	N/A	2.791	2.638	0.045	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	542	542	612	2503	0	0	0	0
normalized size	1	1.	1.13	4.62	0.	0.	0.	0.
time (sec)	N/A	7.265	2.336	0.048	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	42	63	0	78
normalized size	1	1.	1.	1.05	2.1	3.15	0.	3.9
time (sec)	N/A	0.036	0.147	0.014	1.239	1.857	0.	1.197

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	278	265	145	398	317	0	328
normalized size	1	1.32	1.26	0.69	1.9	1.51	0.	1.56
time (sec)	N/A	0.315	1.437	0.006	1.647	1.774	0.	1.255

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	213	232	109	293	238	0	239
normalized size	1	1.34	1.46	0.69	1.84	1.5	0.	1.5
time (sec)	N/A	0.189	1.089	0.007	1.492	1.911	0.	1.148

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	149	194	73	188	162	350	153
normalized size	1	1.37	1.78	0.67	1.72	1.49	3.21	1.4
time (sec)	N/A	0.122	0.703	0.005	1.636	1.817	77.822	1.159

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	151	217	143	0	178	304	0
normalized size	1	1.62	2.33	1.54	0.	1.91	3.27	0.
time (sec)	N/A	0.165	0.891	0.044	0.	1.598	45.044	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	155	233	163	0	215	270	0
normalized size	1	1.57	2.35	1.65	0.	2.17	2.73	0.
time (sec)	N/A	0.252	0.217	0.025	0.	1.297	64.196	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	182	134	222	0	224	253	0
normalized size	1	1.44	1.06	1.76	0.	1.78	2.01	0.
time (sec)	N/A	0.277	0.167	0.025	0.	1.367	89.724	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	248	173	306	0	298	0	0
normalized size	1	1.17	0.82	1.44	0.	1.41	0.	0.
time (sec)	N/A	0.373	0.2	0.033	0.	1.449	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	245	202	273	309	298	362	257
normalized size	1	1.13	0.94	1.26	1.43	1.38	1.68	1.19
time (sec)	N/A	0.205	0.809	0.035	1.56	1.432	125.927	1.152

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	179	157	191	201	227	325	170
normalized size	1	1.4	1.23	1.49	1.57	1.77	2.54	1.33
time (sec)	N/A	0.091	0.582	0.017	1.506	1.58	37.972	1.138

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	155	135	148	0	203	287	0
normalized size	1	1.52	1.32	1.45	0.	1.99	2.81	0.
time (sec)	N/A	0.122	0.565	0.022	0.	1.524	54.216	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	81	146	0	203	257	0
normalized size	1	1.	0.52	0.93	0.	1.29	1.64	0.
time (sec)	N/A	0.125	0.128	0.022	0.	1.382	77.442	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	87	82	0	173	0	1489
normalized size	1	1.	0.54	0.51	0.	1.08	0.	9.31
time (sec)	N/A	0.145	0.125	0.005	0.	1.418	0.	1.84

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	124	118	0	251	0	2048
normalized size	1	1.	0.55	0.52	0.	1.11	0.	9.06
time (sec)	N/A	0.178	0.15	0.005	0.	1.54	0.	2.506

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	158	154	0	327	0	2607
normalized size	1	1.	0.54	0.53	0.	1.12	0.	8.93
time (sec)	N/A	0.242	0.181	0.007	0.	1.861	0.	3.479

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [35] had the largest ratio of [0.4643]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	26	0.038
2	A	2	1	1.	24	0.042
3	A	2	1	1.	23	0.043
4	A	2	1	1.	26	0.038
5	A	2	1	1.	26	0.038
6	A	2	1	1.	26	0.038
7	A	2	1	1.	26	0.038
8	A	2	1	1.	26	0.038
9	A	2	1	1.	26	0.038
10	A	2	1	1.	26	0.038
11	A	2	1	1.	28	0.036
12	A	2	1	1.	26	0.038
13	A	2	1	1.	25	0.04
14	A	2	1	1.	28	0.036
15	A	2	1	1.	28	0.036
16	A	2	1	1.	28	0.036
17	A	2	1	1.	28	0.036
18	A	2	1	1.	28	0.036
19	A	2	1	1.	28	0.036
20	A	2	1	1.	28	0.036
21	A	13	11	1.	28	0.393
22	A	12	11	1.	28	0.393
23	A	11	10	1.	28	0.357
24	A	10	9	1.	26	0.346
25	A	8	7	1.	25	0.28
26	A	12	10	1.	28	0.357
27	A	13	12	1.	28	0.429
28	A	13	11	1.	28	0.393
29	A	11	10	1.	28	0.357
30	A	10	9	1.	28	0.321
31	A	10	9	1.	28	0.321
32	A	10	9	1.	26	0.346
33	A	10	9	1.	25	0.36
34	A	14	12	1.	28	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
35	A	15	13	1.	28	0.464
36	A	15	13	1.	28	0.464
37	A	2	1	1.	30	0.033
38	A	2	1	1.	30	0.033
39	A	2	1	1.	28	0.036
40	A	8	5	1.	30	0.167
41	A	10	6	0.98	30	0.2
42	A	10	9	1.	28	0.321
43	A	11	10	1.	30	0.333
44	A	11	10	1.	31	0.323
45	A	11	10	1.	34	0.294
46	A	11	10	1.	34	0.294
47	A	7	6	1.	30	0.2
48	A	7	6	1.	30	0.2
49	A	7	6	1.	30	0.2
50	A	7	6	1.	28	0.214
51	A	7	6	1.	30	0.2
52	A	7	6	1.	30	0.2
53	A	7	6	1.	30	0.2
54	A	7	6	1.	30	0.2
55	A	5	3	1.	30	0.1
56	A	5	3	1.	30	0.1
57	A	5	3	1.	27	0.111
58	A	5	3	1.	30	0.1
59	A	5	3	1.	30	0.1
60	A	5	3	1.	30	0.1
61	A	8	7	1.	30	0.233
62	A	7	7	1.	30	0.233
63	A	6	6	1.	30	0.2
64	A	5	5	1.	28	0.179
65	A	8	7	1.	30	0.233
66	A	8	7	1.	30	0.233
67	A	8	7	1.	30	0.233
68	A	6	4	1.	30	0.133
69	A	6	4	1.	30	0.133
70	A	4	3	1.	30	0.1
71	A	4	3	1.	27	0.111
72	A	6	4	1.	30	0.133
73	A	6	4	1.	30	0.133
74	A	7	5	1.	31	0.161
75	A	7	5	1.	31	0.161
76	A	7	5	1.	31	0.161
77	A	7	5	1.	31	0.161
78	A	5	4	1.	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
79	A	4	3	1.	31	0.097
80	A	4	3	1.	31	0.097
81	A	4	3	1.	31	0.097
82	A	6	4	1.	31	0.129
83	A	6	4	1.	31	0.129
84	A	6	4	1.	31	0.129
85	A	6	4	1.	31	0.129
86	A	4	3	1.	28	0.107
87	A	5	3	1.	31	0.097
88	A	5	3	1.	31	0.097
89	A	5	3	1.	31	0.097
90	A	5	3	1.	31	0.097
91	A	7	5	1.	31	0.161
92	A	7	5	1.	31	0.161
93	A	7	5	1.	31	0.161
94	A	5	4	1.	31	0.129
95	A	5	4	1.	31	0.129
96	A	5	4	1.	28	0.143
97	A	6	3	1.	31	0.097
98	A	6	3	1.	31	0.097
99	A	6	3	1.	31	0.097
100	A	8	7	1.	31	0.226
101	A	8	7	1.	31	0.226
102	A	8	7	1.	31	0.226
103	A	8	7	1.	31	0.226
104	A	6	6	1.	29	0.207
105	A	8	7	1.	31	0.226
106	A	8	7	1.	31	0.226
107	A	8	7	1.	31	0.226
108	A	8	7	1.	31	0.226
109	A	12	7	1.	31	0.226
110	A	12	7	1.	31	0.226
111	A	12	7	1.	31	0.226
112	A	12	7	1.	31	0.226
113	A	10	6	1.	28	0.214
114	A	12	7	1.	31	0.226
115	A	12	7	1.	31	0.226
116	A	12	7	1.	31	0.226
117	A	13	8	1.	31	0.258
118	A	13	8	1.	31	0.258
119	A	13	8	1.	31	0.258
120	A	11	7	1.	31	0.226
121	A	11	7	1.	31	0.226
122	A	11	7	1.	28	0.25

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
123	A	13	7	1.	31	0.226
124	A	13	7	1.	31	0.226
125	A	7	6	1.	33	0.182
126	A	6	4	1.	35	0.114
127	A	6	4	1.	35	0.114
128	A	4	3	1.	32	0.094
129	A	6	4	1.	35	0.114
130	A	6	4	1.	35	0.114
131	A	1	1	1.	42	0.024
132	A	5	4	1.32	35	0.114
133	A	4	3	1.34	35	0.086
134	A	4	3	1.37	33	0.091
135	A	6	5	1.62	35	0.143
136	A	6	6	1.57	35	0.171
137	A	6	6	1.44	35	0.171
138	A	7	7	1.17	35	0.2
139	A	6	6	1.13	35	0.171
140	A	5	5	1.4	32	0.156
141	A	5	5	1.52	35	0.143
142	A	5	5	1.	35	0.143
143	A	4	4	1.	35	0.114
144	A	5	5	1.	35	0.143
145	A	6	5	1.	35	0.143

Chapter 3

Listing of integrals

3.1 $\int x^2 \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=74

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{7}x^7(Ac+bC)+\frac{1}{6}bBx^6+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + (b*B*x^6)/6 + ((A*c + b*C
)*x^7)/7 + (B*c*x^8)/8 + (c*C*x^9)/9
```

Rubi [A] time = 0.0815531, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1628}

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{7}x^7(Ac+bC)+\frac{1}{6}bBx^6+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + (b*B*x^6)/6 + ((A*c + b*C
)*x^7)/7 + (B*c*x^8)/8 + (c*C*x^9)/9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right) dx &= \int \left(aAx^2+aBx^3+(Ab+aC)x^4+bBx^5+(Ac+bC)x^6+Bcx^7+cCx^8\right) d\\ &= \frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}bBx^6+\frac{1}{7}(Ac+bC)x^7+\frac{1}{8}Bcx^8+\frac{1}{9}\end{aligned}$$

Mathematica [A] time = 0.0154219, size = 74, normalized size = 1.

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{7}x^7(Ac+bC)+\frac{1}{6}bBx^6+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + (b*B*x^6)/6 + ((A*c + b*C
)*x^7)/7 + (B*c*x^8)/8 + (c*C*x^9)/9
```

Maple [A] time = 0.001, size = 61, normalized size = 0.8

$$\frac{aAx^3}{3}+\frac{aBx^4}{4}+\frac{(Ab+aC)\,x^5}{5}+\frac{bBx^6}{6}+\frac{(Ac+bC)\,x^7}{7}+\frac{Bcx^8}{8}+\frac{cCx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*b*B*x^6+1/7*(A*c+C*b)*x^7+1/8
*B*c*x^8+1/9*c*C*x^9
```

Maxima [A] time = 0.931994, size = 81, normalized size = 1.09

$$\frac{1}{9}\,Ccx^9+\frac{1}{8}\,Bcx^8+\frac{1}{6}\,Bbx^6+\frac{1}{7}\,(Cb+Ac)x^7+\frac{1}{4}\,Bax^4+\frac{1}{5}\,(Ca+Ab)x^5+\frac{1}{3}\,Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/6*B*b*x^6 + 1/7*(C*b + A*c)*x^7 + 1/4*B*a*x^4
 + 1/5*(C*a + A*b)*x^5 + 1/3*A*a*x^3
```

Fricas [A] time = 1.0522, size = 169, normalized size = 2.28

$$\frac{1}{9}x^9cC+\frac{1}{8}x^8cB+\frac{1}{7}x^7bC+\frac{1}{7}x^7cA+\frac{1}{6}x^6bB+\frac{1}{5}x^5aC+\frac{1}{5}x^5bA+\frac{1}{4}x^4aB+\frac{1}{3}x^3aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c*C + 1/8*x^8*c*B + 1/7*x^7*b*C + 1/7*x^7*c*A + 1/6*x^6*b*B + 1/5*x
^5*a*C + 1/5*x^5*b*A + 1/4*x^4*a*B + 1/3*x^3*a*A
```

Sympy [A] time = 0.086636, size = 68, normalized size = 0.92

$$\frac{Aax^3}{3}+\frac{Bax^4}{4}+\frac{Bbx^6}{6}+\frac{Bcx^8}{8}+\frac{Ccx^9}{9}+x^7\left(\frac{Ac}{7}+\frac{Cb}{7}\right)+x^5\left(\frac{Ab}{5}+\frac{Ca}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] A*a*x**3/3 + B*a*x**4/4 + B*b*x**6/6 + B*c*x**8/8 + C*c*x**9/9 + x**7*(A*c/
7 + C*b/7) + x**5*(A*b/5 + C*a/5)
```

Giac [A] time = 1.09347, size = 86, normalized size = 1.16

$$\frac{1}{9}\,Ccx^9 + \frac{1}{8}\,Bcx^8 + \frac{1}{7}\,Cbx^7 + \frac{1}{7}\,Acx^7 + \frac{1}{6}\,Bbx^6 + \frac{1}{5}\,Cax^5 + \frac{1}{5}\,Abx^5 + \frac{1}{4}\,Bax^4 + \frac{1}{3}\,Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/7*C*b*x^7 + 1/7*A*c*x^7 + 1/6*B*b*x^6 + 1/5*C
*a*x^5 + 1/5*A*b*x^5 + 1/4*B*a*x^4 + 1/3*A*a*x^3
```

3.2 $\int x\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=74

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{6}x^6(Ac+bC)+\frac{1}{5}bBx^5+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + (b*B*x^5)/5 + ((A*c + b*C
)*x^6)/6 + (B*c*x^7)/7 + (c*C*x^8)/8
```

Rubi [A] time = 0.0565914, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1628}

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{6}x^6(Ac+bC)+\frac{1}{5}bBx^5+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + (b*B*x^5)/5 + ((A*c + b*C
)*x^6)/6 + (B*c*x^7)/7 + (c*C*x^8)/8
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx&=\int\left(aAx+aBx^2+(Ab+aC)x^3+bBx^4+(Ac+bC)x^5+Bcx^6+cCx^7\right)\,dx\\&=\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}bBx^5+\frac{1}{6}(Ac+bC)x^6+\frac{1}{7}Bcx^7+\frac{1}{8}cCx\end{aligned}$$

Mathematica [A] time = 0.0120258, size = 74, normalized size = 1.

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{6}x^6(Ac+bC)+\frac{1}{5}bBx^5+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + (b*B*x^5)/5 + ((A*c + b*C
)*x^6)/6 + (B*c*x^7)/7 + (c*C*x^8)/8
```

Maple [A] time = 0.001, size = 61, normalized size = 0.8

$$\frac{aAx^2}{2}+\frac{aBx^3}{3}+\frac{(Ab+aC)\,x^4}{4}+\frac{bBx^5}{5}+\frac{(Ac+bC)\,x^6}{6}+\frac{Bcx^7}{7}+\frac{cCx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*b*B*x^5+1/6*(A*c+C*b)*x^6+1/7
*B*c*x^7+1/8*c*C*x^8
```

Maxima [A] time = 0.961008, size = 81, normalized size = 1.09

$$\frac{1}{8}\,Ccx^8 + \frac{1}{7}\,Bcx^7 + \frac{1}{5}\,Bbx^5 + \frac{1}{6}\,(Cb + Ac)x^6 + \frac{1}{3}\,Bax^3 + \frac{1}{4}\,(Ca + Ab)x^4 + \frac{1}{2}\,Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/8*C*c*x^8 + 1/7*B*c*x^7 + 1/5*B*b*x^5 + 1/6*(C*b + A*c)*x^6 + 1/3*B*a*x^3
 + 1/4*(C*a + A*b)*x^4 + 1/2*A*a*x^2
```

Fricas [A] time = 1.12357, size = 169, normalized size = 2.28

$$\frac{1}{8}x^8cC + \frac{1}{7}x^7cB + \frac{1}{6}x^6bC + \frac{1}{6}x^6cA + \frac{1}{5}x^5bB + \frac{1}{4}x^4aC + \frac{1}{4}x^4bA + \frac{1}{3}x^3aB + \frac{1}{2}x^2aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*c*C + 1/7*x^7*c*B + 1/6*x^6*b*C + 1/6*x^6*c*A + 1/5*x^5*b*B + 1/4*x
^4*a*C + 1/4*x^4*b*A + 1/3*x^3*a*B + 1/2*x^2*a*A
```

Sympy [A] time = 0.07363, size = 68, normalized size = 0.92

$$\frac{Aax^2}{2} + \frac{Bax^3}{3} + \frac{Bbx^5}{5} + \frac{Bcx^7}{7} + \frac{Ccx^8}{8} + x^6\left(\frac{Ac}{6} + \frac{Cb}{6}\right) + x^4\left(\frac{Ab}{4} + \frac{Ca}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] A*a*x**2/2 + B*a*x**3/3 + B*b*x**5/5 + B*c*x**7/7 + C*c*x**8/8 + x**6*(A*c/
6 + C*b/6) + x**4*(A*b/4 + C*a/4)
```

Giac [A] time = 1.08453, size = 86, normalized size = 1.16

$$\frac{1}{8}\,Ccx^8 + \frac{1}{7}\,Bcx^7 + \frac{1}{6}\,Cbx^6 + \frac{1}{6}\,Acx^6 + \frac{1}{5}\,Bbx^5 + \frac{1}{4}\,Cax^4 + \frac{1}{4}\,Abx^4 + \frac{1}{3}\,Bax^3 + \frac{1}{2}\,Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*C*c*x^8 + 1/7*B*c*x^7 + 1/6*C*b*x^6 + 1/6*A*c*x^6 + 1/5*B*b*x^5 + 1/4*C
*a*x^4 + 1/4*A*b*x^4 + 1/3*B*a*x^3 + 1/2*A*a*x^2
```

3.3 $\int \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=69

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{2}aBx^2+\frac{1}{5}x^5(Ac+bC)+\frac{1}{4}bBx^4+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + ((A*c + b*C)*x^5)
/5 + (B*c*x^6)/6 + (c*C*x^7)/7
```

Rubi [A] time = 0.0355215, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1657}

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{2}aBx^2+\frac{1}{5}x^5(Ac+bC)+\frac{1}{4}bBx^4+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + ((A*c + b*C)*x^5)
/5 + (B*c*x^6)/6 + (c*C*x^7)/7
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx &= \int \left(aA+aBx+(Ab+aC)x^2+bBx^3+(Ac+bC)x^4+Bcx^5+cCx^6\right)\,dx\\ &= aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}bBx^4+\frac{1}{5}(Ac+bC)x^5+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7\end{aligned}$$

Mathematica [A] time = 0.0127978, size = 69, normalized size = 1.

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{2}aBx^2+\frac{1}{5}x^5(Ac+bC)+\frac{1}{4}bBx^4+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + ((A*c + b*C)*x^5)
/5 + (B*c*x^6)/6 + (c*C*x^7)/7
```

Maple [A] time = 0., size = 58, normalized size = 0.8

$$aAx+\frac{aBx^2}{2}+\frac{(Ab+aC)\,x^3}{3}+\frac{bBx^4}{4}+\frac{(Ac+bC)\,x^5}{5}+\frac{Bcx^6}{6}+\frac{cCx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)
```

```
[Out] a*A*x+1/2*a*B*x^2+1/3*(A*b+C*a)*x^3+1/4*b*B*x^4+1/5*(A*c+C*b)*x^5+1/6*B*c*x
^6+1/7*c*C*x^7
```

Maxima [A] time = 0.94561, size = 77, normalized size = 1.12

$$\frac{1}{7}Ccx^7+\frac{1}{6}Bcx^6+\frac{1}{4}Bbx^4+\frac{1}{5}(Cb+Ac)x^5+\frac{1}{2}Bax^2+\frac{1}{3}(Ca+Ab)x^3+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/7*C*c*x^7 + 1/6*B*c*x^6 + 1/4*B*b*x^4 + 1/5*(C*b + A*c)*x^5 + 1/2*B*a*x^2
 + 1/3*(C*a + A*b)*x^3 + A*a*x
```

Fricas [A] time = 1.12176, size = 161, normalized size = 2.33

$$\frac{1}{7}x^7cC+\frac{1}{6}x^6cB+\frac{1}{5}x^5bC+\frac{1}{5}x^5cA+\frac{1}{4}x^4bB+\frac{1}{3}x^3aC+\frac{1}{3}x^3bA+\frac{1}{2}x^2aB+xaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c*C + 1/6*x^6*c*B + 1/5*x^5*b*C + 1/5*x^5*c*A + 1/4*x^4*b*B + 1/3*x
^3*a*C + 1/3*x^3*b*A + 1/2*x^2*a*B + x*a*A
```

Sympy [A] time = 0.11015, size = 65, normalized size = 0.94

$$Aax+\frac{Bax^2}{2}+\frac{Bbx^4}{4}+\frac{Bcx^6}{6}+\frac{Ccx^7}{7}+x^5\left(\frac{Ac}{5}+\frac{Cb}{5}\right)+x^3\left(\frac{Ab}{3}+\frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] A*a*x + B*a*x**2/2 + B*b*x**4/4 + B*c*x**6/6 + C*c*x**7/7 + x**5*(A*c/5 + C
*b/5) + x**3*(A*b/3 + C*a/3)
```

Giac [A] time = 1.09284, size = 82, normalized size = 1.19

$$\frac{1}{7}Ccx^7+\frac{1}{6}Bcx^6+\frac{1}{5}Cbx^5+\frac{1}{5}Acx^5+\frac{1}{4}Bbx^4+\frac{1}{3}Cax^3+\frac{1}{3}Abx^3+\frac{1}{2}Bax^2+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/7*C*c*x^7 + 1/6*B*c*x^6 + 1/5*C*b*x^5 + 1/5*A*c*x^5 + 1/4*B*b*x^4 + 1/3*C
*a*x^3 + 1/3*A*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

3.4 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx$

Optimal. Leaf size=65

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+aBx+\frac{1}{4}x^4(Ac+bC)+\frac{1}{3}bBx^3+\frac{1}{5}Bcx^5+\frac{1}{6}cCx^6$$

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + (b*B*x^3)/3 + ((A*c + b*C)*x^4)/4 + (B*c*x^5)
/5 + (c*C*x^6)/6 + a*A*Log[x]
```

Rubi [A] time = 0.0400262, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1628}

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+aBx+\frac{1}{4}x^4(Ac+bC)+\frac{1}{3}bBx^3+\frac{1}{5}Bcx^5+\frac{1}{6}cCx^6$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + (b*B*x^3)/3 + ((A*c + b*C)*x^4)/4 + (B*c*x^5)
/5 + (c*C*x^6)/6 + a*A*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx &= \int \left(aB+\frac{aA}{x}+(Ab+aC)x+bBx^2+(Ac+bC)x^3+Bcx^4+cCx^5\right)dx\\ &= aBx+\frac{1}{2}(Ab+aC)x^2+\frac{1}{3}bBx^3+\frac{1}{4}(Ac+bC)x^4+\frac{1}{5}Bcx^5+\frac{1}{6}cCx^6+aA\log(x)\end{aligned}$$

Mathematica [A] time = 0.015423, size = 65, normalized size = 1.

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+aBx+\frac{1}{4}x^4(Ac+bC)+\frac{1}{3}bBx^3+\frac{1}{5}Bcx^5+\frac{1}{6}cCx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + (b*B*x^3)/3 + ((A*c + b*C)*x^4)/4 + (B*c*x^5)
/5 + (c*C*x^6)/6 + a*A*Log[x]
```

Maple [A] time = 0.003, size = 60, normalized size = 0.9

$$\frac{cCx^6}{6}+\frac{Bcx^5}{5}+\frac{Ax^4c}{4}+\frac{Cx^4b}{4}+\frac{bBx^3}{3}+\frac{Ax^2b}{2}+\frac{Cx^2a}{2}+aBx+aA\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x)
```

```
[Out] 1/6*c*C*x^6+1/5*B*c*x^5+1/4*A*x^4*c+1/4*C*x^4*b+1/3*b*B*x^3+1/2*A*x^2*b+1/2
*C*x^2*a+a*B*x+a*A*ln(x)
```

Maxima [A] time = 0.940837, size = 74, normalized size = 1.14

$$\frac{1}{6}Ccx^6 + \frac{1}{5}Bcx^5 + \frac{1}{3}Bbx^3 + \frac{1}{4}(Cb + Ac)x^4 + Bax + \frac{1}{2}(Ca + Ab)x^2 + Aa\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="maxima")
```

```
[Out] 1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/4*(C*b + A*c)*x^4 + B*a*x + 1/2
*(C*a + A*b)*x^2 + A*a*log(x)
```

Fricas [A] time = 1.27108, size = 143, normalized size = 2.2

$$\frac{1}{6}Ccx^6 + \frac{1}{5}Bcx^5 + \frac{1}{3}Bbx^3 + \frac{1}{4}(Cb + Ac)x^4 + Bax + \frac{1}{2}(Ca + Ab)x^2 + Aa\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="fricas")
```

```
[Out] 1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/4*(C*b + A*c)*x^4 + B*a*x + 1/2
*(C*a + A*b)*x^2 + A*a*log(x)
```

Sympy [A] time = 0.449987, size = 63, normalized size = 0.97

$$Aa\log(x) + Bax + \frac{Bbx^3}{3} + \frac{Bcx^5}{5} + \frac{Ccx^6}{6} + x^4\left(\frac{Ac}{4} + \frac{Cb}{4}\right) + x^2\left(\frac{Ab}{2} + \frac{Ca}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x,x)
```

```
[Out] A*a*log(x) + B*a*x + B*b*x**3/3 + B*c*x**5/5 + C*c*x**6/6 + x**4*(A*c/4 + C
*b/4) + x**2*(A*b/2 + C*a/2)
```

Giac [A] time = 1.09484, size = 81, normalized size = 1.25

$$\frac{1}{6}Ccx^6 + \frac{1}{5}Bcx^5 + \frac{1}{4}Cbx^4 + \frac{1}{4}Acx^4 + \frac{1}{3}Bbx^3 + \frac{1}{2}Cax^2 + \frac{1}{2}Abx^2 + Bax + Aa\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="giac")
```

```
[Out] 1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/4*C*b*x^4 + 1/4*A*c*x^4 + 1/3*B*b*x^3 + 1/2*C
*a*x^2 + 1/2*A*b*x^2 + B*a*x + A*a*log(abs(x))
```

3.5 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2} dx$

Optimal. Leaf size=63

$$x(aC+Ab)-\frac{aA}{x}+aB\log(x)+\frac{1}{3}x^3(Ac+bC)+\frac{1}{2}bBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

```
[Out] -((a*A)/x) + (A*b + a*C)*x + (b*B*x^2)/2 + ((A*c + b*C)*x^3)/3 + (B*c*x^4)/
4 + (c*C*x^5)/5 + a*B*Log[x]
```

Rubi [A] time = 0.0505412, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1628}

$$x(aC+Ab)-\frac{aA}{x}+aB\log(x)+\frac{1}{3}x^3(Ac+bC)+\frac{1}{2}bBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + (b*B*x^2)/2 + ((A*c + b*C)*x^3)/3 + (B*c*x^4)/
4 + (c*C*x^5)/5 + a*B*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx &= \int \left(Ab\left(1+\frac{aC}{Ab}\right)+\frac{aA}{x^2}+\frac{aB}{x}+bBx+(Ac+bC)x^2+Bcx^3+cCx^4\right)dx\\ &= -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}bBx^2+\frac{1}{3}(Ac+bC)x^3+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5+aB\log(x\end{aligned}$$

Mathematica [A] time = 0.0230996, size = 63, normalized size = 1.

$$x(aC+Ab)-\frac{aA}{x}+aB\log(x)+\frac{1}{3}x^3(Ac+bC)+\frac{1}{2}bBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + (b*B*x^2)/2 + ((A*c + b*C)*x^3)/3 + (B*c*x^4)/
4 + (c*C*x^5)/5 + a*B*Log[x]
```

Maple [A] time = 0.007, size = 57, normalized size = 0.9

$$\frac{cCx^5}{5}+\frac{Bcx^4}{4}+\frac{Ax^3c}{3}+\frac{Cx^3b}{3}+\frac{bBx^2}{2}+Abx+aCx-\frac{Aa}{x}+aB\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x)
```

```
[Out] 1/5*c*C*x^5+1/4*B*c*x^4+1/3*A*x^3*c+1/3*C*x^3*b+1/2*b*B*x^2+A*b*x+a*C*x-a*A
/x+a*B*ln(x)
```

Maxima [A] time = 0.965118, size = 74, normalized size = 1.17

$$\frac{1}{5}Ccx^5 + \frac{1}{4}Bcx^4 + \frac{1}{2}Bbx^2 + \frac{1}{3}(Cb + Ac)x^3 + Ba\log(x) + (Ca + Ab)x - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/2*B*b*x^2 + 1/3*(C*b + A*c)*x^3 + B*a*log(x)
+ (C*a + A*b)*x - A*a/x
```

Fricas [A] time = 1.28934, size = 157, normalized size = 2.49

$$\frac{12\,Ccx^6 + 15\,Bcx^5 + 30\,Bbx^3 + 20\,(Cb + Ac)x^4 + 60\,Bax\log(x) + 60\,(Ca + Ab)x^2 - 60\,Aa}{60\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*c*x^6 + 15*B*c*x^5 + 30*B*b*x^3 + 20*(C*b + A*c)*x^4 + 60*B*a*x*
log(x) + 60*(C*a + A*b)*x^2 - 60*A*a)/x
```

Sympy [A] time = 0.49202, size = 58, normalized size = 0.92

$$-\frac{Aa}{x} + Ba\log(x) + \frac{Bbx^2}{2} + \frac{Bcx^4}{4} + \frac{Ccx^5}{5} + x^3\left(\frac{Ac}{3} + \frac{Cb}{3}\right) + x(Ab + Ca)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -A*a/x + B*a*log(x) + B*b*x**2/2 + B*c*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 +
C*b/3) + x*(A*b + C*a)
```

Giac [A] time = 1.09517, size = 77, normalized size = 1.22

$$\frac{1}{5}Ccx^5 + \frac{1}{4}Bcx^4 + \frac{1}{3}Cbx^3 + \frac{1}{3}Acx^3 + \frac{1}{2}Bbx^2 + Cax + Abx + Ba\log(|x|) - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/3*C*b*x^3 + 1/3*A*c*x^3 + 1/2*B*b*x^2 + C*a*x
 + A*b*x + B*a*log(abs(x)) - A*a/x
```

3.6 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3} \, dx$

Optimal. Leaf size=63

$$\log(x)(aC + Ab) - \frac{aA}{2x^2} - \frac{aB}{x} + \frac{1}{2}x^2(Ac + bC) + bBx + \frac{1}{3}Bcx^3 + \frac{1}{4}cCx^4$$

```
[Out] -(a*A)/(2*x^2) - (a*B)/x + b*B*x + ((A*c + b*C)*x^2)/2 + (B*c*x^3)/3 + (c*C
*x^4)/4 + (A*b + a*C)*Log[x]
```

Rubi [A] time = 0.048346, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1628}

$$\log(x)(aC + Ab) - \frac{aA}{2x^2} - \frac{aB}{x} + \frac{1}{2}x^2(Ac + bC) + bBx + \frac{1}{3}Bcx^3 + \frac{1}{4}cCx^4$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] -(a*A)/(2*x^2) - (a*B)/x + b*B*x + ((A*c + b*C)*x^2)/2 + (B*c*x^3)/3 + (c*C
*x^4)/4 + (A*b + a*C)*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)}{x^3} \, dx &= \int \left(bB + \frac{aA}{x^3} + \frac{aB}{x^2} + \frac{Ab + aC}{x} + (Ac + bC)x + Bcx^2 + cCx^3\right) dx \\ &= -\frac{aA}{2x^2} - \frac{aB}{x} + bBx + \frac{1}{2}(Ac + bC)x^2 + \frac{1}{3}Bcx^3 + \frac{1}{4}cCx^4 + (Ab + aC)\log(x)\end{aligned}$$

Mathematica [A] time = 0.0398518, size = 58, normalized size = 0.92

$$\log(x)(aC + Ab) - \frac{a(A + 2Bx)}{2x^2} + \frac{1}{12}x\left(cx\left(6A + 4Bx + 3Cx^2\right) + 6b(2B + Cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] -(a*(A + 2*B*x))/(2*x^2) + (x*(6*b*(2*B + C*x) + c*x*(6*A + 4*B*x + 3*C*x^2
)))/12 + (A*b + a*C)*Log[x]
```

Maple [A] time = 0.006, size = 58, normalized size = 0.9

$$\frac{cCx^4}{4} + \frac{Bcx^3}{3} + \frac{Ax^2c}{2} + \frac{Cx^2b}{2} + bBx - \frac{Ba}{x} - \frac{Aa}{2x^2} + A\ln(x)b + C\ln(x)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x)
```

```
[Out] 1/4*c*C*x^4+1/3*B*c*x^3+1/2*A*x^2*c+1/2*C*x^2*b+b*B*x-a*B/x-1/2*a*A/x^2+A*l
n(x)*b+C*ln(x)*a
```

Maxima [A] time = 0.945481, size = 74, normalized size = 1.17

$$\frac{1}{4}\,Ccx^4+\frac{1}{3}\,Bcx^3+Bbx+\frac{1}{2}\,(Cb+Ac)x^2+(Ca+Ab)\log(x)-\frac{2\,Bax+Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*C*c*x^4 + 1/3*B*c*x^3 + B*b*x + 1/2*(C*b + A*c)*x^2 + (C*a + A*b)*log(x
) - 1/2*(2*B*a*x + A*a)/x^2
```

Fricas [A] time = 1.21064, size = 154, normalized size = 2.44

$$\frac{3\,Ccx^6+4\,Bcx^5+12\,Bbx^3+6\,(Cb+Ac)x^4+12\,(Ca+Ab)x^2\log(x)-12\,Bax-6\,Aa}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*C*c*x^6 + 4*B*c*x^5 + 12*B*b*x^3 + 6*(C*b + A*c)*x^4 + 12*(C*a + A*
b)*x^2*log(x) - 12*B*a*x - 6*A*a)/x^2
```

Sympy [A] time = 0.670233, size = 60, normalized size = 0.95

$$Bbx+\frac{Bcx^3}{3}+\frac{Ccx^4}{4}+x^2\left(\frac{Ac}{2}+\frac{Cb}{2}\right)+(Ab+Ca)\log(x)-\frac{Aa+2Bax}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] B*b*x + B*c*x**3/3 + C*c*x**4/4 + x**2*(A*c/2 + C*b/2) + (A*b + C*a)*log(x)
 - (A*a + 2*B*a*x)/(2*x**2)
```

Giac [A] time = 1.08334, size = 78, normalized size = 1.24

$$\frac{1}{4}\,Ccx^4+\frac{1}{3}\,Bcx^3+\frac{1}{2}\,Cbx^2+\frac{1}{2}\,Acx^2+Bbx+(Ca+Ab)\log(|x|)-\frac{2\,Bax+Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*C*c*x^4 + 1/3*B*c*x^3 + 1/2*C*b*x^2 + 1/2*A*c*x^2 + B*b*x + (C*a + A*b)
*log(abs(x)) - 1/2*(2*B*a*x + A*a)/x^2
```

3.7 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx$

Optimal. Leaf size=63

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}-\frac{aB}{2x^2}+x(Ac+bC)+bB\log(x)+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3$$

```
[Out] -(a*A)/(3*x^3) - (a*B)/(2*x^2) - (A*b + a*C)/x + (A*c + b*C)*x + (B*c*x^2)/
2 + (c*C*x^3)/3 + b*B*Log[x]
```

Rubi [A] time = 0.0509742, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1628}

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}-\frac{aB}{2x^2}+x(Ac+bC)+bB\log(x)+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x]
```

```
[Out] -(a*A)/(3*x^3) - (a*B)/(2*x^2) - (A*b + a*C)/x + (A*c + b*C)*x + (B*c*x^2)/
2 + (c*C*x^3)/3 + b*B*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx &= \int \left(Ac\left(1+\frac{bC}{Ac}\right)+\frac{aA}{x^4}+\frac{aB}{x^3}+\frac{Ab+aC}{x^2}+\frac{bB}{x}+Bcx+cCx^2\right)dx\\ &= -\frac{aA}{3x^3}-\frac{aB}{2x^2}-\frac{Ab+aC}{x}+(Ac+bC)x+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3+bB\log(x)\end{aligned}$$

Mathematica [A] time = 0.0466659, size = 60, normalized size = 0.95

$$-\frac{a(2A+3x(B+2Cx))}{6x^3}-\frac{Ab}{x}+Acx+bB\log(x)+bCx+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x]
```

```
[Out] -((A*b)/x) + A*c*x + b*C*x + (B*c*x^2)/2 + (c*C*x^3)/3 - (a*(2*A + 3*x*(B +
 2*C*x)))/(6*x^3) + b*B*Log[x]
```

Maple [A] time = 0.006, size = 57, normalized size = 0.9

$$\frac{cCx^3}{3}+\frac{Bcx^2}{2}+Acx+bCx-\frac{Ab}{x}-\frac{aC}{x}-\frac{Ba}{2x^2}-\frac{Aa}{3x^3}+bB\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x)
```

```
[Out] 1/3*c*C*x^3+1/2*B*c*x^2+A*c*x+b*C*x-1/x*A*b-1/x*a*C-1/2*a*B/x^2-1/3*a*A/x^3
+b*B*ln(x)
```

Maxima [A] time = 0.959429, size = 76, normalized size = 1.21

$$\frac{1}{3}\,Ccx^3+\frac{1}{2}\,Bcx^2+Bb\log\left(x\right)+\left(Cb+Ac\right)x-\frac{3\,Bax+6\,\left(Ca+Ab\right)x^2+2\,Aa}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*C*c*x^3 + 1/2*B*c*x^2 + B*b*log(x) + (C*b + A*c)*x - 1/6*(3*B*a*x + 6*(
C*a + A*b)*x^2 + 2*A*a)/x^3
```

Fricas [A] time = 1.26255, size = 149, normalized size = 2.37

$$\frac{2\,Ccx^6+3\,Bcx^5+6\,Bbx^3\log\left(x\right)+6\,\left(Cb+Ac\right)x^4-3\,Bax-6\,\left(Ca+Ab\right)x^2-2\,Aa}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*C*c*x^6 + 3*B*c*x^5 + 6*B*b*x^3*log(x) + 6*(C*b + A*c)*x^4 - 3*B*a*x
 - 6*(C*a + A*b)*x^2 - 2*A*a)/x^3
```

Sympy [A] time = 1.01312, size = 61, normalized size = 0.97

$$Bb\log\left(x\right)+\frac{Bcx^2}{2}+\frac{Ccx^3}{3}+x\left(Ac+Cb\right)-\frac{2Aa+3Bax+x^2\left(6Ab+6Ca\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**4,x)
```

```
[Out] B*b*log(x) + B*c*x**2/2 + C*c*x**3/3 + x*(A*c + C*b) - (2*A*a + 3*B*a*x + x
**2*(6*A*b + 6*C*a))/(6*x**3)
```

Giac [A] time = 1.10642, size = 76, normalized size = 1.21

$$\frac{1}{3}\,Ccx^3+\frac{1}{2}\,Bcx^2+Cbx+Acx+Bb\log\left(|x|\right)-\frac{3\,Bax+6\,\left(Ca+Ab\right)x^2+2\,Aa}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*C*c*x^3 + 1/2*B*c*x^2 + C*b*x + A*c*x + B*b*log(abs(x)) - 1/6*(3*B*a*x
+ 6*(C*a + A*b)*x^2 + 2*A*a)/x^3
```

3.8 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx$

Optimal. Leaf size=63

$$-\frac{aC+Ab}{2x^2}-\frac{aA}{4x^4}-\frac{aB}{3x^3}+\log(x)(Ac+bC)-\frac{bB}{x}+Bcx+\frac{1}{2}cCx^2$$

```
[Out] -(a*A)/(4*x^4) - (a*B)/(3*x^3) - (A*b + a*C)/(2*x^2) - (b*B)/x + B*c*x + (c
*C*x^2)/2 + (A*c + b*C)*Log[x]
```

Rubi [A] time = 0.0510479, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1628}

$$-\frac{aC+Ab}{2x^2}-\frac{aA}{4x^4}-\frac{aB}{3x^3}+\log(x)(Ac+bC)-\frac{bB}{x}+Bcx+\frac{1}{2}cCx^2$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x]
```

```
[Out] -(a*A)/(4*x^4) - (a*B)/(3*x^3) - (A*b + a*C)/(2*x^2) - (b*B)/x + B*c*x + (c
*C*x^2)/2 + (A*c + b*C)*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx &= \int \left(Bc+\frac{aA}{x^5}+\frac{aB}{x^4}+\frac{Ab+aC}{x^3}+\frac{bB}{x^2}+\frac{Ac+bC}{x}+cCx\right)dx\\ &= -\frac{aA}{4x^4}-\frac{aB}{3x^3}-\frac{Ab+aC}{2x^2}-\frac{bB}{x}+Bcx+\frac{1}{2}cCx^2+(Ac+bC)\log(x)\end{aligned}$$

Mathematica [A] time = 0.0296124, size = 62, normalized size = 0.98

$$-\frac{a(3A+4Bx+6Cx^2)}{12x^4}+\frac{-Ab-2bBx+cx^3(2B+Cx)}{2x^2}+\log(x)(Ac+bC)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x]
```

```
[Out] -(a*(3*A + 4*B*x + 6*C*x^2))/(12*x^4) + (-(A*b) - 2*b*B*x + c*x^3*(2*B + C*
x))/(2*x^2) + (A*c + b*C)*Log[x]
```

Maple [A] time = 0.007, size = 58, normalized size = 0.9

$$\frac{cCx^2}{2} + Bcx - \frac{Bb}{x} - \frac{Aa}{4\,x^4} - \frac{Ab}{2\,x^2} - \frac{aC}{2\,x^2} - \frac{Ba}{3\,x^3} + A\ln(x)\,c + C\ln(x)\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x)
```

```
[Out] 1/2*c*C*x^2+B*c*x-b*B/x-1/4*a*A/x^4-1/2/x^2*A*b-1/2/x^2*a*C-1/3*a*B/x^3+A*l
n(x)*c+C*ln(x)*b
```

Maxima [A] time = 0.969201, size = 76, normalized size = 1.21

$$\frac{1}{2}\,Ccx^2 + Bcx + (Cb + Ac)\log(x) - \frac{12\,Bbx^3 + 4\,Bax + 6\,(Ca + Ab)x^2 + 3\,Aa}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*C*c*x^2 + B*c*x + (C*b + A*c)*log(x) - 1/12*(12*B*b*x^3 + 4*B*a*x + 6*(
C*a + A*b)*x^2 + 3*A*a)/x^4
```

Fricas [A] time = 1.29137, size = 154, normalized size = 2.44

$$\frac{6\,Ccx^6 + 12\,Bcx^5 + 12\,(Cb + Ac)x^4\log(x) - 12\,Bbx^3 - 4\,Bax - 6\,(Ca + Ab)x^2 - 3\,Aa}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(6*C*c*x^6 + 12*B*c*x^5 + 12*(C*b + A*c)*x^4*log(x) - 12*B*b*x^3 - 4*B
*a*x - 6*(C*a + A*b)*x^2 - 3*A*a)/x^4
```

Sympy [A] time = 2.98161, size = 61, normalized size = 0.97

$$Bcx + \frac{Ccx^2}{2} + (Ac + Cb)\log(x) - \frac{3Aa + 4Bax + 12Bbx^3 + x^2\,(6Ab + 6Ca)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**5,x)
```

```
[Out] B*c*x + C*c*x**2/2 + (A*c + C*b)*log(x) - (3*A*a + 4*B*a*x + 12*B*b*x**3 +
x**2*(6*A*b + 6*C*a))/(12*x**4)
```

Giac [A] time = 1.09652, size = 77, normalized size = 1.22

$$\frac{1}{2}\,Ccx^2 + Bcx + (Cb + Ac)\log(|x|) - \frac{12\,Bbx^3 + 4\,Bax + 6\,(Ca + Ab)x^2 + 3\,Aa}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*C*c*x^2 + B*c*x + (C*b + A*c)*log(abs(x)) - 1/12*(12*B*b*x^3 + 4*B*a*x
+ 6*(C*a + A*b)*x^2 + 3*A*a)/x^4
```

3.9 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6}\,dx$

Optimal. Leaf size=63

$$-\frac{aC+Ab}{3x^3}-\frac{aA}{5x^5}-\frac{aB}{4x^4}-\frac{Ac+bC}{x}-\frac{bB}{2x^2}+Bc\log(x)+cCx$$

```
[Out] -(a*A)/(5*x^5) - (a*B)/(4*x^4) - (A*b + a*C)/(3*x^3) - (b*B)/(2*x^2) - (A*c
 + b*C)/x + c*C*x + B*c*Log[x]
```

Rubi [A] time = 0.0516926, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1628}

$$-\frac{aC+Ab}{3x^3}-\frac{aA}{5x^5}-\frac{aB}{4x^4}-\frac{Ac+bC}{x}-\frac{bB}{2x^2}+Bc\log(x)+cCx$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x]
```

```
[Out] -(a*A)/(5*x^5) - (a*B)/(4*x^4) - (A*b + a*C)/(3*x^3) - (b*B)/(2*x^2) - (A*c
 + b*C)/x + c*C*x + B*c*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6}\,dx &= \int \left(cC+\frac{aA}{x^6}+\frac{aB}{x^5}+\frac{Ab+aC}{x^4}+\frac{bB}{x^3}+\frac{Ac+bC}{x^2}+\frac{Bc}{x}\right)dx\\ &= -\frac{aA}{5x^5}-\frac{aB}{4x^4}-\frac{Ab+aC}{3x^3}-\frac{bB}{2x^2}-\frac{Ac+bC}{x}+cCx+Bc\log(x)\end{aligned}$$

Mathematica [A] time = 0.0584929, size = 63, normalized size = 1.

$$Bc\log(x)-\frac{12aA+5ax(3B+4Cx)+20Ax^2(b+3cx^2)+30bx^3(B+2Cx)-60cCx^6}{60x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x]
```

```
[Out] -(12*a*A - 60*c*C*x^6 + 30*b*x^3*(B + 2*C*x) + 5*a*x*(3*B + 4*C*x) + 20*A*x
^2*(b + 3*c*x^2))/(60*x^5) + B*c*Log[x]
```

Maple [A] time = 0.007, size = 60, normalized size = 1.

$$cCx - \frac{Ac}{x} - \frac{bC}{x} - \frac{Bb}{2\,x^2} - \frac{Aa}{5\,x^5} - \frac{Ba}{4\,x^4} - \frac{Ab}{3\,x^3} - \frac{aC}{3\,x^3} + Bc\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x)
```

```
[Out] c*C*x-1/x*A*c-1/x*b*C-1/2*b*B/x^2-1/5*a*A/x^5-1/4*a*B/x^4-1/3/x^3*A*b-1/3/x
^3*a*C+B*c*ln(x)
```

Maxima [A] time = 0.952459, size = 76, normalized size = 1.21

$$Ccx + Bc\log(x) - \frac{30\,Bbx^3 + 60\,(Cb + Ac)x^4 + 15\,Bax + 20\,(Ca + Ab)x^2 + 12\,Aa}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="maxima")
```

```
[Out] C*c*x + B*c*log(x) - 1/60*(30*B*b*x^3 + 60*(C*b + A*c)*x^4 + 15*B*a*x + 20*
(C*a + A*b)*x^2 + 12*A*a)/x^5
```

Fricas [A] time = 1.20798, size = 159, normalized size = 2.52

$$\frac{60\,Ccx^6 + 60\,Bcx^5\log(x) - 30\,Bbx^3 - 60\,(Cb + Ac)x^4 - 15\,Bax - 20\,(Ca + Ab)x^2 - 12\,Aa}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(60*C*c*x^6 + 60*B*c*x^5*log(x) - 30*B*b*x^3 - 60*(C*b + A*c)*x^4 - 15
*B*a*x - 20*(C*a + A*b)*x^2 - 12*A*a)/x^5
```

Sympy [A] time = 8.34503, size = 63, normalized size = 1.

$$Bc\log(x) + Ccx - \frac{12Aa + 15Bax + 30Bbx^3 + x^4\,(60Ac + 60Cb) + x^2\,(20Ab + 20Ca)}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**6,x)
```

```
[Out] B*c*log(x) + C*c*x - (12*A*a + 15*B*a*x + 30*B*b*x**3 + x**4*(60*A*c + 60*C
*b) + x**2*(20*A*b + 20*C*a))/(60*x**5)
```

Giac [A] time = 1.09146, size = 77, normalized size = 1.22

$$Ccx + Bc\log(|x|) - \frac{30\,Bbx^3 + 60\,(Cb + Ac)x^4 + 15\,Bax + 20\,(Ca + Ab)x^2 + 12\,Aa}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="giac")
```

```
[Out] C*c*x + B*c*log(abs(x)) - 1/60*(30*B*b*x^3 + 60*(C*b + A*c)*x^4 + 15*B*a*x
+ 20*(C*a + A*b)*x^2 + 12*A*a)/x^5
```

3.10 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx$

Optimal. Leaf size=68

$$-\frac{aC+Ab}{4x^4}-\frac{aA}{6x^6}-\frac{aB}{5x^5}-\frac{Ac+bC}{2x^2}-\frac{bB}{3x^3}-\frac{Bc}{x}+cC\log(x)$$

```
[Out] -(a*A)/(6*x^6) - (a*B)/(5*x^5) - (A*b + a*C)/(4*x^4) - (b*B)/(3*x^3) - (A*c
 + b*C)/(2*x^2) - (B*c)/x + c*C*Log[x]
```

Rubi [A] time = 0.0482934, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1628}

$$-\frac{aC+Ab}{4x^4}-\frac{aA}{6x^6}-\frac{aB}{5x^5}-\frac{Ac+bC}{2x^2}-\frac{bB}{3x^3}-\frac{Bc}{x}+cC\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x]
```

```
[Out] -(a*A)/(6*x^6) - (a*B)/(5*x^5) - (A*b + a*C)/(4*x^4) - (b*B)/(3*x^3) - (A*c
 + b*C)/(2*x^2) - (B*c)/x + c*C*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx &= \int \left(\frac{aA}{x^7}+\frac{aB}{x^6}+\frac{Ab+aC}{x^5}+\frac{bB}{x^4}+\frac{Ac+bC}{x^3}+\frac{Bc}{x^2}+\frac{cC}{x}\right)dx\\ &= -\frac{aA}{6x^6}-\frac{aB}{5x^5}-\frac{Ab+aC}{4x^4}-\frac{bB}{3x^3}-\frac{Ac+bC}{2x^2}-\frac{Bc}{x}+cC\log(x)\end{aligned}$$

Mathematica [A] time = 0.0475365, size = 68, normalized size = 1.

$$cC\log(x)-\frac{a(10A+3x(4B+5Cx))+5x^2\left(3A\left(b+2cx^2\right)+2x\left(2bB+3bCx+6Bcx^2\right)\right)}{60x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x]
```

```
[Out] -(a*(10*A + 3*x*(4*B + 5*C*x)) + 5*x^2*(3*A*(b + 2*c*x^2) + 2*x*(2*b*B + 3*
b*C*x + 6*B*c*x^2)))/(60*x^6) + c*C*Log[x]
```

Maple [A] time = 0.006, size = 63, normalized size = 0.9

$$-\frac{Bc}{x}-\frac{Ac}{2\,x^{2}}-\frac{bC}{2\,x^{2}}-\frac{Ba}{5\,x^{5}}-\frac{Ab}{4\,x^{4}}-\frac{aC}{4\,x^{4}}-\frac{Aa}{6\,x^{6}}-\frac{Bb}{3\,x^{3}}+cC\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x)
```

```
[Out] -B*c/x-1/2/x^2*A*c-1/2/x^2*b*C-1/5*a*B/x^5-1/4/x^4*A*b-1/4/x^4*a*C-1/6*a*A/
x^6-1/3*b*B/x^3+c*C*ln(x)
```

Maxima [A] time = 0.967971, size = 80, normalized size = 1.18

$$Cc\log\left(x\right)-\frac{60\,Bcx^{5}+20\,Bbx^{3}+30\,(Cb+Ac)x^{4}+12\,Bax+15\,(Ca+Ab)x^{2}+10\,Aa}{60\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="maxima")
```

```
[Out] C*c*log(x) - 1/60*(60*B*c*x^5 + 20*B*b*x^3 + 30*(C*b + A*c)*x^4 + 12*B*a*x
+ 15*(C*a + A*b)*x^2 + 10*A*a)/x^6
```

Fricas [A] time = 1.27163, size = 159, normalized size = 2.34

$$\frac{60\,Ccx^{6}\log\left(x\right)-60\,Bcx^{5}-20\,Bbx^{3}-30\,(Cb+Ac)x^{4}-12\,Bax-15\,(Ca+Ab)x^{2}-10\,Aa}{60\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*C*c*x^6*log(x) - 60*B*c*x^5 - 20*B*b*x^3 - 30*(C*b + A*c)*x^4 - 12
*B*a*x - 15*(C*a + A*b)*x^2 - 10*A*a)/x^6
```

Sympy [A] time = 16.138, size = 66, normalized size = 0.97

$$Cc\log\left(x\right)-\frac{10Aa+12Bax+20Bbx^{3}+60Bcx^{5}+x^{4}\left(30Ac+30Cb\right)+x^{2}\left(15Ab+15Ca\right)}{60x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**7,x)
```

```
[Out] C*c*log(x) - (10*A*a + 12*B*a*x + 20*B*b*x**3 + 60*B*c*x**5 + x**4*(30*A*c
+ 30*C*b) + x**2*(15*A*b + 15*C*a))/(60*x**6)
```

Giac [A] time = 1.09615, size = 81, normalized size = 1.19

$$Cc\log\left(|x|\right)-\frac{60\,Bcx^{5}+20\,Bbx^{3}+30\,(Cb+Ac)x^{4}+12\,Bax+15\,(Ca+Ab)x^{2}+10\,Aa}{60\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="giac")
```

```
[Out] C*c*log(abs(x)) - 1/60*(60*B*c*x^5 + 20*B*b*x^3 + 30*(C*b + A*c)*x^4 + 12*B
*a*x + 15*(C*a + A*b)*x^2 + 10*A*a)/x^6
```

3.11 $\int x^2 \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=159

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{9}x^9\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{7}x^7\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}Bx^8\left(2ac+b^2\right)$$

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*b*B*x^6)/3 + (
(A*(b^2 + 2*a*c) + 2*a*b*C)*x^7)/7 + (B*(b^2 + 2*a*c)*x^8)/8 + ((2*A*b*c +
(b^2 + 2*a*c)*C)*x^9)/9 + (b*B*c*x^10)/5 + (c*(A*c + 2*b*C)*x^11)/11 + (B*c
^2*x^12)/12 + (c^2*C*x^13)/13
```

Rubi [A] time = 0.21447, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1628}

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{9}x^9\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{7}x^7\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}Bx^8\left(2ac+b^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*b*B*x^6)/3 + (
(A*(b^2 + 2*a*c) + 2*a*b*C)*x^7)/7 + (B*(b^2 + 2*a*c)*x^8)/8 + ((2*A*b*c +
(b^2 + 2*a*c)*C)*x^9)/9 + (b*B*c*x^10)/5 + (c*(A*c + 2*b*C)*x^11)/11 + (B*c
^2*x^12)/12 + (c^2*C*x^13)/13
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx &= \int \left(a^2Ax^2+a^2Bx^3+a(2Ab+aC)x^4+2abBx^5+\left(A\left(b^2+2ac\right)+2abC\right)\right.\\ &= \frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{5}a(2Ab+aC)x^5+\frac{1}{3}abBx^6+\frac{1}{7}\left(A\left(b^2+2ac\right)+2a\right.\end{aligned}$$

Mathematica [A] time = 0.0472205, size = 159, normalized size = 1.

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{9}x^9\left(2acC+2Abc+b^2C\right)+\frac{1}{7}x^7\left(2aAc+2abC+Ab^2\right)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}Bx^8\left(2ac+b^2\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*b*B*x^6)/3 + (
(A*b^2 + 2*a*A*c + 2*a*b*C)*x^7)/7 + (B*(b^2 + 2*a*c)*x^8)/8 + ((2*A*b*c +
b^2*C + 2*a*c*C)*x^9)/9 + (b*B*c*x^10)/5 + (c*(A*c + 2*b*C)*x^11)/11 + (B*c
^2*x^12)/12 + (c^2*C*x^13)/13
```

Maple [A] time = 0.002, size = 142, normalized size = 0.9

$$\frac{c^2Cx^{13}}{13}+\frac{Bc^2x^{12}}{12}+\frac{\left(Ac^2+2\,Cbc\right)x^{11}}{11}+\frac{bBcx^{10}}{5}+\frac{\left(2\,Abc+\left(2\,ac+b^2\right)C\right)x^9}{9}+\frac{B\left(2\,ac+b^2\right)x^8}{8}+\frac{\left(A\left(2\,ac+b^2\right)+2\,ab\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/13*c^2*C*x^13+1/12*B*c^2*x^12+1/11*(A*c^2+2*C*b*c)*x^11+1/5*b*B*c*x^10+1/
9*(2*A*b*c+(2*a*c+b^2)*C)*x^9+1/8*B*(2*a*c+b^2)*x^8+1/7*(A*(2*a*c+b^2)+2*a*
b*C)*x^7+1/3*a*b*B*x^6+1/5*(2*A*a*b+C*a^2)*x^5+1/4*a^2*B*x^4+1/3*a^2*A*x^3
```

Maxima [A] time = 0.942833, size = 193, normalized size = 1.21

$$\frac{1}{13}\,Cc^2x^{13}+\frac{1}{12}\,Bc^2x^{12}+\frac{1}{5}\,Bbcx^{10}+\frac{1}{11}\left(2\,Cbc+Ac^2\right)x^{11}+\frac{1}{9}\left(Cb^2+2\,(Ca+Ab)c\right)x^9+\frac{1}{3}\,Babx^6+\frac{1}{8}\left(Bb^2+2\,Bac\right)x^8+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 1/5*B*b*c*x^10 + 1/11*(2*C*b*c + A*c^2)
*x^11 + 1/9*(C*b^2 + 2*(C*a + A*b)*c)*x^9 + 1/3*B*a*b*x^6 + 1/8*(B*b^2 + 2*
B*a*c)*x^8 + 1/7*(2*C*a*b + A*b^2 + 2*A*a*c)*x^7 + 1/4*B*a^2*x^4 + 1/3*A*a^
2*x^3 + 1/5*(C*a^2 + 2*A*a*b)*x^5
```

Fricas [A] time = 1.11294, size = 400, normalized size = 2.52

$$\frac{1}{13}x^{13}c^2C+\frac{1}{12}x^{12}c^2B+\frac{2}{11}x^{11}cbC+\frac{1}{11}x^{11}c^2A+\frac{1}{5}x^{10}cbB+\frac{1}{9}x^9b^2C+\frac{2}{9}x^9caC+\frac{2}{9}x^9cbA+\frac{1}{8}x^8b^2B+\frac{1}{4}x^8caB+\frac{2}{7}x^7ba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*c^2*C + 1/12*x^12*c^2*B + 2/11*x^11*c*b*C + 1/11*x^11*c^2*A + 1/5
*x^10*c*b*B + 1/9*x^9*b^2*C + 2/9*x^9*c*a*C + 2/9*x^9*c*b*A + 1/8*x^8*b^2*B
 + 1/4*x^8*c*a*B + 2/7*x^7*b*a*C + 1/7*x^7*b^2*A + 2/7*x^7*c*a*A + 1/3*x^6*
b*a*B + 1/5*x^5*a^2*C + 2/5*x^5*b*a*A + 1/4*x^4*a^2*B + 1/3*x^3*a^2*A
```

Sympy [A] time = 0.09443, size = 168, normalized size = 1.06

$$\frac{Aa^2x^3}{3}+\frac{Ba^2x^4}{4}+\frac{Babx^6}{3}+\frac{Bbcx^{10}}{5}+\frac{Bc^2x^{12}}{12}+\frac{Cc^2x^{13}}{13}+x^{11}\left(\frac{Ac^2}{11}+\frac{2Cbc}{11}\right)+x^9\left(\frac{2Abc}{9}+\frac{2Cac}{9}+\frac{Cb^2}{9}\right)+x^8\left(\frac{Bac}{4}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*a**2*x**3/3 + B*a**2*x**4/4 + B*a*b*x**6/3 + B*b*c*x**10/5 + B*c**2*x**12
/12 + C*c**2*x**13/13 + x**11*(A*c**2/11 + 2*C*b*c/11) + x**9*(2*A*b*c/9 +
2*C*a*c/9 + C*b**2/9) + x**8*(B*a*c/4 + B*b**2/8) + x**7*(2*A*a*c/7 + A*b**
2/7 + 2*C*a*b/7) + x**5*(2*A*a*b/5 + C*a**2/5)
```

Giac [A] time = 1.11528, size = 208, normalized size = 1.31

$$\frac{1}{13}Cc^2x^{13} + \frac{1}{12}Bc^2x^{12} + \frac{2}{11}Cbcx^{11} + \frac{1}{11}Ac^2x^{11} + \frac{1}{5}Bbcx^{10} + \frac{1}{9}Cb^2x^9 + \frac{2}{9}Cacx^9 + \frac{2}{9}Abcx^9 + \frac{1}{8}Bb^2x^8 + \frac{1}{4}Bacx^8 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 2/11*C*b*c*x^11 + 1/11*A*c^2*x^11 + 1/5
*B*b*c*x^10 + 1/9*C*b^2*x^9 + 2/9*C*a*c*x^9 + 2/9*A*b*c*x^9 + 1/8*B*b^2*x^8
 + 1/4*B*a*c*x^8 + 2/7*C*a*b*x^7 + 1/7*A*b^2*x^7 + 2/7*A*a*c*x^7 + 1/3*B*a*
b*x^6 + 1/5*C*a^2*x^5 + 2/5*A*a*b*x^5 + 1/4*B*a^2*x^4 + 1/3*A*a^2*x^3
```

3.12 $\int x\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=159

$$\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{8}x^8\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{6}x^6\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{4}ax^4(aC+2Ab)+\frac{1}{7}Bx^7\left(2ac+b^2\right)+\frac{2}{5}$$

```
[Out] (a^2*A*x^2)/2 + (a^2*B*x^3)/3 + (a*(2*A*b + a*C)*x^4)/4 + (2*a*b*B*x^5)/5 +
 ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^6)/6 + (B*(b^2 + 2*a*c)*x^7)/7 + ((2*A*b*c
+ (b^2 + 2*a*c)*C)*x^8)/8 + (2*b*B*c*x^9)/9 + (c*(A*c + 2*b*C)*x^10)/10 + (
B*c^2*x^11)/11 + (c^2*C*x^12)/12
```

Rubi [A] time = 0.143461, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1628}

$$\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{8}x^8\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{6}x^6\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{4}ax^4(aC+2Ab)+\frac{1}{7}Bx^7\left(2ac+b^2\right)+\frac{2}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^2)/2 + (a^2*B*x^3)/3 + (a*(2*A*b + a*C)*x^4)/4 + (2*a*b*B*x^5)/5 +
 ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^6)/6 + (B*(b^2 + 2*a*c)*x^7)/7 + ((2*A*b*c
+ (b^2 + 2*a*c)*C)*x^8)/8 + (2*b*B*c*x^9)/9 + (c*(A*c + 2*b*C)*x^10)/10 + (
B*c^2*x^11)/11 + (c^2*C*x^12)/12
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx &= \int\left(a^2Ax+a^2Bx^2+a(2Ab+aC)x^3+2abBx^4+\left(A\left(b^2+2ac\right)+2abC\right)x^5+\right.\\ &= \frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{4}a(2Ab+aC)x^4+\frac{2}{5}abBx^5+\frac{1}{6}\left(A\left(b^2+2ac\right)+2abC\right)\end{aligned}$$

Mathematica [A] time = 0.035737, size = 159, normalized size = 1.

$$\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{8}x^8\left(2acC+2Abc+b^2C\right)+\frac{1}{6}x^6\left(2aAc+2abC+Ab^2\right)+\frac{1}{4}ax^4(aC+2Ab)+\frac{1}{7}Bx^7\left(2ac+b^2\right)+\frac{2}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^2)/2 + (a^2*B*x^3)/3 + (a*(2*A*b + a*C)*x^4)/4 + (2*a*b*B*x^5)/5 +
 ((A*b^2 + 2*a*A*c + 2*a*b*C)*x^6)/6 + (B*(b^2 + 2*a*c)*x^7)/7 + ((2*A*b*c
+ b^2*C + 2*a*c*C)*x^8)/8 + (2*b*B*c*x^9)/9 + (c*(A*c + 2*b*C)*x^10)/10 + (
B*c^2*x^11)/11 + (c^2*C*x^12)/12
```

Maple [A] time = 0.001, size = 142, normalized size = 0.9

$$\frac{c^2Cx^{12}}{12}+\frac{Bc^2x^{11}}{11}+\frac{\left(Ac^2+2\,Cbc\right)x^{10}}{10}+\frac{2\,bBcx^9}{9}+\frac{\left(2\,Abc+\left(2\,ac+b^2\right)C\right)x^8}{8}+\frac{B\left(2\,ac+b^2\right)x^7}{7}+\frac{\left(A\left(2\,ac+b^2\right)+\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/12*c^2*C*x^12+1/11*B*c^2*x^11+1/10*(A*c^2+2*C*b*c)*x^10+2/9*b*B*c*x^9+1/8
*(2*A*b*c+(2*a*c+b^2)*C)*x^8+1/7*B*(2*a*c+b^2)*x^7+1/6*(A*(2*a*c+b^2)+2*a*b
*C)*x^6+2/5*a*b*B*x^5+1/4*(2*A*a*b+C*a^2)*x^4+1/3*a^2*B*x^3+1/2*a^2*A*x^2
```

Maxima [A] time = 0.949302, size = 193, normalized size = 1.21

$$\frac{1}{12}\,Cc^2x^{12}+\frac{1}{11}\,Bc^2x^{11}+\frac{2}{9}\,Bbcx^9+\frac{1}{10}\left(2\,Cbc+Ac^2\right)x^{10}+\frac{1}{8}\left(Cb^2+2\,(Ca+Ab)c\right)x^8+\frac{2}{5}\,Babx^5+\frac{1}{7}\left(Bb^2+2\,Bac\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*C*c^2*x^12 + 1/11*B*c^2*x^11 + 2/9*B*b*c*x^9 + 1/10*(2*C*b*c + A*c^2)*
x^10 + 1/8*(C*b^2 + 2*(C*a + A*b)*c)*x^8 + 2/5*B*a*b*x^5 + 1/7*(B*b^2 + 2*B
*a*c)*x^7 + 1/6*(2*C*a*b + A*b^2 + 2*A*a*c)*x^6 + 1/3*B*a^2*x^3 + 1/2*A*a^2
*x^2 + 1/4*(C*a^2 + 2*A*a*b)*x^4
```

Fricas [A] time = 1.12957, size = 397, normalized size = 2.5

$$\frac{1}{12}x^{12}c^2C+\frac{1}{11}x^{11}c^2B+\frac{1}{5}x^{10}cbC+\frac{1}{10}x^{10}c^2A+\frac{2}{9}x^9cbB+\frac{1}{8}x^8b^2C+\frac{1}{4}x^8caC+\frac{1}{4}x^8cbA+\frac{1}{7}x^7b^2B+\frac{2}{7}x^7caB+\frac{1}{3}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*c^2*C + 1/11*x^11*c^2*B + 1/5*x^10*c*b*C + 1/10*x^10*c^2*A + 2/9*
x^9*c*b*B + 1/8*x^8*b^2*C + 1/4*x^8*c*a*C + 1/4*x^8*c*b*A + 1/7*x^7*b^2*B +
 2/7*x^7*c*a*B + 1/3*x^6*b*a*C + 1/6*x^6*b^2*A + 1/3*x^6*c*a*A + 2/5*x^5*b*
a*B + 1/4*x^4*a^2*C + 1/2*x^4*b*a*A + 1/3*x^3*a^2*B + 1/2*x^2*a^2*A
```

Sympy [A] time = 0.095328, size = 163, normalized size = 1.03

$$\frac{Aa^2x^2}{2}+\frac{Ba^2x^3}{3}+\frac{2Babx^5}{5}+\frac{2Bbcx^9}{9}+\frac{Bc^2x^{11}}{11}+\frac{Cc^2x^{12}}{12}+x^{10}\left(\frac{Ac^2}{10}+\frac{Cbc}{5}\right)+x^8\left(\frac{Abc}{4}+\frac{Cac}{4}+\frac{Cb^2}{8}\right)+x^7\left(\frac{2Bac}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*a**2*x**2/2 + B*a**2*x**3/3 + 2*B*a*b*x**5/5 + 2*B*b*c*x**9/9 + B*c**2*x*
*11/11 + C*c**2*x**12/12 + x**10*(A*c**2/10 + C*b*c/5) + x**8*(A*b*c/4 + C*
a*c/4 + C*b**2/8) + x**7*(2*B*a*c/7 + B*b**2/7) + x**6*(A*a*c/3 + A*b**2/6
+ C*a*b/3) + x**4*(A*a*b/2 + C*a**2/4)
```

Giac [A] time = 1.10008, size = 208, normalized size = 1.31

$$\frac{1}{12}Cc^2x^{12} + \frac{1}{11}Bc^2x^{11} + \frac{1}{5}Cbcx^{10} + \frac{1}{10}Ac^2x^{10} + \frac{2}{9}Bbcx^9 + \frac{1}{8}Cb^2x^8 + \frac{1}{4}Cacx^8 + \frac{1}{4}Abcx^8 + \frac{1}{7}Bb^2x^7 + \frac{2}{7}Bacx^7 + \frac{1}{3}C$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*C*c^2*x^12 + 1/11*B*c^2*x^11 + 1/5*C*b*c*x^10 + 1/10*A*c^2*x^10 + 2/9*
B*b*c*x^9 + 1/8*C*b^2*x^8 + 1/4*C*a*c*x^8 + 1/4*A*b*c*x^8 + 1/7*B*b^2*x^7 +
 2/7*B*a*c*x^7 + 1/3*C*a*b*x^6 + 1/6*A*b^2*x^6 + 1/3*A*a*c*x^6 + 2/5*B*a*b*
x^5 + 1/4*C*a^2*x^4 + 1/2*A*a*b*x^4 + 1/3*B*a^2*x^3 + 1/2*A*a^2*x^2
```

3.13 $\int \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=154

$$a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{7}x^7\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{5}x^5\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{3}ax^3(aC+2Ab)+\frac{1}{6}Bx^6\left(2ac+b^2\right)+$$

```
[Out] a^2*A*x + (a^2*B*x^2)/2 + (a*(2*A*b + a*C)*x^3)/3 + (a*b*B*x^4)/2 + ((A*(b^
2 + 2*a*c) + 2*a*b*C)*x^5)/5 + (B*(b^2 + 2*a*c)*x^6)/6 + ((2*A*b*c + (b^2 +
 2*a*c)*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c + 2*b*C)*x^9)/9 + (B*c^2*x^10)/
10 + (c^2*C*x^11)/11
```

Rubi [A] time = 0.110979, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1657}

$$a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{7}x^7\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{5}x^5\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{3}ax^3(aC+2Ab)+\frac{1}{6}Bx^6\left(2ac+b^2\right)+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*A*x + (a^2*B*x^2)/2 + (a*(2*A*b + a*C)*x^3)/3 + (a*b*B*x^4)/2 + ((A*(b^
2 + 2*a*c) + 2*a*b*C)*x^5)/5 + (B*(b^2 + 2*a*c)*x^6)/6 + ((2*A*b*c + (b^2 +
 2*a*c)*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c + 2*b*C)*x^9)/9 + (B*c^2*x^10)/
10 + (c^2*C*x^11)/11
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx=\int \left(a^2A+a^2Bx+a(2Ab+aC)x^2+2abBx^3+\left(A\left(b^2+2ac\right)+2abC\right)x^4+\right.$$
$$=a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{3}a(2Ab+aC)x^3+\frac{1}{2}abBx^4+\frac{1}{5}\left(A\left(b^2+2ac\right)+2abC\right)x$$

Mathematica [A] time = 0.0305159, size = 154, normalized size = 1.

$$a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{7}x^7\left(2acC+2Abc+b^2C\right)+\frac{1}{5}x^5\left(2aAc+2abC+Ab^2\right)+\frac{1}{3}ax^3(aC+2Ab)+\frac{1}{6}Bx^6\left(2ac+b^2\right)+\frac{1}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*A*x + (a^2*B*x^2)/2 + (a*(2*A*b + a*C)*x^3)/3 + (a*b*B*x^4)/2 + ((A*b^2
 + 2*a*A*c + 2*a*b*C)*x^5)/5 + (B*(b^2 + 2*a*c)*x^6)/6 + ((2*A*b*c + b^2*C
+ 2*a*c*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c + 2*b*C)*x^9)/9 + (B*c^2*x^10)/
10 + (c^2*C*x^11)/11
```

Maple [A] time = 0.001, size = 139, normalized size = 0.9

$$\frac{c^2Cx^{11}}{11}+\frac{Bc^2x^{10}}{10}+\frac{\left(Ac^2+2\,Cbc\right)x^9}{9}+\frac{bBcx^8}{4}+\frac{\left(2\,Abc+\left(2\,ac+b^2\right)C\right)x^7}{7}+\frac{B\left(2\,ac+b^2\right)x^6}{6}+\frac{\left(A\left(2\,ac+b^2\right)+2\,abC\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/11*c^2*C*x^11+1/10*B*c^2*x^10+1/9*(A*c^2+2*C*b*c)*x^9+1/4*b*B*c*x^8+1/7*(
2*A*b*c+(2*a*c+b^2)*C)*x^7+1/6*B*(2*a*c+b^2)*x^6+1/5*(A*(2*a*c+b^2)+2*a*b*C
)*x^5+1/2*a*b*B*x^4+1/3*(2*A*a*b+C*a^2)*x^3+1/2*a^2*B*x^2+a^2*A*x
```

Maxima [A] time = 0.972651, size = 189, normalized size = 1.23

$$\frac{1}{11}\,Cc^2x^{11}+\frac{1}{10}\,Bc^2x^{10}+\frac{1}{4}\,Bbcx^8+\frac{1}{9}\left(2\,Cbc+Ac^2\right)x^9+\frac{1}{7}\left(Cb^2+2\,(Ca+Ab)c\right)x^7+\frac{1}{2}\,Babx^4+\frac{1}{6}\left(Bb^2+2\,Bac\right)x^6+\frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*C*c^2*x^11 + 1/10*B*c^2*x^10 + 1/4*B*b*c*x^8 + 1/9*(2*C*b*c + A*c^2)*x
^9 + 1/7*(C*b^2 + 2*(C*a + A*b)*c)*x^7 + 1/2*B*a*b*x^4 + 1/6*(B*b^2 + 2*B*a
*c)*x^6 + 1/5*(2*C*a*b + A*b^2 + 2*A*a*c)*x^5 + 1/2*B*a^2*x^2 + A*a^2*x + 1
/3*(C*a^2 + 2*A*a*b)*x^3
```

Fricas [A] time = 1.10337, size = 385, normalized size = 2.5

$$\frac{1}{11}x^{11}c^2C+\frac{1}{10}x^{10}c^2B+\frac{2}{9}x^9cbC+\frac{1}{9}x^9c^2A+\frac{1}{4}x^8cbB+\frac{1}{7}x^7b^2C+\frac{2}{7}x^7caC+\frac{2}{7}x^7cbA+\frac{1}{6}x^6b^2B+\frac{1}{3}x^6caB+\frac{2}{5}x^5baC+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*c^2*C + 1/10*x^10*c^2*B + 2/9*x^9*c*b*C + 1/9*x^9*c^2*A + 1/4*x^8
*c*b*B + 1/7*x^7*b^2*C + 2/7*x^7*c*a*C + 2/7*x^7*c*b*A + 1/6*x^6*b^2*B + 1/
3*x^6*c*a*B + 2/5*x^5*b*a*C + 1/5*x^5*b^2*A + 2/5*x^5*c*a*A + 1/2*x^4*b*a*B
 + 1/3*x^3*a^2*C + 2/3*x^3*b*a*A + 1/2*x^2*a^2*B + x*a^2*A
```

Sympy [A] time = 0.095202, size = 165, normalized size = 1.07

$$Aa^2x+\frac{Ba^2x^2}{2}+\frac{Babx^4}{2}+\frac{Bbcx^8}{4}+\frac{Bc^2x^{10}}{10}+\frac{Cc^2x^{11}}{11}+x^9\left(\frac{Ac^2}{9}+\frac{2Cbc}{9}\right)+x^7\left(\frac{2Abc}{7}+\frac{2Cac}{7}+\frac{Cb^2}{7}\right)+x^6\left(\frac{Bac}{3}+\frac{Bb^2}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*a**2*x + B*a**2*x**2/2 + B*a*b*x**4/2 + B*b*c*x**8/4 + B*c**2*x**10/10 +
C*c**2*x**11/11 + x**9*(A*c**2/9 + 2*C*b*c/9) + x**7*(2*A*b*c/7 + 2*C*a*c/7
 + C*b**2/7) + x**6*(B*a*c/3 + B*b**2/6) + x**5*(2*A*a*c/5 + A*b**2/5 + 2*C
*a*b/5) + x**3*(2*A*a*b/3 + C*a**2/3)
```

Giac [A] time = 1.09106, size = 204, normalized size = 1.32

$$\frac{1}{11}Cc^2x^{11} + \frac{1}{10}Bc^2x^{10} + \frac{2}{9}Cbcx^9 + \frac{1}{9}Ac^2x^9 + \frac{1}{4}Bbcx^8 + \frac{1}{7}Cb^2x^7 + \frac{2}{7}Cacx^7 + \frac{2}{7}Abcx^7 + \frac{1}{6}Bb^2x^6 + \frac{1}{3}Bacx^6 + \frac{2}{5}C$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*C*c^2*x^11 + 1/10*B*c^2*x^10 + 2/9*C*b*c*x^9 + 1/9*A*c^2*x^9 + 1/4*B*b
*c*x^8 + 1/7*C*b^2*x^7 + 2/7*C*a*c*x^7 + 2/7*A*b*c*x^7 + 1/6*B*b^2*x^6 + 1/
3*B*a*c*x^6 + 2/5*C*a*b*x^5 + 1/5*A*b^2*x^5 + 2/5*A*a*c*x^5 + 1/2*B*a*b*x^4
 + 1/3*C*a^2*x^3 + 2/3*A*a*b*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

3.14 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x}\,dx$

Optimal. Leaf size=150

$$a^2A\log(x)+a^2Bx+\frac{1}{6}x^6\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{4}x^4\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{2}ax^2(aC+2Ab)+\frac{1}{5}Bx^5\left(2ac+b^2\right)+\frac{2}{3}$$

```
[Out] a^2*B*x + (a*(2*A*b + a*C)*x^2)/2 + (2*a*b*B*x^3)/3 + ((A*(b^2 + 2*a*c) + 2
*a*b*C)*x^4)/4 + (B*(b^2 + 2*a*c)*x^5)/5 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^6
)/6 + (2*b*B*c*x^7)/7 + (c*(A*c + 2*b*C)*x^8)/8 + (B*c^2*x^9)/9 + (c^2*C*x^
10)/10 + a^2*A*Log[x]
```

Rubi [A] time = 0.106612, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1628}

$$a^2A\log(x)+a^2Bx+\frac{1}{6}x^6\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{4}x^4\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{2}ax^2(aC+2Ab)+\frac{1}{5}Bx^5\left(2ac+b^2\right)+\frac{2}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x]
```

```
[Out] a^2*B*x + (a*(2*A*b + a*C)*x^2)/2 + (2*a*b*B*x^3)/3 + ((A*(b^2 + 2*a*c) + 2
*a*b*C)*x^4)/4 + (B*(b^2 + 2*a*c)*x^5)/5 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^6
)/6 + (2*b*B*c*x^7)/7 + (c*(A*c + 2*b*C)*x^8)/8 + (B*c^2*x^9)/9 + (c^2*C*x^
10)/10 + a^2*A*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2}{x}\,dx=\int \left(a^2B+\frac{a^2A}{x}+a(2Ab+aC)x+2abBx^2+\left(A\left(b^2+2ac\right)+2abC\right)x^3+B\left(b^2\right.\right.$$
$$=a^2Bx+\frac{1}{2}a(2Ab+aC)x^2+\frac{2}{3}abBx^3+\frac{1}{4}\left(A\left(b^2+2ac\right)+2abC\right)x^4+\frac{1}{5}B\left(b^2+\right.$$

Mathematica [A] time = 0.039684, size = 150, normalized size = 1.

$$a^2A\log(x)+a^2Bx+\frac{1}{6}x^6\left(2acC+2Abc+b^2C\right)+\frac{1}{4}x^4\left(2aAc+2abC+Ab^2\right)+\frac{1}{2}ax^2(aC+2Ab)+\frac{1}{5}Bx^5\left(2ac+b^2\right)+\frac{2}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x]
```

```
[Out] a^2*B*x + (a*(2*A*b + a*C)*x^2)/2 + (2*a*b*B*x^3)/3 + ((A*b^2 + 2*a*A*c + 2
*a*b*C)*x^4)/4 + (B*(b^2 + 2*a*c)*x^5)/5 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^6
)/6 + (2*b*B*c*x^7)/7 + (c*(A*c + 2*b*C)*x^8)/8 + (B*c^2*x^9)/9 + (c^2*C*x^
```

```
10)/10 + a^2*A*Log[x]
```

Maple [A] time = 0.003, size = 149, normalized size = 1.

$$\frac{c^2Cx^{10}}{10}+\frac{Bc^2x^9}{9}+\frac{Ax^8c^2}{8}+\frac{Cx^8bc}{4}+\frac{2\,bBcx^7}{7}+\frac{Ax^6bc}{3}+\frac{Cx^6ac}{3}+\frac{Cx^6b^2}{6}+\frac{2\,Bx^5ac}{5}+\frac{Bx^5b^2}{5}+\frac{Ax^4ac}{2}+\frac{Ax^4b^2}{4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x)
```

```
[Out] 1/10*c^2*C*x^10+1/9*B*c^2*x^9+1/8*A*x^8*c^2+1/4*C*x^8*b*c+2/7*b*B*c*x^7+1/3
*A*x^6*b*c+1/3*C*x^6*a*c+1/6*C*x^6*b^2+2/5*B*x^5*a*c+1/5*B*x^5*b^2+1/2*A*x^
4*a*c+1/4*A*x^4*b^2+1/2*C*x^4*a*b+2/3*a*b*B*x^3+A*x^2*a*b+1/2*C*x^2*a^2+a^2
*B*x+a^2*A*ln(x)
```

Maxima [A] time = 0.955496, size = 186, normalized size = 1.24

$$\frac{1}{10}\,Cc^2x^{10}+\frac{1}{9}\,Bc^2x^9+\frac{2}{7}\,Bbcx^7+\frac{1}{8}\left(2\,Cbc+Ac^2\right)x^8+\frac{1}{6}\left(Cb^2+2\,(Ca+Ab)c\right)x^6+\frac{2}{3}\,Babx^3+\frac{1}{5}\left(Bb^2+2\,Bac\right)x^5+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/8*(2*C*b*c + A*c^2)*x^8
 + 1/6*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2/3*B*a*b*x^3 + 1/5*(B*b^2 + 2*B*a*c
)*x^5 + 1/4*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + B*a^2*x + A*a^2*log(x) + 1/2*
(C*a^2 + 2*A*a*b)*x^2
```

Fricas [A] time = 1.24675, size = 335, normalized size = 2.23

$$\frac{1}{10}\,Cc^2x^{10}+\frac{1}{9}\,Bc^2x^9+\frac{2}{7}\,Bbcx^7+\frac{1}{8}\left(2\,Cbc+Ac^2\right)x^8+\frac{1}{6}\left(Cb^2+2\,(Ca+Ab)c\right)x^6+\frac{2}{3}\,Babx^3+\frac{1}{5}\left(Bb^2+2\,Bac\right)x^5+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/8*(2*C*b*c + A*c^2)*x^8
 + 1/6*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2/3*B*a*b*x^3 + 1/5*(B*b^2 + 2*B*a*c
)*x^5 + 1/4*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + B*a^2*x + A*a^2*log(x) + 1/2*
(C*a^2 + 2*A*a*b)*x^2
```

Sympy [A] time = 0.499565, size = 156, normalized size = 1.04

$$Aa^2\log{(x)}+Ba^2x+\frac{2Babx^3}{3}+\frac{2Bbcx^7}{7}+\frac{Bc^2x^9}{9}+\frac{Cc^2x^{10}}{10}+x^8\left(\frac{Ac^2}{8}+\frac{Cbc}{4}\right)+x^6\left(\frac{Abc}{3}+\frac{Cac}{3}+\frac{Cb^2}{6}\right)+x^5\left(\frac{2Bac}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x,x)
```

```
[Out] A*a**2*log(x) + B*a**2*x + 2*B*a*b*x**3/3 + 2*B*b*c*x**7/7 + B*c**2*x**9/9
+ C*c**2*x**10/10 + x**8*(A*c**2/8 + C*b*c/4) + x**6*(A*b*c/3 + C*a*c/3 + C
*b**2/6) + x**5*(2*B*a*c/5 + B*b**2/5) + x**4*(A*a*c/2 + A*b**2/4 + C*a*b/2
) + x**2*(A*a*b + C*a**2/2)
```

Giac [A] time = 1.09492, size = 201, normalized size = 1.34

$$\frac{1}{10}Cc^2x^{10} + \frac{1}{9}Bc^2x^9 + \frac{1}{4}Cbcx^8 + \frac{1}{8}Ac^2x^8 + \frac{2}{7}Bbcx^7 + \frac{1}{6}Cb^2x^6 + \frac{1}{3}Cacx^6 + \frac{1}{3}Abcx^6 + \frac{1}{5}Bb^2x^5 + \frac{2}{5}Bacx^5 + \frac{1}{2}Cabx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 1/4*C*b*c*x^8 + 1/8*A*c^2*x^8 + 2/7*B*b*c
*x^7 + 1/6*C*b^2*x^6 + 1/3*C*a*c*x^6 + 1/3*A*b*c*x^6 + 1/5*B*b^2*x^5 + 2/5*
B*a*c*x^5 + 1/2*C*a*b*x^4 + 1/4*A*b^2*x^4 + 1/2*A*a*c*x^4 + 2/3*B*a*b*x^3 +
 1/2*C*a^2*x^2 + A*a*b*x^2 + B*a^2*x + A*a^2*log(abs(x))
```

3.15 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2}\,dx$

Optimal. Leaf size=145

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{5}x^5\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{3}x^3\left(A\left(2ac+b^2\right)+2abC\right)+ax(aC+2Ab)+\frac{1}{4}Bx^4\left(2ac+b^2\right)+$$

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + ((A*(b^2 + 2*a*c) + 2*a*b*C)
*x^3)/3 + (B*(b^2 + 2*a*c)*x^4)/4 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^5)/5 + (
b*B*c*x^6)/3 + (c*(A*c + 2*b*C)*x^7)/7 + (B*c^2*x^8)/8 + (c^2*C*x^9)/9 + a^
2*B*Log[x]
```

Rubi [A] time = 0.121105, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1628}

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{5}x^5\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{3}x^3\left(A\left(2ac+b^2\right)+2abC\right)+ax(aC+2Ab)+\frac{1}{4}Bx^4\left(2ac+b^2\right)+$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + ((A*(b^2 + 2*a*c) + 2*a*b*C)
*x^3)/3 + (B*(b^2 + 2*a*c)*x^4)/4 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^5)/5 + (
b*B*c*x^6)/3 + (c*(A*c + 2*b*C)*x^7)/7 + (B*c^2*x^8)/8 + (c^2*C*x^9)/9 + a^
2*B*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2}{x^2}\,dx=\int \left(a(2Ab+aC)+\frac{a^2A}{x^2}+\frac{a^2B}{x}+2abBx+\left(A\left(b^2+2ac\right)+2abC\right)x^2+B\left(b\right.\right.$$
$$=-\frac{a^2A}{x}+a(2Ab+aC)x+abBx^2+\frac{1}{3}\left(A\left(b^2+2ac\right)+2abC\right)x^3+\frac{1}{4}B\left(b^2+2\right.$$

Mathematica [A] time = 0.0969736, size = 145, normalized size = 1.

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{5}x^5\left(2acC+2Abc+b^2C\right)+\frac{1}{3}x^3\left(2aAc+2abC+Ab^2\right)+ax(aC+2Ab)+\frac{1}{4}Bx^4\left(2ac+b^2\right)+a$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + ((A*b^2 + 2*a*A*c + 2*a*b*C)
*x^3)/3 + (B*(b^2 + 2*a*c)*x^4)/4 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^5)/5 + (
```

```
b*B*c*x^6)/3 + (c*(A*c + 2*b*C)*x^7)/7 + (B*c^2*x^8)/8 + (c^2*C*x^9)/9 + a^
2*B*Log[x]
```

Maple [A] time = 0.007, size = 147, normalized size = 1.

$$\frac{c^2Cx^9}{9}+\frac{Bc^2x^8}{8}+\frac{Ax^7c^2}{7}+\frac{2\,Cx^7bc}{7}+\frac{bBcx^6}{3}+\frac{2\,Ax^5bc}{5}+\frac{2\,Cx^5ac}{5}+\frac{Cx^5b^2}{5}+\frac{Bx^4ac}{2}+\frac{Bx^4b^2}{4}+\frac{2\,Ax^3ac}{3}+\frac{Ax^3b^2}{3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x)
```

```
[Out] 1/9*c^2*C*x^9+1/8*B*c^2*x^8+1/7*A*x^7*c^2+2/7*C*x^7*b*c+1/3*b*B*c*x^6+2/5*A
*x^5*b*c+2/5*C*x^5*a*c+1/5*C*x^5*b^2+1/2*B*x^4*a*c+1/4*B*x^4*b^2+2/3*A*x^3*
a*c+1/3*A*x^3*b^2+2/3*C*x^3*a*b+a*b*B*x^2+2*A*a*b*x+C*a^2*x-a^2*A/x+a^2*B*l
n(x)
```

Maxima [A] time = 0.930936, size = 185, normalized size = 1.28

$$\frac{1}{9}\,Cc^2x^9+\frac{1}{8}\,Bc^2x^8+\frac{1}{3}\,Bbcx^6+\frac{1}{7}\left(2\,Cbc+Ac^2\right)x^7+\frac{1}{5}\left(Cb^2+2\,(Ca+Ab)c\right)x^5+Babx^2+\frac{1}{4}\left(Bb^2+2\,Bac\right)x^4+\frac{1}{3}\left(2\,Ca\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*C*c^2*x^9 + 1/8*B*c^2*x^8 + 1/3*B*b*c*x^6 + 1/7*(2*C*b*c + A*c^2)*x^7 +
 1/5*(C*b^2 + 2*(C*a + A*b)*c)*x^5 + B*a*b*x^2 + 1/4*(B*b^2 + 2*B*a*c)*x^4
+ 1/3*(2*C*a*b + A*b^2 + 2*A*a*c)*x^3 + B*a^2*log(x) - A*a^2/x + (C*a^2 + 2
*A*a*b)*x
```

Fricas [A] time = 1.25102, size = 365, normalized size = 2.52

$$\frac{280\,Cc^2x^{10}+315\,Bc^2x^9+840\,Bbcx^7+360\left(2\,Cbc+Ac^2\right)x^8+504\left(Cb^2+2\,(Ca+Ab)c\right)x^6+2520\,Babx^3+630\left(Bb^2+2\right.}{2520\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/2520*(280*C*c^2*x^10 + 315*B*c^2*x^9 + 840*B*b*c*x^7 + 360*(2*C*b*c + A*c
^2)*x^8 + 504*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2520*B*a*b*x^3 + 630*(B*b^2 +
 2*B*a*c)*x^5 + 840*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 2520*B*a^2*x*log(x) -
 2520*A*a^2 + 2520*(C*a^2 + 2*A*a*b)*x^2)/x
```

Sympy [A] time = 0.503331, size = 156, normalized size = 1.08

$$-\frac{Aa^2}{x}+Ba^2\log{(x)}+Babx^2+\frac{Bbcx^6}{3}+\frac{Bc^2x^8}{8}+\frac{Cc^2x^9}{9}+x^7\left(\frac{Ac^2}{7}+\frac{2Cbc}{7}\right)+x^5\left(\frac{2Abc}{5}+\frac{2Cac}{5}+\frac{Cb^2}{5}\right)+x^4\left(\frac{Bac}{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**2,x)
```

```
[Out] -A*a**2/x + B*a**2*log(x) + B*a*b*x**2 + B*b*c*x**6/3 + B*c**2*x**8/8 + C*c
**2*x**9/9 + x**7*(A*c**2/7 + 2*C*b*c/7) + x**5*(2*A*b*c/5 + 2*C*a*c/5 + C*
b**2/5) + x**4*(B*a*c/2 + B*b**2/4) + x**3*(2*A*a*c/3 + A*b**2/3 + 2*C*a*b/
3) + x*(2*A*a*b + C*a**2)
```

Giac [A] time = 1.08822, size = 198, normalized size = 1.37

$$\frac{1}{9}Cc^2x^9 + \frac{1}{8}Bc^2x^8 + \frac{2}{7}Cbcx^7 + \frac{1}{7}Ac^2x^7 + \frac{1}{3}Bbcx^6 + \frac{1}{5}Cb^2x^5 + \frac{2}{5}Cacx^5 + \frac{2}{5}Abcx^5 + \frac{1}{4}Bb^2x^4 + \frac{1}{2}Bacx^4 + \frac{2}{3}Cabx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/9*C*c^2*x^9 + 1/8*B*c^2*x^8 + 2/7*C*b*c*x^7 + 1/7*A*c^2*x^7 + 1/3*B*b*c*x
^6 + 1/5*C*b^2*x^5 + 2/5*C*a*c*x^5 + 2/5*A*b*c*x^5 + 1/4*B*b^2*x^4 + 1/2*B*
a*c*x^4 + 2/3*C*a*b*x^3 + 1/3*A*b^2*x^3 + 2/3*A*a*c*x^3 + B*a*b*x^2 + C*a^2
*x + 2*A*a*b*x + B*a^2*log(abs(x)) - A*a^2/x
```

3.16 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3}\,dx$

Optimal. Leaf size=149

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{4}x^4\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{2}x^2\left(A\left(2ac+b^2\right)+2abC\right)+a\log(x)(aC+2Ab)+\frac{1}{3}Bx^3\left(2ac+b^2\right)+2ab$$

```
[Out] -(a^2*A)/(2*x^2) - (a^2*B)/x + 2*a*b*B*x + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^2
)/2 + (B*(b^2 + 2*a*c)*x^3)/3 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^4)/4 + (2*b*
B*c*x^5)/5 + (c*(A*c + 2*b*C)*x^6)/6 + (B*c^2*x^7)/7 + (c^2*C*x^8)/8 + a*(2
*A*b + a*C)*Log[x]
```

Rubi [A] time = 0.122592, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1628}

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{4}x^4\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{2}x^2\left(A\left(2ac+b^2\right)+2abC\right)+a\log(x)(aC+2Ab)+\frac{1}{3}Bx^3\left(2ac+b^2\right)+2ab$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x]
```

```
[Out] -(a^2*A)/(2*x^2) - (a^2*B)/x + 2*a*b*B*x + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^2
)/2 + (B*(b^2 + 2*a*c)*x^3)/3 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^4)/4 + (2*b*
B*c*x^5)/5 + (c*(A*c + 2*b*C)*x^6)/6 + (B*c^2*x^7)/7 + (c^2*C*x^8)/8 + a*(2
*A*b + a*C)*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2}{x^3}\,dx &= \int \left(2abB+\frac{a^2A}{x^3}+\frac{a^2B}{x^2}+\frac{a(2Ab+aC)}{x}+\left(A\left(b^2+2ac\right)+2abC\right)x+B\left(b^2+2\right.\right.\\ &= -\frac{a^2A}{2x^2}-\frac{a^2B}{x}+2abBx+\frac{1}{2}\left(A\left(b^2+2ac\right)+2abC\right)x^2+\frac{1}{3}B\left(b^2+2ac\right)x^3+\frac{1}{4}\left(\right.\end{aligned}$$

Mathematica [A] time = 0.108635, size = 139, normalized size = 0.93

$$-\frac{a^2(A+2Bx)}{2x^2}+\frac{1}{6}ax\left(cx\left(6A+4Bx+3Cx^2\right)+6b(2B+Cx)\right)+a\log(x)(aC+2Ab)+\frac{1}{840}x^2\left(140A\left(3b^2+3bcx^2+c^2x^4\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x]
```

```
[Out] -(a^2*(A + 2*B*x))/(2*x^2) + (a*x*(6*b*(2*B + C*x) + c*x*(6*A + 4*B*x + 3*C
*x^2)))/6 + (x^2*(70*b^2*x*(4*B + 3*C*x) + 56*b*c*x^3*(6*B + 5*C*x) + 15*c^
```

```
2*x^5*(8*B + 7*C*x) + 140*A*(3*b^2 + 3*b*c*x^2 + c^2*x^4)))/840 + a*(2*A*b
+ a*C)*Log[x]
```

Maple [A] time = 0.008, size = 148, normalized size = 1.

$$\frac{c^2Cx^8}{8}+\frac{Bc^2x^7}{7}+\frac{Ax^6c^2}{6}+\frac{Cx^6bc}{3}+\frac{2\,bBcx^5}{5}+\frac{Ax^4bc}{2}+\frac{Cx^4ac}{2}+\frac{Cx^4b^2}{4}+\frac{2\,Bx^3ac}{3}+\frac{Bx^3b^2}{3}+Ax^2ac+\frac{Ax^2b^2}{2}+C$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x)
```

```
[Out] 1/8*c^2*C*x^8+1/7*B*c^2*x^7+1/6*A*x^6*c^2+1/3*C*x^6*b*c+2/5*b*B*c*x^5+1/2*A
*x^4*b*c+1/2*C*x^4*a*c+1/4*C*x^4*b^2+2/3*B*x^3*a*c+1/3*B*x^3*b^2+A*x^2*a*c+
1/2*A*x^2*b^2+C*x^2*a*b+2*a*b*B*x-a^2*B/x-1/2*a^2*A/x^2+2*A*ln(x)*a*b+C*ln(
x)*a^2
```

Maxima [A] time = 0.943822, size = 188, normalized size = 1.26

$$\frac{1}{8}Cc^2x^8+\frac{1}{7}Bc^2x^7+\frac{2}{5}Bbcx^5+\frac{1}{6}\left(2\,Cbc+Ac^2\right)x^6+\frac{1}{4}\left(Cb^2+2\,(Ca+Ab)c\right)x^4+2\,Babx+\frac{1}{3}\left(Bb^2+2\,Bac\right)x^3+\frac{1}{2}\,(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*C*c^2*x^8 + 1/7*B*c^2*x^7 + 2/5*B*b*c*x^5 + 1/6*(2*C*b*c + A*c^2)*x^6 +
 1/4*(C*b^2 + 2*(C*a + A*b)*c)*x^4 + 2*B*a*b*x + 1/3*(B*b^2 + 2*B*a*c)*x^3
+ 1/2*(2*C*a*b + A*b^2 + 2*A*a*c)*x^2 + (C*a^2 + 2*A*a*b)*log(x) - 1/2*(2*B
*a^2*x + A*a^2)/x^2
```

Fricas [A] time = 1.30031, size = 362, normalized size = 2.43

$$\frac{105\,Cc^2x^{10}+120\,Bc^2x^9+336\,Bbcx^7+140\left(2\,Cbc+Ac^2\right)x^8+210\left(Cb^2+2\,(Ca+Ab)c\right)x^6+1680\,Babx^3+280\left(Bb^2\right.}{840\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/840*(105*C*c^2*x^10 + 120*B*c^2*x^9 + 336*B*b*c*x^7 + 140*(2*C*b*c + A*c^
2)*x^8 + 210*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 1680*B*a*b*x^3 + 280*(B*b^2 +
2*B*a*c)*x^5 + 420*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 840*B*a^2*x + 840*(C*a
^2 + 2*A*a*b)*x^2*log(x) - 420*A*a^2)/x^2
```

Sympy [A] time = 0.641083, size = 151, normalized size = 1.01

$$2Babx+\frac{2Bbcx^5}{5}+\frac{Bc^2x^7}{7}+\frac{Cc^2x^8}{8}+a\,(2Ab+Ca)\log{(x)}+x^6\left(\frac{Ac^2}{6}+\frac{Cbc}{3}\right)+x^4\left(\frac{Abc}{2}+\frac{Cac}{2}+\frac{Cb^2}{4}\right)+x^3\left(\frac{2Bac}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**3,x)
```

```
[Out] 2*B*a*b*x + 2*B*b*c*x**5/5 + B*c**2*x**7/7 + C*c**2*x**8/8 + a*(2*A*b + C*a
)*log(x) + x**6*(A*c**2/6 + C*b*c/3) + x**4*(A*b*c/2 + C*a*c/2 + C*b**2/4)
+ x**3*(2*B*a*c/3 + B*b**2/3) + x**2*(A*a*c + A*b**2/2 + C*a*b) - (A*a**2 +
 2*B*a**2*x)/(2*x**2)
```

Giac [A] time = 1.13592, size = 200, normalized size = 1.34

$$\frac{1}{8}Cc^2x^8 + \frac{1}{7}Bc^2x^7 + \frac{1}{3}Cbcx^6 + \frac{1}{6}Ac^2x^6 + \frac{2}{5}Bbcx^5 + \frac{1}{4}Cb^2x^4 + \frac{1}{2}Cacx^4 + \frac{1}{2}Abcx^4 + \frac{1}{3}Bb^2x^3 + \frac{2}{3}Bacx^3 + Cabx^2 + \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/8*C*c^2*x^8 + 1/7*B*c^2*x^7 + 1/3*C*b*c*x^6 + 1/6*A*c^2*x^6 + 2/5*B*b*c*x
^5 + 1/4*C*b^2*x^4 + 1/2*C*a*c*x^4 + 1/2*A*b*c*x^4 + 1/3*B*b^2*x^3 + 2/3*B*
a*c*x^3 + C*a*b*x^2 + 1/2*A*b^2*x^2 + A*a*c*x^2 + 2*B*a*b*x + (C*a^2 + 2*A*
a*b)*log(abs(x)) - 1/2*(2*B*a^2*x + A*a^2)/x^2
```

3.17 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx$

Optimal. Leaf size=149

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+\frac{1}{3}x^3\left(C\left(2ac+b^2\right)+2Abc\right)+x\left(A\left(2ac+b^2\right)+2abC\right)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}Bx^2\left(2ac+b^2\right)+2abB\log($$

```
[Out] -(a^2*A)/(3*x^3) - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + (A*(b^2 + 2*a*c)
 + 2*a*b*C)*x + (B*(b^2 + 2*a*c)*x^2)/2 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^3)
/3 + (b*B*c*x^4)/2 + (c*(A*c + 2*b*C)*x^5)/5 + (B*c^2*x^6)/6 + (c^2*C*x^7)/
7 + 2*a*b*B*Log[x]
```

Rubi [A] time = 0.137291, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1628}

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+\frac{1}{3}x^3\left(C\left(2ac+b^2\right)+2Abc\right)+x\left(A\left(2ac+b^2\right)+2abC\right)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}Bx^2\left(2ac+b^2\right)+2abB\log($$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x]
```

```
[Out] -(a^2*A)/(3*x^3) - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + (A*(b^2 + 2*a*c)
 + 2*a*b*C)*x + (B*(b^2 + 2*a*c)*x^2)/2 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^3)
/3 + (b*B*c*x^4)/2 + (c*(A*c + 2*b*C)*x^5)/5 + (B*c^2*x^6)/6 + (c^2*C*x^7)/
7 + 2*a*b*B*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2}{x^4}\,dx=\int \left(Ab^2\left(1+\frac{2a(Ac+bC)}{Ab^2}\right)+\frac{a^2A}{x^4}+\frac{a^2B}{x^3}+\frac{a(2Ab+aC)}{x^2}+\frac{2abB}{x}+B\left(b^2\right.\right.$$
$$=-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}-\frac{a(2Ab+aC)}{x}+\left(A\left(b^2+2ac\right)+2abC\right)x+\frac{1}{2}B\left(b^2+2ac\right)x^2$$

Mathematica [A] time = 0.0817475, size = 151, normalized size = 1.01

$$\frac{a^2(-C)-2aAb}{x}-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+\frac{1}{3}x^3\left(2acC+2Abc+b^2C\right)+x\left(2aAc+2abC+Ab^2\right)+\frac{1}{2}Bx^2\left(2ac+b^2\right)+2abB\log($$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x]
```

```
[Out] -(a^2*A)/(3*x^3) - (a^2*B)/(2*x^2) + (-2*a*A*b - a^2*C)/x + (A*b^2 + 2*a*A*
c + 2*a*b*C)*x + (B*(b^2 + 2*a*c)*x^2)/2 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^3
```

```
)/3 + (b*B*c*x^4)/2 + (c*(A*c + 2*b*C)*x^5)/5 + (B*c^2*x^6)/6 + (c^2*C*x^7)
/7 + 2*a*b*B*Log[x]
```

Maple [A] time = 0.006, size = 146, normalized size = 1.

$$\frac{c^2Cx^7}{7}+\frac{Bc^2x^6}{6}+\frac{Ax^5c^2}{5}+\frac{2\,Cx^5bc}{5}+\frac{bBcx^4}{2}+\frac{2\,Ax^3bc}{3}+\frac{2\,Cx^3ac}{3}+\frac{Cx^3b^2}{3}+Bx^2ac+\frac{Bx^2b^2}{2}+2\,aAcx+Ab^2x+2\,ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x)
```

```
[Out] 1/7*c^2*C*x^7+1/6*B*c^2*x^6+1/5*A*x^5*c^2+2/5*C*x^5*b*c+1/2*b*B*c*x^4+2/3*A
*x^3*b*c+2/3*C*x^3*a*c+1/3*C*x^3*b^2+B*x^2*a*c+1/2*B*x^2*b^2+2*a*A*c*x+A*b^
2*x+2*a*b*C*x-2*a/x*A*b-a^2/x*C-1/2*a^2*B/x^2-1/3*a^2*A/x^3+2*a*b*B*ln(x)
```

Maxima [A] time = 0.956365, size = 189, normalized size = 1.27

$$\frac{1}{7}\,Cc^2x^7+\frac{1}{6}\,Bc^2x^6+\frac{1}{2}\,Bbcx^4+\frac{1}{5}\left(2\,Cbc+Ac^2\right)x^5+\frac{1}{3}\left(Cb^2+2\,(Ca+Ab)c\right)x^3+2\,Bab\log\left(x\right)+\frac{1}{2}\left(Bb^2+2\,Bac\right)x^2+($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 1/2*B*b*c*x^4 + 1/5*(2*C*b*c + A*c^2)*x^5 +
 1/3*(C*b^2 + 2*(C*a + A*b)*c)*x^3 + 2*B*a*b*log(x) + 1/2*(B*b^2 + 2*B*a*c)
*x^2 + (2*C*a*b + A*b^2 + 2*A*a*c)*x - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2
+ 2*A*a*b)*x^2)/x^3
```

Fricas [A] time = 1.28773, size = 354, normalized size = 2.38

$$\frac{30\,Cc^2x^{10}+35\,Bc^2x^9+105\,Bbcx^7+42\left(2\,Cbc+Ac^2\right)x^8+70\left(Cb^2+2\,(Ca+Ab)c\right)x^6+420\,Babx^3\log\left(x\right)+105\left(Bb^2+\right.}{210\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/210*(30*C*c^2*x^10 + 35*B*c^2*x^9 + 105*B*b*c*x^7 + 42*(2*C*b*c + A*c^2)*
x^8 + 70*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 420*B*a*b*x^3*log(x) + 105*(B*b^2
+ 2*B*a*c)*x^5 + 210*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 105*B*a^2*x - 70*A*a
^2 - 210*(C*a^2 + 2*A*a*b)*x^2)/x^3
```

Sympy [A] time = 0.886277, size = 158, normalized size = 1.06

$$2Bab\log\left(x\right)+\frac{Bbcx^4}{2}+\frac{Bc^2x^6}{6}+\frac{Cc^2x^7}{7}+x^5\left(\frac{Ac^2}{5}+\frac{2Cbc}{5}\right)+x^3\left(\frac{2Abc}{3}+\frac{2Cac}{3}+\frac{Cb^2}{3}\right)+x^2\left(Bac+\frac{Bb^2}{2}\right)+x\left(2Aac+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**4,x)
```

```
[Out] 2*B*a*b*log(x) + B*b*c*x**4/2 + B*c**2*x**6/6 + C*c**2*x**7/7 + x**5*(A*c**
2/5 + 2*C*b*c/5) + x**3*(2*A*b*c/3 + 2*C*a*c/3 + C*b**2/3) + x**2*(B*a*c +
B*b**2/2) + x*(2*A*a*c + A*b**2 + 2*C*a*b) - (2*A*a**2 + 3*B*a**2*x + x**2*
(12*A*a*b + 6*C*a**2))/(6*x**3)
```

Giac [A] time = 1.08759, size = 197, normalized size = 1.32

$$\frac{1}{7}Cc^2x^7 + \frac{1}{6}Bc^2x^6 + \frac{2}{5}Cbcx^5 + \frac{1}{5}Ac^2x^5 + \frac{1}{2}Bbcx^4 + \frac{1}{3}Cb^2x^3 + \frac{2}{3}Cacx^3 + \frac{2}{3}Abcx^3 + \frac{1}{2}Bb^2x^2 + Bacx^2 + 2\,Cabx +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 2/5*C*b*c*x^5 + 1/5*A*c^2*x^5 + 1/2*B*b*c*x
^4 + 1/3*C*b^2*x^3 + 2/3*C*a*c*x^3 + 2/3*A*b*c*x^3 + 1/2*B*b^2*x^2 + B*a*c*
x^2 + 2*C*a*b*x + A*b^2*x + 2*A*a*c*x + 2*B*a*b*log(abs(x)) - 1/6*(3*B*a^2*
x + 2*A*a^2 + 6*(C*a^2 + 2*A*a*b)*x^2)/x^3
```

3.18 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5} dx$

Optimal. Leaf size=148

$$-\frac{a^2A}{4x^4}-\frac{a^2B}{3x^3}+\frac{1}{2}x^2\left(C\left(2ac+b^2\right)+2Abc\right)+\log(x)\left(A\left(2ac+b^2\right)+2abC\right)-\frac{a(aC+2Ab)}{2x^2}+Bx\left(2ac+b^2\right)-\frac{2abB}{x}+\frac{1}{4}c$$

```
[Out] -(a^2*A)/(4*x^4) - (a^2*B)/(3*x^3) - (a*(2*A*b + a*C))/(2*x^2) - (2*a*b*B)/
x + B*(b^2 + 2*a*c)*x + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^2)/2 + (2*b*B*c*x^3)
/3 + (c*(A*c + 2*b*C)*x^4)/4 + (B*c^2*x^5)/5 + (c^2*C*x^6)/6 + (A*(b^2 + 2*
a*c) + 2*a*b*C)*Log[x]
```

Rubi [A] time = 0.141568, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1628}

$$-\frac{a^2A}{4x^4}-\frac{a^2B}{3x^3}+\frac{1}{2}x^2\left(C\left(2ac+b^2\right)+2Abc\right)+\log(x)\left(A\left(2ac+b^2\right)+2abC\right)-\frac{a(aC+2Ab)}{2x^2}+Bx\left(2ac+b^2\right)-\frac{2abB}{x}+\frac{1}{4}c$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x]
```

```
[Out] -(a^2*A)/(4*x^4) - (a^2*B)/(3*x^3) - (a*(2*A*b + a*C))/(2*x^2) - (2*a*b*B)/
x + B*(b^2 + 2*a*c)*x + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^2)/2 + (2*b*B*c*x^3)
/3 + (c*(A*c + 2*b*C)*x^4)/4 + (B*c^2*x^5)/5 + (c^2*C*x^6)/6 + (A*(b^2 + 2*
a*c) + 2*a*b*C)*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2}{x^5}\,dx=\int \left(B\left(b^2+2ac\right)+\frac{a^2A}{x^5}+\frac{a^2B}{x^4}+\frac{a(2Ab+aC)}{x^3}+\frac{2abB}{x^2}+\frac{A\left(b^2+2ac\right)+2abC}{x}\right.$$
$$=-\frac{a^2A}{4x^4}-\frac{a^2B}{3x^3}-\frac{a(2Ab+aC)}{2x^2}-\frac{2abB}{x}+B\left(b^2+2ac\right)x+\frac{1}{2}\left(2Abc+\left(b^2+2ac\right)\right.$$

Mathematica [A] time = 0.0826919, size = 130, normalized size = 0.88

$$-\frac{a^2\left(3A+4Bx+6Cx^2\right)}{12x^4}+\log(x)\left(A\left(2ac+b^2\right)+2abC\right)+\frac{a\left(-Ab-2bBx+cx^3(2B+Cx)\right)}{x^2}+\frac{1}{60}x\left(10bcx(6A+x(4B+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x]
```

```
[Out] -(a^2*(3*A + 4*B*x + 6*C*x^2))/(12*x^4) + (a*(-(A*b) - 2*b*B*x + c*x^3*(2*B
 + C*x)))/x^2 + (x*(30*b^2*(2*B + C*x) + 10*b*c*x*(6*A + x*(4*B + 3*C*x)) +
```

```
 c^2*x^3*(15*A + 2*x*(6*B + 5*C*x))))/60 + (A*(b^2 + 2*a*c) + 2*a*b*C)*Log[
x]
```

Maple [A] time = 0.008, size = 144, normalized size = 1.

$$\frac{c^2Cx^6}{6}+\frac{Bc^2x^5}{5}+\frac{Ax^4c^2}{4}+\frac{Cx^4bc}{2}+\frac{2\,bBcx^3}{3}+Ax^2bc+Cx^2ac+\frac{Cx^2b^2}{2}+2\,Bacx+Bb^2x-2\,\frac{Bab}{x}-\frac{Aab}{x^2}-\frac{Ca^2}{2\,x^2}-\frac{A}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x)
```

```
[Out] 1/6*c^2*C*x^6+1/5*B*c^2*x^5+1/4*A*x^4*c^2+1/2*C*x^4*b*c+2/3*b*B*c*x^3+A*x^2
*b*c+C*x^2*a*c+1/2*C*x^2*b^2+2*B*a*c*x+B*b^2*x-2*a*b*B/x-a/x^2*A*b-1/2*a^2/
x^2*C-1/4*a^2*A/x^4-1/3*a^2*B/x^3+2*A*ln(x)*a*c+A*ln(x)*b^2+2*C*ln(x)*a*b
```

Maxima [A] time = 0.983737, size = 188, normalized size = 1.27

$$\frac{1}{6}\,Cc^2x^6+\frac{1}{5}\,Bc^2x^5+\frac{2}{3}\,Bbcx^3+\frac{1}{4}\left(2\,Cbc+Ac^2\right)x^4+\frac{1}{2}\left(Cb^2+2\,(Ca+Ab)c\right)x^2+\left(Bb^2+2\,Bac\right)x+\left(2\,Cab+Ab^2+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*C*c^2*x^6 + 1/5*B*c^2*x^5 + 2/3*B*b*c*x^3 + 1/4*(2*C*b*c + A*c^2)*x^4 +
 1/2*(C*b^2 + 2*(C*a + A*b)*c)*x^2 + (B*b^2 + 2*B*a*c)*x + (2*C*a*b + A*b^2
 + 2*A*a*c)*log(x) - 1/12*(24*B*a*b*x^3 + 4*B*a^2*x + 3*A*a^2 + 6*(C*a^2 +
2*A*a*b)*x^2)/x^4
```

Fricas [A] time = 1.23705, size = 346, normalized size = 2.34

$$\frac{10\,Cc^2x^{10}+12\,Bc^2x^9+40\,Bbcx^7+15\left(2\,Cbc+Ac^2\right)x^8+30\left(Cb^2+2\,(Ca+Ab)c\right)x^6-120\,Babx^3+60\left(Bb^2+2\,Bac\right)x}{60\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/60*(10*C*c^2*x^10 + 12*B*c^2*x^9 + 40*B*b*c*x^7 + 15*(2*C*b*c + A*c^2)*x^
8 + 30*(C*b^2 + 2*(C*a + A*b)*c)*x^6 - 120*B*a*b*x^3 + 60*(B*b^2 + 2*B*a*c)
*x^5 + 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4*log(x) - 20*B*a^2*x - 15*A*a^2 -
30*(C*a^2 + 2*A*a*b)*x^2)/x^4
```

Sympy [A] time = 2.62166, size = 151, normalized size = 1.02

$$\frac{2Bbcx^3}{3}+\frac{Bc^2x^5}{5}+\frac{Cc^2x^6}{6}+x^4\left(\frac{Ac^2}{4}+\frac{Cbc}{2}\right)+x^2\left(Abc+Cac+\frac{Cb^2}{2}\right)+x\left(2Bac+Bb^2\right)+\left(2Aac+Ab^2+2Cab\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**5,x)
```

```
[Out] 2*B*b*c*x**3/3 + B*c**2*x**5/5 + C*c**2*x**6/6 + x**4*(A*c**2/4 + C*b*c/2)
+ x**2*(A*b*c + C*a*c + C*b**2/2) + x*(2*B*a*c + B*b**2) + (2*A*a*c + A*b**
2 + 2*C*a*b)*log(x) - (3*A*a**2 + 4*B*a**2*x + 24*B*a*b*x**3 + x**2*(12*A*a
*b + 6*C*a**2))/(12*x**4)
```

Giac [A] time = 1.11442, size = 192, normalized size = 1.3

$$\frac{1}{6}Cc^2x^6 + \frac{1}{5}Bc^2x^5 + \frac{1}{2}Cbcx^4 + \frac{1}{4}Ac^2x^4 + \frac{2}{3}Bbcx^3 + \frac{1}{2}Cb^2x^2 + Cacx^2 + Abcx^2 + Bb^2x + 2Bacx + \Big(2Cab + Ab^2 + 2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/6*C*c^2*x^6 + 1/5*B*c^2*x^5 + 1/2*C*b*c*x^4 + 1/4*A*c^2*x^4 + 2/3*B*b*c*x
^3 + 1/2*C*b^2*x^2 + C*a*c*x^2 + A*b*c*x^2 + B*b^2*x + 2*B*a*c*x + (2*C*a*b
 + A*b^2 + 2*A*a*c)*log(abs(x)) - 1/12*(24*B*a*b*x^3 + 4*B*a^2*x + 3*A*a^2
+ 6*(C*a^2 + 2*A*a*b)*x^2)/x^4
```

3.19 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx$

Optimal. Leaf size=143

$$-\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}+x\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{A\left(2ac+b^2\right)+2abC}{x}-\frac{a(aC+2Ab)}{3x^3}+B\log(x)\left(2ac+b^2\right)-\frac{abB}{x^2}+\frac{1}{3}cx^3(\ldots$$

```
[Out] -(a^2*A)/(5*x^5) - (a^2*B)/(4*x^4) - (a*(2*A*b + a*C))/(3*x^3) - (a*b*B)/x^
2 - (A*(b^2 + 2*a*c) + 2*a*b*C)/x + (2*A*b*c + (b^2 + 2*a*c)*C)*x + b*B*c*x
^2 + (c*(A*c + 2*b*C)*x^3)/3 + (B*c^2*x^4)/4 + (c^2*C*x^5)/5 + B*(b^2 + 2*a
*c)*Log[x]
```

Rubi [A] time = 0.146662, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1628}

$$-\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}+x\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{A\left(2ac+b^2\right)+2abC}{x}-\frac{a(aC+2Ab)}{3x^3}+B\log(x)\left(2ac+b^2\right)-\frac{abB}{x^2}+\frac{1}{3}cx^3(\ldots$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x]
```

```
[Out] -(a^2*A)/(5*x^5) - (a^2*B)/(4*x^4) - (a*(2*A*b + a*C))/(3*x^3) - (a*b*B)/x^
2 - (A*(b^2 + 2*a*c) + 2*a*b*C)/x + (2*A*b*c + (b^2 + 2*a*c)*C)*x + b*B*c*x
^2 + (c*(A*c + 2*b*C)*x^3)/3 + (B*c^2*x^4)/4 + (c^2*C*x^5)/5 + B*(b^2 + 2*a
*c)*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2}{x^6}\,dx=\int \left(2Abc\left(1+\frac{b\left(1+\frac{2ac}{b^2}\right)C}{2Ac}\right)+\frac{a^2A}{x^6}+\frac{a^2B}{x^5}+\frac{a(2Ab+aC)}{x^4}+\frac{2abB}{x^3}+\frac{A(b\ldots$$

$$=-\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}-\frac{a(2Ab+aC)}{3x^3}-\frac{abB}{x^2}-\frac{A\left(b^2+2ac\right)+2abC}{x}+\left(2Abc+\left(b^2\ldots\right.\right.$$

Mathematica [A] time = 0.0823089, size = 142, normalized size = 0.99

$$-\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}-\frac{2aAc+2abC+Ab^2}{x}-\frac{a(aC+2Ab)}{3x^3}+B\log(x)\left(2ac+b^2\right)+Cx\left(2ac+b^2\right)-\frac{abB}{x^2}+\frac{1}{3}cx^3(Ac+2bC)\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x]
```

```
[Out] -(a^2*A)/(5*x^5) - (a^2*B)/(4*x^4) - (a*(2*A*b + a*C))/(3*x^3) - (a*b*B)/x^
2 - (A*b^2 + 2*a*A*c + 2*a*b*C)/x + 2*A*b*c*x + (b^2 + 2*a*c)*C*x + b*B*c*x
^2 + (c*(A*c + 2*b*C)*x^3)/3 + (B*c^2*x^4)/4 + (c^2*C*x^5)/5 + B*(b^2 + 2*a
*c)*Log[x]
```

Maple [A] time = 0.007, size = 144, normalized size = 1.

$$\frac{c^2Cx^5}{5}+\frac{Bc^2x^4}{4}+\frac{Ax^3c^2}{3}+\frac{2\,Cx^3bc}{3}+bBcx^2+2\,Abcx+2\,acCx+b^2Cx-2\,\frac{aAc}{x}-\frac{Ab^2}{x}-2\,\frac{abC}{x}-\frac{Aa^2}{5\,x^5}-\frac{Bab}{x^2}-\frac{Ba^2}{4\,x^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x)
```

```
[Out] 1/5*c^2*C*x^5+1/4*B*c^2*x^4+1/3*A*x^3*c^2+2/3*C*x^3*b*c+b*B*c*x^2+2*A*b*c*x
+2*a*c*C*x+b^2*C*x-2/x*a*A*c-1/x*A*b^2-2/x*a*b*C-1/5*a^2*A/x^5-a*b*B/x^2-1/
4*a^2*B/x^4-2/3*a/x^3*A*b-1/3*a^2/x^3*C+2*B*ln(x)*a*c+B*ln(x)*b^2
```

Maxima [A] time = 0.970014, size = 186, normalized size = 1.3

$$\frac{1}{5}\,Cc^2x^5+\frac{1}{4}\,Bc^2x^4+Bbcx^2+\frac{1}{3}\left(2\,Cbc+Ac^2\right)x^3+\left(Cb^2+2\,(Ca+Ab)c\right)x+\left(Bb^2+2\,Bac\right)\log\left(x\right)-\frac{60\,Babx^3+60\left(2\,C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*C*c^2*x^5 + 1/4*B*c^2*x^4 + B*b*c*x^2 + 1/3*(2*C*b*c + A*c^2)*x^3 + (C*
b^2 + 2*(C*a + A*b)*c)*x + (B*b^2 + 2*B*a*c)*log(x) - 1/60*(60*B*a*b*x^3 +
60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 15*B*a^2*x + 12*A*a^2 + 20*(C*a^2 + 2*
A*a*b)*x^2)/x^5
```

Fricas [A] time = 1.20422, size = 344, normalized size = 2.41

$$\frac{12\,Cc^2x^{10}+15\,Bc^2x^9+60\,Bbcx^7+20\left(2\,Cbc+Ac^2\right)x^8+60\left(Cb^2+2\,(Ca+Ab)c\right)x^6+60\left(Bb^2+2\,Bac\right)x^5\log\left(x\right)-60\,B}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*c^2*x^10 + 15*B*c^2*x^9 + 60*B*b*c*x^7 + 20*(2*C*b*c + A*c^2)*x^
8 + 60*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 60*(B*b^2 + 2*B*a*c)*x^5*log(x) - 60
*B*a*b*x^3 - 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 15*B*a^2*x - 12*A*a^2 - 2
0*(C*a^2 + 2*A*a*b)*x^2)/x^5
```

Sympy [A] time = 8.27827, size = 151, normalized size = 1.06

$$Bbcx^2+\frac{Bc^2x^4}{4}+B\left(2ac+b^2\right)\log\left(x\right)+\frac{Cc^2x^5}{5}+x^3\left(\frac{Ac^2}{3}+\frac{2Cbc}{3}\right)+x\left(2Abc+2Cac+Cb^2\right)-\frac{12Aa^2+15Ba^2x+60Ba}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**6,x)
```

```
[Out] B*b*c*x**2 + B*c**2*x**4/4 + B*(2*a*c + b**2)*log(x) + C*c**2*x**5/5 + x**3
*(A*c**2/3 + 2*C*b*c/3) + x*(2*A*b*c + 2*C*a*c + C*b**2) - (12*A*a**2 + 15*
B*a**2*x + 60*B*a*b*x**3 + x**4*(120*A*a*c + 60*A*b**2 + 120*C*a*b) + x**2*
(40*A*a*b + 20*C*a**2))/(60*x**5)
```

Giac [A] time = 1.12074, size = 189, normalized size = 1.32

$$\frac{1}{5}Cc^2x^5 + \frac{1}{4}Bc^2x^4 + \frac{2}{3}Cbcx^3 + \frac{1}{3}Ac^2x^3 + Bbcx^2 + Cb^2x + 2\,Cacx + 2\,Abcx + \left(Bb^2 + 2\,Bac\right)\log\left(|x|\right) - \frac{60\,Babx^3 + 6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/5*C*c^2*x^5 + 1/4*B*c^2*x^4 + 2/3*C*b*c*x^3 + 1/3*A*c^2*x^3 + B*b*c*x^2 +
 C*b^2*x + 2*C*a*c*x + 2*A*b*c*x + (B*b^2 + 2*B*a*c)*log(abs(x)) - 1/60*(60
*B*a*b*x^3 + 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 15*B*a^2*x + 12*A*a^2 + 2
0*(C*a^2 + 2*A*a*b)*x^2)/x^5
```

3.20 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx$

Optimal. Leaf size=149

$$-\frac{a^2A}{6x^6}-\frac{a^2B}{5x^5}-\frac{A\left(2ac+b^2\right)+2abC}{2x^2}+\log(x)\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{a(aC+2Ab)}{4x^4}-\frac{B\left(2ac+b^2\right)}{x}-\frac{2abB}{3x^3}+\frac{1}{2}cx^2(Ac$$

```
[Out] -(a^2*A)/(6*x^6) - (a^2*B)/(5*x^5) - (a*(2*A*b + a*C))/(4*x^4) - (2*a*b*B)/
(3*x^3) - (A*(b^2 + 2*a*c) + 2*a*b*C)/(2*x^2) - (B*(b^2 + 2*a*c))/x + 2*b*B
*c*x + (c*(A*c + 2*b*C)*x^2)/2 + (B*c^2*x^3)/3 + (c^2*C*x^4)/4 + (2*A*b*c +
 (b^2 + 2*a*c)*C)*Log[x]
```

Rubi [A] time = 0.143141, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1628}

$$-\frac{a^2A}{6x^6}-\frac{a^2B}{5x^5}-\frac{A\left(2ac+b^2\right)+2abC}{2x^2}+\log(x)\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{a(aC+2Ab)}{4x^4}-\frac{B\left(2ac+b^2\right)}{x}-\frac{2abB}{3x^3}+\frac{1}{2}cx^2(Ac$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x]
```

```
[Out] -(a^2*A)/(6*x^6) - (a^2*B)/(5*x^5) - (a*(2*A*b + a*C))/(4*x^4) - (2*a*b*B)/
(3*x^3) - (A*(b^2 + 2*a*c) + 2*a*b*C)/(2*x^2) - (B*(b^2 + 2*a*c))/x + 2*b*B
*c*x + (c*(A*c + 2*b*C)*x^2)/2 + (B*c^2*x^3)/3 + (c^2*C*x^4)/4 + (2*A*b*c +
 (b^2 + 2*a*c)*C)*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2}{x^7}\,dx=\int \left(2bBc+\frac{a^2A}{x^7}+\frac{a^2B}{x^6}+\frac{a(2Ab+aC)}{x^5}+\frac{2abB}{x^4}+\frac{A\left(b^2+2ac\right)+2abC}{x^3}+\frac{B(b$$

$$=-\frac{a^2A}{6x^6}-\frac{a^2B}{5x^5}-\frac{a(2Ab+aC)}{4x^4}-\frac{2abB}{3x^3}-\frac{A\left(b^2+2ac\right)+2abC}{2x^2}-\frac{B\left(b^2+2ac\right)}{x}+$$

Mathematica [A] time = 0.10065, size = 144, normalized size = 0.97

$$-\frac{a^2(10A+3x(4B+5Cx))}{60x^6}+\log(x)\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{a\left(3A\left(b+2cx^2\right)+2x\left(2bB+3bCx+6Bcx^2\right)\right)}{6x^4}+\frac{A\left(c^2x^4-\right.}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x]
```

```
[Out] -((b^2*B)/x) + b*c*x*(2*B + C*x) + (c^2*x^3*(4*B + 3*C*x))/12 + (A*(-b^2 +
c^2*x^4))/(2*x^2) - (a^2*(10*A + 3*x*(4*B + 5*C*x)))/(60*x^6) - (a*(3*A*(b
```

```
+ 2*c*x^2) + 2*x*(2*b*B + 3*b*C*x + 6*B*c*x^2)))/(6*x^4) + (2*A*b*c + (b^2
+ 2*a*c)*C)*Log[x]
```

Maple [A] time = 0.009, size = 148, normalized size = 1.

$$\frac{c^2Cx^4}{4}+\frac{Bc^2x^3}{3}+\frac{Ax^2c^2}{2}+Cx^2bc+2\,bBcx-\frac{Aab}{2\,x^4}-\frac{Ca^2}{4\,x^4}-2\,\frac{Bac}{x}-\frac{Bb^2}{x}-\frac{aAc}{x^2}-\frac{Ab^2}{2\,x^2}-\frac{abC}{x^2}-\frac{Aa^2}{6\,x^6}-\frac{Ba^2}{5\,x^5}-\frac{2\,Bab}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x)
```

```
[Out] 1/4*c^2*C*x^4+1/3*B*c^2*x^3+1/2*A*x^2*c^2+C*x^2*b*c+2*b*B*c*x-1/2*a/x^4*A*b
-1/4*a^2/x^4*C-2*B/x*a*c-B/x*b^2-1/x^2*a*A*c-1/2/x^2*A*b^2-1/x^2*a*b*C-1/6*
a^2*A/x^6-1/5*a^2*B/x^5-2/3*a*b*B/x^3+2*A*ln(x)*b*c+2*C*ln(x)*a*c+C*ln(x)*b
^2
```

Maxima [A] time = 0.952514, size = 189, normalized size = 1.27

$$\frac{1}{4}\,Cc^2x^4+\frac{1}{3}\,Bc^2x^3+2\,Bbcx+\frac{1}{2}\left(2\,Cbc+Ac^2\right)x^2+\left(Cb^2+2\,(Ca+Ab)c\right)\log\left(x\right)-\frac{40\,Babx^3+60\left(Bb^2+2\,Bac\right)x^5+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*C*c^2*x^4 + 1/3*B*c^2*x^3 + 2*B*b*c*x + 1/2*(2*C*b*c + A*c^2)*x^2 + (C*
b^2 + 2*(C*a + A*b)*c)*log(x) - 1/60*(40*B*a*b*x^3 + 60*(B*b^2 + 2*B*a*c)*x
^5 + 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 12*B*a^2*x + 10*A*a^2 + 15*(C*a^2
 + 2*A*a*b)*x^2)/x^6
```

Fricas [A] time = 1.21679, size = 346, normalized size = 2.32

$$\frac{15\,Cc^2x^{10}+20\,Bc^2x^9+120\,Bbcx^7+30\left(2\,Cbc+Ac^2\right)x^8+60\left(Cb^2+2\,(Ca+Ab)c\right)x^6\log\left(x\right)-40\,Babx^3-60\left(Bb^2+\cdots\right)}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(15*C*c^2*x^10 + 20*B*c^2*x^9 + 120*B*b*c*x^7 + 30*(2*C*b*c + A*c^2)*x
^8 + 60*(C*b^2 + 2*(C*a + A*b)*c)*x^6*log(x) - 40*B*a*b*x^3 - 60*(B*b^2 + 2
*B*a*c)*x^5 - 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 12*B*a^2*x - 10*A*a^2 -
15*(C*a^2 + 2*A*a*b)*x^2)/x^6
```

Sympy [A] time = 29.7179, size = 153, normalized size = 1.03

$$2Bbcx+\frac{Bc^2x^3}{3}+\frac{Cc^2x^4}{4}+x^2\left(\frac{Ac^2}{2}+Cbc\right)+\left(2Abc+2Cac+Cb^2\right)\log\left(x\right)-\frac{10Aa^2+12Ba^2x+40Babx^3+x^5\left(120B\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**7,x)
```

```
[Out] 2*B*b*c*x + B*c**2*x**3/3 + C*c**2*x**4/4 + x**2*(A*c**2/2 + C*b*c) + (2*A*
b*c + 2*C*a*c + C*b**2)*log(x) - (10*A*a**2 + 12*B*a**2*x + 40*B*a*b*x**3 +
 x**5*(120*B*a*c + 60*B*b**2) + x**4*(60*A*a*c + 30*A*b**2 + 60*C*a*b) + x*
*2*(30*A*a*b + 15*C*a**2))/(60*x**6)
```

Giac [A] time = 1.12944, size = 190, normalized size = 1.28

$$\frac{1}{4}Cc^2x^4 + \frac{1}{3}Bc^2x^3 + Cbcx^2 + \frac{1}{2}Ac^2x^2 + 2Bbcx + \left(Cb^2 + 2Cac + 2Abc\right)\log\left(|x|\right) - \frac{40Babx^3 + 60\left(Bb^2 + 2Bac\right)x^5 + 30}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/4*C*c^2*x^4 + 1/3*B*c^2*x^3 + C*b*c*x^2 + 1/2*A*c^2*x^2 + 2*B*b*c*x + (C*
b^2 + 2*C*a*c + 2*A*b*c)*log(abs(x)) - 1/60*(40*B*a*b*x^3 + 60*(B*b^2 + 2*B
*a*c)*x^5 + 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 12*B*a^2*x + 10*A*a^2 + 15
*(C*a^2 + 2*A*a*b)*x^2)/x^6
```

3.21 $\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=339

$$-\frac{\left(-\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\mathrm{ta}}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] ((A*c - b*C)*x)/c^2 + (B*x^2)/(2*c) + (C*x^3)/(3*c) - ((A*b*c - b^2*C + a*c
*C - (A*c*(b^2 - 2*a*c) - b*(b^2 - 3*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) - ((A*b*c - b^2*C + a*c*C + (A*c*(b^2 - 2*a*c) - b*(b^2 -
3*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (B*(b^2 - 2*a*c)
*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - (b*B
*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] time = 1.85593, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1662, 1279, 1166, 205, 12, 1114, 703, 634, 618, 206, 628}

$$-\frac{\left(-\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\mathrm{ta}}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((A*c - b*C)*x)/c^2 + (B*x^2)/(2*c) + (C*x^3)/(3*c) - ((A*b*c - b^2*C + a*c
*C - (A*c*(b^2 - 2*a*c) - b*(b^2 - 3*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) - ((A*b*c - b^2*C + a*c*C + (A*c*(b^2 - 2*a*c) - b*(b^2 -
3*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (B*(b^2 - 2*a*c)
*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - (b*B
*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
```

```
0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx = \int \frac{Bx^5}{a+bx^2+cx^4}\,dx + \int \frac{x^4\left(A+Cx^2\right)}{a+bx^2+cx^4}\,dx$$

$$= \frac{Cx^3}{3c} + B\int \frac{x^5}{a+bx^2+cx^4}\,dx - \frac{\int \frac{x^2\left(3aC-3(Ac-bC)x^2\right)}{a+bx^2+cx^4}\,dx}{3c}$$

$$= \frac{(Ac-bC)x}{c^2} + \frac{Cx^3}{3c} + \frac{1}{2}B\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx, x, x^2\right) + \frac{\int \frac{-3a(Ac-bC)-3\left(Abc-b^2C+acC\right)x^2}{a+bx^2+cx^4}}{3c^2}$$

$$= \frac{(Ac-bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c} + \frac{B\,\mathrm{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx, x, x^2\right)}{2c} - \frac{\left(Abc-b^2C+acC-\frac{Ac\left(b^2-2\right.}{}\right.}{}$$

$$= \frac{(Ac-bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c} - \frac{\left(Abc-b^2C+acC-\frac{Ac\left(b^2-2ac\right)-b\left(b^2-3ac\right)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

$$= \frac{(Ac-bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c} - \frac{\left(Abc-b^2C+acC-\frac{Ac\left(b^2-2ac\right)-b\left(b^2-3ac\right)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

$$= \frac{(Ac-bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c} - \frac{\left(Abc-b^2C+acC-\frac{Ac\left(b^2-2ac\right)-b\left(b^2-3ac\right)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.643908, size = 460, normalized size = 1.36

$$\frac{\frac{6\sqrt{2}\left(Ac\left(-b\sqrt{b^2-4ac}-2ac+b^2\right)+C\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}+3abc-b^3\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{6\sqrt{2}\left(C\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}-3abc+b^3\right)-Ac\left(b\sqrt{b^2-4ac}-2a\right.\right.}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (12*Sqrt[c]*(A*c - b*C)*x + 6*B*c^(3/2)*x^2 + 4*c^(3/2)*C*x^3 + (6*Sqrt[2]*
(A*c*(b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c]) + (-b^3 + 3*a*b*c + b^2*Sqrt[b^2 -
 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (6*Sqrt
[2]*(-(A*c*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])) + (b^3 - 3*a*b*c + b^2*Sqrt
[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (
3*B*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c]
 - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - (3*B*Sqrt[c]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4
*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(12*c^(5/2)
)
```

Maple [B] time = 0.046, size = 1622, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 3/2/c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*a*b+3/2/c/(4*a*c
-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*a*b-1/c/(4*a*c-b^2)*B*ln(-2*c
*x^2+(-4*a*c+b^2)^(1/2)-b)*a*b+2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b
)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*a^2-2/(4
*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*C*a^2-1/2/c/(4*a*c-b^2)*B*ln(2*c*x^2+(-4*a*c+b
^2)^(1/2)+b)*(-4*a*c+b^2)^(1/2)*a+1/4/c^2/(4*a*c-b^2)*B*ln(2*c*x^2+(-4*a*c+
b^2)^(1/2)+b)*(-4*a*c+b^2)^(1/2)*b^2-1/c/(4*a*c-b^2)*B*ln(2*c*x^2+(-4*a*c+b
^2)^(1/2)+b)*a*b+1/2/c/(4*a*c-b^2)*B*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-4*
a*c+b^2)^(1/2)*a-1/4/c^2/(4*a*c-b^2)*B*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-
4*a*c+b^2)^(1/2)*b^2+2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*a*b-1/(4*a*c-b^2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*A*(-4*a*c+b^2)^(1/2)*a-2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
A*a*b-1/2/c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*
x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*b^3+1/2/c^2/(4*a*c-b^2)*2^(1/
2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2
)-b)*c)^(1/2))*b^4*C+1/2/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^3-1/2/c^2/(4*
a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*C-1/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2
)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*(-4*a
*c+b^2)^(1/2)*a+1/4/c^2/(4*a*c-b^2)*B*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b^3
+1/4/c^2/(4*a*c-b^2)*B*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^3+1/c*A*x-1/c^2*b
*C*x-1/2/c^2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c
*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b^3-5/2/c
/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(
((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*C*a+1/2/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*A*(-4*a*c+b^2)^(1/2)*b^2-1/2/c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*
c+b^2)^(1/2)*b^3+5/2/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*C*a+1/2*B*x^2/c+1
/3*C*x^3/c+1/2/c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arcta
nh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.22 $\int \frac{x^3\left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=278

$$\frac{\left(2acC+Abc+b^2(-C)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{(Ac-bC)\log\left(a+bx^2+cx^4\right)}{4c^2}-\frac{B\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\left(\frac{b^2}{\sqrt{b}}\cdots\right.}{\cdots}$$

```
[Out] (B*x)/c + (C*x^2)/(2*c) - (B*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) - (B*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt
[b^2 - 4*a*c]]) + ((A*b*c - b^2*C + 2*a*c*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) + ((A*c - b*C)*Log[a + b*x^2 + c*x^4]
)/(4*c^2)
```

Rubi [A] time = 0.465966, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1662, 1251, 773, 634, 618, 206, 628, 12, 1122, 1166, 205}

$$\frac{\left(2acC+Abc+b^2(-C)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{(Ac-bC)\log\left(a+bx^2+cx^4\right)}{4c^2}-\frac{B\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\left(\frac{b^2}{\sqrt{b}}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c + (C*x^2)/(2*c) - (B*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) - (B*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt
[b^2 - 4*a*c]]) + ((A*b*c - b^2*C + 2*a*c*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) + ((A*c - b*C)*Log[a + b*x^2 + c*x^4]
)/(4*c^2)
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^3 \left(A + Bx + Cx^2\right)}{a + bx^2 + cx^4} dx = \int \frac{Bx^4}{a + bx^2 + cx^4} dx + \int \frac{x^3 \left(A + Cx^2\right)}{a + bx^2 + cx^4} dx$$
$$= \frac{1}{2} \text{Subst}\left(\int \frac{x(A + Cx)}{a + bx + cx^2} dx, x, x^2\right) + B \int \frac{x^4}{a + bx^2 + cx^4} dx$$
$$= \frac{Bx}{c} + \frac{Cx^2}{2c} + \frac{\text{Subst}\left(\int \frac{-aC+(Ac-bC)x}{a+bx+cx^2} dx, x, x^2\right)}{2c} - \frac{B \int \frac{a+bx^2}{a+bx^2+cx^4} dx}{c}$$
$$= \frac{Bx}{c} + \frac{Cx^2}{2c} - \frac{\left(B\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx}{2c} - \frac{\left(B\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx}{2c} -$$
$$= \frac{Bx}{c} + \frac{Cx^2}{2c} - \frac{B\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{B\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{(Ac}{}$$
$$= \frac{Bx}{c} + \frac{Cx^2}{2c} - \frac{B\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{B\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{(A}{}$$

Mathematica [A] time = 0.45116, size = 377, normalized size = 1.36

$$\frac{\frac{\left(Ac\left(\sqrt{b^2-4ac}-b\right)+C\left(-b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\sqrt{b^2-4ac}} - \frac{\left(C\left(b\sqrt{b^2-4ac}-2ac+b^2\right)-Ac\left(\sqrt{b^2-4ac}+b\right)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}} - \frac{2\sqrt{2}B\sqrt{c}\left(b\sqrt{b^2-4a}\right.}{\sqrt{b^2-}}}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*B*c*x + 2*c*C*x^2 - (2*Sqrt[2]*B*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*B*Sqrt[c]*(b^2 - 2*a*c + b
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*c*(-b + Sqrt[b^2 -
4*a*c]) + (b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c]
 - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - ((-(A*c*(b + Sqrt[b^2 - 4*a*c])) + (b^2 -
2*a*c + b*Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[
b^2 - 4*a*c])/(4*c^2)
```

Maple [B] time = 0.033, size = 1171, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*C*x^2/c+B*x/c+1/4/c/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*(-4
*a*c+b^2)^(1/2)*b+1/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*a-1/4/c
/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*b^2+1/2/c/(4*a*c-b^2)*ln(-
2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*C*(-4*a*c+b^2)^(1/2)*a-1/4/c^2/(4*a*c-b^2)*ln
```

```
(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*C*(-4*a*c+b^2)^(1/2)*b^2-1/c/(4*a*c-b^2)*ln
(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*C*a*b+1/4/c^2/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a
*c+b^2)^(1/2)-b)*C*b^3-1/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/
2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*B*(-4*a*c+b^2)^(1/
2)*a+1/2/c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*B*(-4*a*c+b^2)^(1/2)*b^2+2/(4*a*
c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a
*c+b^2)^(1/2)-b)*c)^(1/2))*B*a*b-1/2/c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*B*b^
3-1/4/c/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*(-4*a*c+b^2)^(1/2)*b
+1/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*a-1/4/c/(4*a*c-b^2)*ln(2*
c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*b^2-1/2/c/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)
^(1/2)+b)*C*(-4*a*c+b^2)^(1/2)*a+1/4/c^2/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2
)^(1/2)+b)*C*(-4*a*c+b^2)^(1/2)*b^2-1/c/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)
^(1/2)+b)*C*a*b+1/4/c^2/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*C*b^3-
1/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*(-4*a*c+b^2)^(1/2)*a+1/2/c/(4*a*c-b^2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*B*(-4*a*c+b^2)^(1/2)*b^2-2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*B*a*b+1/2/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Cx^2 + 2\,Bx}{2\,c} + \frac{-\int \frac{Bbx^2+(Cb-Ac)x^3+Cax+Ba}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(C*x^2 + 2*B*x)/c + integrate(-(B*b*x^2 + (C*b - A*c)*x^3 + C*a*x + B*a
)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.23 $\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=270

$$\frac{\left(-\frac{Abc-C(b^2-2ac)}{\sqrt{b^2-4ac}}+Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2acC+Abc+b^2(-C)}{\sqrt{b^2-4ac}}+Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{bB\tanh^{-1}\left(\frac{b+2}{\sqrt{b^2-}}\right.}{2c\sqrt{b^2-4ac}}$$

```
[Out] (C*x)/c + ((A*c - b*C - (A*b*c - (b^2 - 2*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) + ((A*c - b*C + (A*b*c - b^2*C + 2*a*c*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rubi [A] time = 0.834809, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1662, 1279, 1166, 205, 12, 1114, 634, 618, 206, 628}

$$\frac{\left(-\frac{Abc-C(b^2-2ac)}{\sqrt{b^2-4ac}}+Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2acC+Abc+b^2(-C)}{\sqrt{b^2-4ac}}+Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{bB\tanh^{-1}\left(\frac{b+2}{\sqrt{b^2-}}\right.}{2c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (C*x)/c + ((A*c - b*C - (A*b*c - (b^2 - 2*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) + ((A*c - b*C + (A*b*c - b^2*C + 2*a*c*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx = \int \frac{Bx^3}{a+bx^2+cx^4}\,dx + \int \frac{x^2\left(A+Cx^2\right)}{a+bx^2+cx^4}\,dx$$

$$= \frac{Cx}{c} + B\int \frac{x^3}{a+bx^2+cx^4}\,dx - \frac{\int \frac{aC+(-Ac+bC)x^2}{a+bx^2+cx^4}\,dx}{c}$$

$$= \frac{Cx}{c} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, x^2\right) - \frac{\left(-Ac+bC+\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx}}{2c}$$

$$= \frac{Cx}{c} + \frac{\left(Ac-bC-\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(Ac-bC+\frac{Abc-(b^2-2ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\right.}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

$$= \frac{Cx}{c} + \frac{\left(Ac-bC-\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(Ac-bC+\frac{Abc-(b^2-2ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\right.}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

$$= \frac{Cx}{c} + \frac{\left(Ac-bC-\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(Ac-bC+\frac{Abc-(b^2-2ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\right.}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.396241, size = 360, normalized size = 1.33

$$\frac{-\frac{2\sqrt{2}\left(Ac\left(b-\sqrt{b^2-4ac}\right)+C\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-2ac+b^2\right)-Ac\left(\sqrt{b^2-4ac}+b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{B\sqrt{c}\left(\sqrt{b^2-\right.}{}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*Sqrt[c]*C*x - (2*Sqrt[2]*(A*c*(b - Sqrt[b^2 - 4*a*c]) + (-b^2 + 2*a*c +
b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*(-(A*c*(
b + Sqrt[b^2 - 4*a*c])) + (b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
Sqrt[b^2 - 4*a*c]]) + (B*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2
 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (B*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c]
)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*c^(3/2))
```

Maple [B] time = 0.036, size = 1327, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] C*x/c+1/4/c/(4*a*c-b^2)*B*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b*(-4*a*c+b^2)^
(1/2)+1/(4*a*c-b^2)*B*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*a-1/4/c/(4*a*c-b^2)
*B*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b^2-1/2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+
```

```
b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
)*A*b*(-4*a*c+b^2)^(1/2)-2*c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)
^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*a+1/2/(4*a*c
-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*
c+b^2)^(1/2)-b)*c)^(1/2))*A*b^2-1/4/c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1
/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*(-4
*a*c+b^2)*b-1/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(
c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*a+1/2/c/
(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((
(-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b^2+1/(4*a*c-b^2)*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/
2)-b)*c)^(1/2))*b*C*a-1/4/c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^
(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*C-1/4/c/(4*
a*c-b^2)*B*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b*(-4*a*c+b^2)^(1/2)+1/(4*a*c-b
^2)*B*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*a-1/4/c/(4*a*c-b^2)*B*ln(2*c*x^2+(-4
*a*c+b^2)^(1/2)+b)*b^2-1/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b*(-4*a*c+b^2)^
(1/2)+2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*a-1/2/(4*a*c-b^2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*A*b^2+1/4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)*b-1/(4*a*c-b
^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*a+1/2/c/(4*a*c-b^2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*C*(-4*a*c+b^2)^(1/2)*b^2-1/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*C*a+1/4
/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*C
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.24 $\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4} dx$

Optimal. Leaf size=223

$$-\frac{(2Ac-bC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}-\frac{B\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{B\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{C\log}{}$$

```
[Out] -((B*Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (B*Sqrt[b + Sqrt[b^2
- 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*Sqrt[c]*Sqrt[b^2 - 4*a*c]) - ((2*A*c - b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (C*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rubi [A] time = 0.212854, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1662, 1247, 634, 618, 206, 628, 12, 1130, 205}

$$-\frac{(2Ac-bC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}-\frac{B\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{B\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{C\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((B*Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (B*Sqrt[b + Sqrt[b^2
- 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*Sqrt[c]*Sqrt[b^2 - 4*a*c]) - ((2*A*c - b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (C*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x\left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx = \int \frac{Bx^2}{a+bx^2+cx^4}\,dx + \int \frac{x\left(A+Cx^2\right)}{a+bx^2+cx^4}\,dx$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Cx}{a+bx+cx^2}\,dx, x, x^2\right) + B\int \frac{x^2}{a+bx^2+cx^4}\,dx$$

$$= \frac{1}{2}\left(B\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx + \frac{1}{2}\left(B\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\cdots}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2\cdots}}$$

$$= -\frac{B\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{B\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{C\log(\cdots}{\cdots}$$

$$= -\frac{B\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{B\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{(2Ac-\cdots}{\cdots}$$

Mathematica [A] time = 0.396297, size = 240, normalized size = 1.08

$$\frac{\left(C\left(\sqrt{b^2-4ac}-b\right)+2Ac\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)-\left(2Ac-C\left(\sqrt{b^2-4ac}+b\right)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)-2\sqrt{2}B}{4c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-2*Sqrt[2]*B*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]] + 2*Sqrt[2]*B*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*
a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]] + (2*A*c + (-
b + Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - (2*A*c -
(b + Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*c*Sqrt[
b^2 - 4*a*c])
```

Maple [B] time = 0.023, size = 728, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*(-4*a*c+b^2)^(1/2)+1/4
/c/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*C*(-4*a*c+b^2)^(1/2)*b+1/(
4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*a*C-1/4/c/(4*a*c-b^2)*ln(-2*c*
x^2+(-4*a*c+b^2)^(1/2)-b)*b^2*C-1/2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2
)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*B*(-4*a
*c+b^2)^(1/2)*b-2*c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*ar
ctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*B*a+1/2/(4*a*c-b^2)*2^(
1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1
/2)-b)*c)^(1/2))*B*b^2+1/2/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*(
-4*a*c+b^2)^(1/2)-1/4/c/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*C*(-4*
a*c+b^2)^(1/2)*b+1/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*a*C-1/4/c/(
4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^2*C-1/2/(4*a*c-b^2)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*B*(-4*a*c+b^2)^(1/2)*b+2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*a-1/2
/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2+Bx+A\right)x}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*x/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 3.23748, size = 7461, normalized size = 33.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 2*(3*(a*c^3)^(3/4)*B*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5
/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(3/4)*
B*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/
2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*(a*c^3)^(3/4)*B*cos(
5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c))))) + 3*(a*c^3)^(3/4)*B*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^3
)^(3/4)*B*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*r
eal_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*B*cosh(1/2*imag_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^2 - 3*(a*c^3)^(3/4)*B*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(3/
4)*B*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh
(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - sqrt(a*c)*C*b*c*cos
(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcs
```

```
in(1/2*sqrt(a*c)*b/(a*abs(c))))) - 2*sqrt(a*c)*A*c^2*cos(5/4*pi + 1/2*real_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c))))) - 2*sqrt(a*c)*C*b*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - 4*sqrt(a*c)*A*c^2*cos(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c))))) - sqrt(a*c)*C*b*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + 2*sqrt(a*c
)*A*c^2*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin
(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2)*arctan(-((a/c)^(1/4)*cos(5/4*p
i + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4*pi +
1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*b*c*abs(c) - (
b^2 - 4*a*c)*c^2) + 2*(3*(a*c^3)^(3/4)*B*cos(1/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
))) - (a*c^3)^(3/4)*B*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*
(a*c^3)^(3/4)*B*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi
 + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*B*cosh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c))))) + 9*(a*c^3)^(3/4)*B*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*
imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*B*cosh(1
/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*B*cos(1/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))^3 + (a*c^3)^(3/4)*B*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 -
 sqrt(a*c)*C*b*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi
+ 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - 2*sqrt(a*c)*A*c^2*co
s(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c))))) - 2*sqrt(a*c)*C*b*c*cos(1/4*pi + 1/2*real
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - 4*sq
rt(a*c)*A*c^2*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c))))) + sqrt(a*c)*C*b*c*cos(1/4*pi + 1/2*real_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^2 + 2*sqrt(a*c)*A*c^2*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2)*arctan(-((
a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)
```

```
^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4
*a*c)*b*c*abs(c) - (b^2 - 4*a*c)*c^2) - 1/2*(2*(a*c^3)^(3/4)*B*cos(5/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 6*(a*c^3)^(3/4)*B*cos(5/4*pi + 1/2*r
eal_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^2 - 6*(a*c^3)^(3/4)*B*cos(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 18*
(a*c^3)^(3/4)*B*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 6*(a*c^3)^(3/4)*B*cos(5/4*pi + 1/2*rea
l_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))^2 - 18*(a*c^3)^(3/4)*B*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 2*(a*c^3)^(3/4)*B*cos(5
/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 6*(a*c^3)^(3/4)*B*cos(5/4*pi
+ 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^3 + sqrt(a*c)*C*b*c*cos(5/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))^2 + 2*sqrt(a*c)*A*c^2*cos(5/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2 - sqrt(a*c)*C*b*c*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))^2 - 2*sqrt(a*c)*A*c^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 +
 2*sqrt(a*c)*C*b*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*
imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - 4*sqrt(a*c)*A*c^2*cos(5/4*
pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c))))) + 2*sqrt(a*c)*C*b*c*cosh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 4*sqr
t(a*c)*A*c^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + sqrt(a*c)*C*b*c*cos(5/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 2*sqrt(a*c)*A*c^2*cos(5/4*pi + 1/2*real_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^2 - sqrt(a*c)*C*b*c*sin(5/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))^2 + 2*sqrt(a*c)*A*c^2*sin(5/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2)*log(-2*x*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*b*c*abs(c) - (b^2 - 4*a*
c)*c^2) - 1/2*(2*(a*c^3)^(3/4)*B*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))^3 - 6*(a*c^3)^(3/4)*B*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*s
in(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 6*(a*c^3
)^(3/4)*B*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3
*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 18*(a*c^3)^(3/4)*B*cos(1/4*pi + 1
```

```
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
)))) + 6*(a*c^3)^(3/4)*B*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 18*(a*c^3)^(3/4)*B
*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^2 - 2*(a*c^3)^(3/4)*B*cos(1/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^3 + 6*(a*c^3)^(3/4)*B*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + s
qrt(a*c)*C*b*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + 2*sqrt(a*
c)*A*c^2*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*
cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - sqrt(a*c)*C*b*c
*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 2*sqrt(a*c)*A*c^2*cosh(
1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + 2*sqrt(a*c)*C*b*c*cos(1/4*pi
+ 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c))))) - 4*sqrt(a*c)*A*c^2*cos(1/4*pi + 1/2*real_part(arcsin(1/2*
sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 2*sqrt(a
*c)*C*b*c*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - 4*sqrt(a*c)*A*c^2*cosh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c))))) - sqrt(a*c)*C*b*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
^2 - 2*sqrt(a*c)*A*c^2*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - s
qrt(a*c)*C*b*c*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + 2*sqrt(a*
c)*A*c^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*
sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2)*log(-2*x*(a/c)^(
1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c)
)/(sqrt(b^2 - 4*a*c)*b*c*abs(c) - (b^2 - 4*a*c)*c^2) + 1/4*C*log(abs(c*x^4
+ b*x^2 + a))/c
```

3.25 $\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=211

$$\frac{\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] ((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((C -
(2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (B*ArcTanh[(
b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.266254, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1673, 1166, 205, 12, 1107, 618, 206}

$$\frac{\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((C -
(2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (B*ArcTanh[(
b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx &= \int \frac{Bx}{a+bx^2+cx^4}\,dx+\int \frac{A+Cx^2}{a+bx^2+cx^4}\,dx\\ &= B\int \frac{x}{a+bx^2+cx^4}\,dx+\frac{1}{2}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx+\frac{1}{2}\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int\\ &= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}+\frac{1}{2}B\,\mathrm{Subst}\left(\int \frac{1}{a+bx+}\right.\\ &= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}-B\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-}\right.\\ &= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}-\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] time = 0.222659, size = 234, normalized size = 1.11

$$\frac{\frac{\sqrt{2}\left(C\left(\sqrt{b^2-4ac}-b\right)+2Ac\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(C\left(\sqrt{b^2-4ac}+b\right)-2Ac\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}+B\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)-B\log\left(\sqrt{b^2-\right.}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*A*c + (-b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[
2]*(-2*A*c + (b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + B*Log[-b + Sq
```

```
rt[b^2 - 4*a*c] - 2*c*x^2] - B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(2*Sqr
t[b^2 - 4*a*c])
```

Maple [B] time = 0.017, size = 616, normalized size = 2.9

$$-\frac{B}{8\,ac-2\,b^2}\sqrt{-4\,ac+b^2}\ln\left(-2\,cx^2+\sqrt{-4\,ac+b^2}-b\right)+\frac{c\sqrt{2}A}{4\,ac-b^2}\sqrt{-4\,ac+b^2}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(\sqrt{-4\,ac+b^2}-b\right)c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*B*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+c*(
-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arct
anh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A-2*c/(4*a*c-b^2)*2^(1/2)
/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b)*c)^(1/2))*C*a+1/2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*a
rctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*b^2-1/2*(-4*a*c+b^2)
^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(
1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*C+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^
2)*B*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*A+2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*a-1/2/(4*a*c-b^2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*C*b^2-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*b*C
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Cx^2+Bx+A}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 2.94584, size = 9080, normalized size = 43.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
4*a*c)*b)*C*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 - 4*
(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cosh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c
 + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cos(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/
4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cosh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^
3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 -
 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cosh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)
*sqrt(b^2 - 4*a*c)*b)*C*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3
/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 2*(sqrt(a*c)*b^2*c^2 + 4*sqrt(
a*c)*a*c^3 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*B*cos(5/4*pi + 1/2*real_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c))))) - 4*(sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*s
qrt(a*c)*b*c^2)*B*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c))))) - 2*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3
```

```
 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*B*cos(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
 + ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2
- 4*a*c)*b*c^2)*A*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*s
in(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(
1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^
2)*A*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1
/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos
(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4
*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(a*b^2*c^3 - 4*a^2*c^4) + 1
/2*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4
*a*c)*b)*C*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/
2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 - 4*(
a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cosh(1/2*imag_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c
+ (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cos(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4
)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cosh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3
)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 -
4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cosh(1/2*imag_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*
sqrt(b^2 - 4*a*c)*b)*C*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3/
4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*sin(1/4
*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 2*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a
*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*B*cos(1/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c))))) + 4*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sq
rt(a*c)*b*c^2)*B*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi +
1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c))))) + 2*(sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)*a*c^3
+ sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*B*cos(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
+ ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 -
 4*a*c)*b*c^2)*A*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*si
n(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1
/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2
)*A*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(
1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(1/4*
pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(a*b^2*c^3 - 4*a^2*c^4) - 1/
```

```
4*(((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*
c)*b)*C*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*c
osh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*((a*c^3)^(3/4)
*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cos(5/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c +
 (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cos(5/4*pi + 1/2*real_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a
*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*
C*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*
imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)
^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*((a*c^3)^(3
/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cos(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2 - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqr
t(b^2 - 4*a*c)*b)*C*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*((
a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)
*C*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*
pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - (sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)
*a*c^3 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*B*cos(5/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^2 - (sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4
*a*c)*sqrt(a*c)*b*c^2)*B*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 -
 2*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c
^2)*B*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cos
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 2*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^
3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*B*cosh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + (sqr
t(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*B*c
os(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + (sqrt(a*c)*b^2*c^2 + 4*sq
rt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*B*sin(5/4*pi + 1/2*real_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^2 + ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3
 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*A*cos(5/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c))))) - ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1
/4)*sqrt(b^2 - 4*a*c)*b*c^2)*A*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
)*log(-2*x*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
 + x^2 + sqrt(a/c))/(a*b^2*c^3 - 4*a^2*c^4) - 1/4*(((a*c^3)^(3/4)*b^2 - 4*(
a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cos(1/4*pi + 1/2*re
al_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c
+ (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cos(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
```

```
s(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4
*a*c)*b)*C*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)
^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cos(1/4*pi + 1/2*real_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((
a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)
*C*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1
/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a
*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cos(1/4*pi + 1/2*real_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - ((a*c^3)^(3/
4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cos(1/4
*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3
)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*C*cos(1/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^3 - (sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*
sqrt(a*c)*b*c^2)*B*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - (sqrt
(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*B*co
sh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*re
al_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 2*(sqrt(a*c)*b^2*c^2 - 4*s
qrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*B*cos(1/4*pi + 1/2*real
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*
sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c))))) - 2*(sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(
a*c)*b*c^2)*B*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1
/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c
)*a*c^3 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*B*cos(1/4*pi + 1/2*real_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^2 - (sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)*a*c^3 + sqrt(b^2 -
4*a*c)*sqrt(a*c)*b*c^2)*B*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
+ ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 -
 4*a*c)*b*c^2)*A*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1
/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2
)*A*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(
1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*b^2*
c^3 - 4*a^2*c^4)
```

3.26 $\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=229

$$\frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}+\frac{A\log(x)}{a}+\frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (Sqrt[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]) + ((A*b - 2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + (A*Log[x])/a - (A*Log[a + b*x^2 + c*x^4]
)/(4*a)
```

Rubi [A] time = 0.259478, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1662, 1251, 800, 634, 618, 206, 628, 12, 1093, 205}

$$\frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}+\frac{A\log(x)}{a}+\frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]) + ((A*b - 2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + (A*Log[x])/a - (A*Log[a + b*x^2 + c*x^4]
)/(4*a)
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
```

```
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}dx = \int \frac{B}{a+bx^2+cx^4}dx + \int \frac{A+Cx^2}{x\left(a+bx^2+cx^4\right)}dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{A+Cx}{x\left(a+bx+cx^2\right)}dx, x, x^2\right) + B\int \frac{1}{a+bx^2+cx^4}dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax} + \frac{-Ab+aC-Acx}{a\left(a+bx+cx^2\right)}\right)dx, x, x^2\right) + \frac{(Bc)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}dx}{\sqrt{b^2-4ac}} - \frac{(Bc)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4}}}{\sqrt{b^2-4a}}$$
$$= \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{A\log(x)}{a} + \frac{\text{Subst}\left(\int \frac{-Ab+aC-Acx}{a+bx+cx^2}\right.}{2a}$$
$$= \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{A\log(x)}{a} - \frac{A\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\right.}{4a}$$
$$= \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{A\log(x)}{a} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a}$$
$$= \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} + $$

Mathematica [A] time = 0.468039, size = 285, normalized size = 1.24

$$-\frac{\left(A\left(\sqrt{b^2-4ac}+b\right)-2aC\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{4a\sqrt{b^2-4ac}} - \frac{\left(A\left(\sqrt{b^2-4ac}-b\right)+2aC\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{4a\sqrt{b^2-4ac}} + \frac{A\,\text{l}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]) + (A*Log[x])/a - ((A*(b + Sqrt[b^2 - 4*a*c]) - 2*a
*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c]) - ((A*(-
b + Sqrt[b^2 - 4*a*c]) + 2*a*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*
Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.027, size = 488, normalized size = 2.1

$$\frac{A\ln(x)}{a} - 4\frac{c\ln\left(-2cx^2+\sqrt{-4ac+b^2}-b\right)A}{16ac-4b^2} + \frac{Ab^2}{a\left(16ac-4b^2\right)}\ln\left(-2cx^2+\sqrt{-4ac+b^2}-b\right) + \frac{Ab}{a\left(16ac-4b^2\right)}\sqrt{-4ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] A*ln(x)/a-4*c/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A+1/a/(16*a*
c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*b^2+1/a*(-4*a*c+b^2)^(1/2)/(16
*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*b-2*(-4*a*c+b^2)^(1/2)/(16*
a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*C+4*c*(-4*a*c+b^2)^(1/2)/(16*a
*c-4*b^2)*B*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((
(-4*a*c+b^2)^(1/2)-b)*c)^(1/2))-4*c/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^
(1/2)+b)*A+1/a/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*b^2-1/a*(-
4*a*c+b^2)^(1/2)/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*b+2*(-4*
a*c+b^2)^(1/2)/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*C+4*c*(-4*a*
c+b^2)^(1/2)/(16*a*c-4*b^2)*B*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] A*log(x)/a - integrate((A*c*x^3 - B*a - (C*a - A*b)*x)/(c*x^4 + b*x^2 + a),
 x)/a
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 2.44713, size = 4740, normalized size = 20.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 2*(2*sqrt(a*c)*C*a*c^2*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5
/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + sqrt(a*c)*A*b*
c^2*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 4*sqrt(a*c)*C*a*c^2*cos(5/4*pi +
1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
) + 2*sqrt(a*c)*A*b*c^2*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 2*sqrt(a*c)*C*a*c^2*cos(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2 - sqrt(a*c)*A*b*c^2*cos(5/4*pi + 1/2*real_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
^2 + (a*c^3)^(1/4)*B*a*c^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (
a*c^3)^(1/4)*B*a*c^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-
((a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/
c)^(1/4)*sin(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 -
 4*a*c)*a*b*c*abs(c) - (a*b^2 - 4*a^2*c)*c^2) + 2*(2*sqrt(a*c)*C*a*c^2*cos(
1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c))))) + sqrt(a*c)*A*b*c^2*cos(1/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c))))) + 4*sqrt(a*c)*C*a*c^2*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1
/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 2*sqrt(a*c)*A*b*c^2*cos
(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c))))) - 2*sqrt(a*c)*C*a*c^2*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - sqr
t(a*c)*A*b*c^2*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + (a*c^3)^(1/4)*B*a*c^2*
cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*re
al_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*B*a*c^2*sin(1/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(1/4*pi + 1
/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(1/4*pi + 1/2*a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*a*b*c*abs(c) - (a*b
^2 - 4*a^2*c)*c^2) + 1/2*(2*sqrt(a*c)*C*a*c^2*cos(5/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^2 + sqrt(a*c)*A*b*c^2*cos(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^2 - 2*sqrt(a*c)*C*a*c^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))^2 - sqrt(a*c)*A*b*c^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
```

```
)^2 + 4*sqrt(a*c)*C*a*c^2*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*si
nh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - 2*sqrt(a*c)*A*b*c^2
*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c))))) + 4*sqrt(a*c)*C*a*c^2*cosh(1/2*imag_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
))) + 2*sqrt(a*c)*A*b*c^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*si
nh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 2*sqrt(a*c)*C*a*c^2
*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - sqrt(a*c)*A*b*c^2*cos(5
/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 2*sqrt(a*c)*C*a*c^2*sin(5/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + sqrt(a*c)*A*b*c^2*sin(5/4*pi + 1/
2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 2*(a*c^3)^(1/4)*B*a*c^2*cos(5/4*pi + 1/
2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c))))) + 2*(a*c^3)^(1/4)*B*a*c^2*cos(5/4*pi + 1/2*re
al_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*
sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*a*b*c*abs(c
) - (a*b^2 - 4*a^2*c)*c^2) + 1/2*(2*sqrt(a*c)*C*a*c^2*cos(1/4*pi + 1/2*real
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*
sqrt(a*c)*b/(a*abs(c)))))^2 + sqrt(a*c)*A*b*c^2*cos(1/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2 - 2*sqrt(a*c)*C*a*c^2*cosh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^2 - sqrt(a*c)*A*b*c^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^2 + 4*sqrt(a*c)*C*a*c^2*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - 2*sqrt(a*c)
*A*b*c^2*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*
cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 4*sqrt(a*c)*C*a*c^2*cosh(1/2*imag_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c))))) - 2*sqrt(a*c)*A*b*c^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - 2*sqrt(a*c)
*C*a*c^2*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*
sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - sqrt(a*c)*A*b*c
^2*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1
/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 2*sqrt(a*c)*C*a*c^2*s
in(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + sqrt(a*c)*A*b*c^2*sin(1/4
*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 2*(a*c^3)^(1/4)*B*a*c^2*cos(1/4
*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 2*(a*c^3)^(1/4)*B*a*c^2*cos(1/4*pi
+ 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(1/4*pi + 1/2*ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*a*b
*c*abs(c) - (a*b^2 - 4*a^2*c)*c^2) - 1/4*A*log(abs(c*x^4 + b*x^2 + a))/a +
A*log(abs(x))/a
```

3.27 $\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=260

$$-\frac{\sqrt{c}\left(\frac{Ab-2aC}{\sqrt{b^2-4ac}}+A\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{A}{ax}+\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{B\log\left(a+\right.}{4}$$

```
[Out] -(A/(a*x)) - (Sqrt[c]*(A + (A*b - 2*a*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) - (Sqrt[c]*(A - (A*b - 2*a*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]
]) + (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c])
 + (B*Log[x])/a - (B*Log[a + b*x^2 + c*x^4])/(4*a)
```

Rubi [A] time = 0.471352, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1662, 1281, 1166, 205, 12, 1114, 705, 29, 634, 618, 206, 628}

$$-\frac{\sqrt{c}\left(\frac{Ab-2aC}{\sqrt{b^2-4ac}}+A\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{A}{ax}+\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{B\log\left(a+\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(A/(a*x)) - (Sqrt[c]*(A + (A*b - 2*a*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) - (Sqrt[c]*(A - (A*b - 2*a*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]
]) + (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c])
 + (B*Log[x])/a - (B*Log[a + b*x^2 + c*x^4])/(4*a)
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx=\int \frac{B}{x\left(a+bx^2+cx^4\right)}\,dx+\int \frac{A+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

$$=-\frac{A}{ax}-\frac{\int \frac{Ab-aC+Acx^2}{a+bx^2+cx^4}\,dx}{a}+B\int \frac{1}{x\left(a+bx^2+cx^4\right)}\,dx$$

$$=-\frac{A}{ax}+\frac{1}{2}B\,\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx,x,x^2\right)-\frac{\left(c\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}-\,-\,(c$$

$$=-\frac{A}{ax}-\frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}+\frac{B\,\text{Sub}}{}$$

$$=-\frac{A}{ax}-\frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}+\frac{B\log(}{a}$$

$$=-\frac{A}{ax}-\frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}+\frac{B\log(}{a}$$

$$=-\frac{A}{ax}-\frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}+\frac{bB\tan}{2a}$$

Mathematica [A] time = 1.1354, size = 315, normalized size = 1.21

$$-\frac{\frac{2\sqrt{2}\sqrt{c}\left(A\left(\sqrt{b^2-4ac}+b\right)-2aC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{2\sqrt{2}\sqrt{c}\left(A\left(\sqrt{b^2-4ac}-b\right)+2aC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{B\left(\sqrt{b^2-4ac}+b\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{B(\sqrt{}}{}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((4*A)/x + (2*Sqrt[2]*Sqrt[c]*(A*(b + Sqrt[b^2 - 4*a*c]) - 2*a*C)*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) + (2*Sqrt[2]*Sqrt[c]*(A*(-b + Sqrt[b^2 - 4*a*c]) + 2*
a*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4
*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - 4*B*Log[x] + (B*(b + Sqrt[b^2 - 4*a*c]
)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (B*(-b + Sqrt[
b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a
)
```

Maple [B] time = 0.027, size = 811, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -A/a/x+B*ln(x)/a+1/a/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*B*(-4
*a*c+b^2)^(1/2)*b-4*c/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*B+1/
a/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*B*b^2-2/a*c/(16*a*c-4*b^
2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b
^2)^(1/2)-b)*c)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b+8*c^2/(16*a*c-4*b^2)*2^(1/2)/
(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b
)*c)^(1/2))*A-2/a*c/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*b^2+4*c/(16*a*c-4*
b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c
+b^2)^(1/2)-b)*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)-1/a/(16*a*c-4*b^2)*ln(2*c*x^2
+(-4*a*c+b^2)^(1/2)+b)*B*(-4*a*c+b^2)^(1/2)*b-4*c/(16*a*c-4*b^2)*ln(2*c*x^2
+(-4*a*c+b^2)^(1/2)+b)*B+1/a/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b
)*B*b^2-2/a*c/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b-8*c^
2/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A+2/a*c/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*A*b^2+4*c/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] B*log(x)/a - integrate((B*c*x^3 + A*c*x^2 + B*b*x - C*a + A*b)/(c*x^4 + b*x
^2 + a), x)/a - A/(a*x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**2/(c*x**4+b*x**2+a),x)
```

[Out] Timed out

Giac [C] time = 3.03701, size = 7089, normalized size = 27.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")

```
[Out] -2*(3*(a*c^3)^(3/4)*A*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*si
n(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(3/
4)*A*c*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi
 + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*(a*c^3)^(3/4)*A
*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1
/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*A*c*cosh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) +
 9*(a*c^3)^(3/4)*A*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4
*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*A*c*cosh(1/2*imag
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*A*c*cos(5/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
3 + (a*c^3)^(3/4)*A*c*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + sq
rt(a*c)*B*b*c^2*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 2*sqrt(a*c)*B*b*c^2*c
os(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*ima
g_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c))))) - sqrt(a*c)*B*b*c^2*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - (a*
c^3)^(1/4)*C*a*c^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*
sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^(
1/4)*A*b*c^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^(1/4)*C
*a*c^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh
(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*A*b*c^2
*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(5/4
*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4*pi
+ 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*a*b*c*abs(c)
 - (a*b^2 - 4*a^2*c)*c^2) - 2*(3*(a*c^3)^(3/4)*A*c*cos(1/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c))))) - (a*c^3)^(3/4)*A*c*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^3 - 9*(a*c^3)^(3/4)*A*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
```

```
c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*A*c*cosh
(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*
sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^3)^(3/4)*A*c*cos(1/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c
^3)^(3/4)*A*c*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1
/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*A*c*cos(1/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*r
eal_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(3/4)*A*c*sin(1/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))^3 - sqrt(a*c)*B*b*c^2*cos(1/4*pi + 1/2*real_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
))) - 2*sqrt(a*c)*B*b*c^2*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(
1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + sqrt(a*c)*B*b*c^2*cos(1/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2 - (a*c^3)^(1/4)*C*a*c^2*cosh(1/2*imag_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c))))) + (a*c^3)^(1/4)*A*b*c^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c))))) + (a*c^3)^(1/4)*C*a*c^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
))) - (a*c^3)^(1/4)*A*b*c^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*a
rctan(-((a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) -
x)/((a/c)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqr
t(b^2 - 4*a*c)*a*b*c*abs(c) - (a*b^2 - 4*a^2*c)*c^2) + 1/2*(2*(a*c^3)^(3/4)
*A*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh
(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 6*(a*c^3)^(3/4)*A*c
*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 6*(a*c^3)^(3/4)*A*c*cos(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c))))) + 18*(a*c^3)^(3/4)*A*c*cos(5/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 6*(a*c^3)^(3/4)
*A*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh
(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 18*(a*c^3)^(3/4)*A*c*cos(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
2 - 2*(a*c^3)^(3/4)*A*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 +
6*(a*c^3)^(3/4)*A*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*
sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - sqrt(a*c)*B*b*c
^2*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1
```

```
/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + sqrt(a*c)*B*b*c^2*cos
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*rea
l_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 2*sqrt(a*c)*B*b*c^2*cos(5/4
*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c))))) - 2*sqrt(a*c)*B*b*c^2*cosh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + sq
rt(a*c)*B*b*c^2*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + sqrt(a*c
)*B*b*c^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 2*(a*c^3)^(1/4
)*C*a*c^2*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*c
osh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 2*(a*c^3)^(1/4)*A*
b*c^2*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(
1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 2*(a*c^3)^(1/4)*C*a*c^
2*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*
imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - 2*(a*c^3)^(1/4)*A*b*c^2*co
s(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(5/4*pi
 + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4
*a*c)*a*b*c*abs(c) - (a*b^2 - 4*a^2*c)*c^2) + 1/2*(2*(a*c^3)^(3/4)*A*c*cos(
1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 6*(a*c^3)^(3/4)*A*c*cos(1/4*
pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2 - 6*(a*c^3)^(3/4)*A*c*cos(1/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c))))) + 18*(a*c^3)^(3/4)*A*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 6*(a*c^3)^(3/4)*A*c*cos(
1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^2 - 18*(a*c^3)^(3/4)*A*c*cos(1/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 2*(a*
c^3)^(3/4)*A*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 6*(a*c^3)
^(3/4)*A*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*
sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*
imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + sqrt(a*c)*B*b*c^2*cos(1/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - sqrt(a*c)*B*b*c^2*cosh(1/2*ima
g_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + 2*sqrt(a*c)*B*b*c^2*cos(1/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c))))) + 2*sqrt(a*c)*B*b*c^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - sqrt(a*c)*B
*b*c^2*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*si
nh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + sqrt(a*c)*B*b*c^2
*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 2*(a*c^3)^(1/4)*C*a*c^2
*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 2*(a*c^3)^(1/4)*A*b*c^2*cos
```

```
(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 2*(a*c^3)^(1/4)*C*a*c^2*cos(1/4
*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - 2*(a*c^3)^(1/4)*A*b*c^2*cos(1/4*pi
+ 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(1/4*pi + 1/2*ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*a*b
*c*abs(c) - (a*b^2 - 4*a^2*c)*c^2) - 1/4*B*log(abs(c*x^4 + b*x^2 + a))/a +
B*log(abs(x))/a - A/(a*x)
```

3.28 $\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=288

$$-\frac{\left(A\left(b^2-2ac\right)-abC\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(Ab-aC)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{A}{2ax^2}-\frac{B\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right.}{\sqrt{2}a\sqrt{b}}$$

```
[Out] -A/(2*a*x^2) - B/(a*x) - (B*Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 -
4*a*c]]) - (B*Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((A
*(b^2 - 2*a*c) - a*b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sq
rt[b^2 - 4*a*c]) - ((A*b - a*C)*Log[x])/a^2 + ((A*b - a*C)*Log[a + b*x^2 +
c*x^4])/(4*a^2)
```

Rubi [A] time = 0.473922, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1662, 1251, 800, 634, 618, 206, 628, 12, 1123, 1166, 205}

$$-\frac{\left(A\left(b^2-2ac\right)-abC\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(Ab-aC)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{A}{2ax^2}-\frac{B\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right.}{\sqrt{2}a\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -A/(2*a*x^2) - B/(a*x) - (B*Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 -
4*a*c]]) - (B*Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((A
*(b^2 - 2*a*c) - a*b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sq
rt[b^2 - 4*a*c]) - ((A*b - a*C)*Log[x])/a^2 + ((A*b - a*C)*Log[a + b*x^2 +
c*x^4])/(4*a^2)
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx=\int \frac{B}{x^2\left(a+bx^2+cx^4\right)}\,dx+\int \frac{A+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx$$

$$=\frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Cx}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^2\right)+B\int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

$$=-\frac{B}{ax}+\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{ax^2}+\frac{-Ab+aC}{a^2x}+\frac{A\left(b^2-ac\right)-abC+c(Ab-aC)x}{a^2\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)+\frac{B\int}{}$$

$$=-\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{\operatorname{Subst}\left(\int \frac{A\left(b^2-ac\right)-abC+c(Ab-aC)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^2}-\frac{\left(Bc\left(1-\frac{b}{\sqrt{b^2-4}}\right.\right.}{}$$

$$=-\frac{A}{2ax^2}-\frac{B}{ax}-\frac{B\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

$$=-\frac{A}{2ax^2}-\frac{B}{ax}-\frac{B\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

$$=-\frac{A}{2ax^2}-\frac{B}{ax}-\frac{B\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.983659, size = 377, normalized size = 1.31

$$\frac{\frac{\left(A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)-aC\left(\sqrt{b^2-4ac}+b\right)\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(A\left(b\sqrt{b^2-4ac}+2ac-b^2\right)+aC\left(b-\sqrt{b^2-4ac}\right)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+4\log(x)(aC-}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*A)/x^2 - (4*a*B)/x - (2*Sqrt[2]*a*B*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]
*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*a*B*Sqrt[c]*(-b + Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4
*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 4*(-(A*b) + a*C)*Log[x] + ((A*(b^2 - 2
*a*c + b*Sqrt[b^2 - 4*a*c]) - a*(b + Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^
2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((A*(-b^2 + 2*a*c + b*Sqrt[b^2 -
 4*a*c]) + a*(b - Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Maple [B] time = 0.037, size = 1054, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*A/a/x^2-B/a/x-1/a^2*ln(x)*A*b+1/a*ln(x)*C+8/a*c/(32*a*c-8*b^2)*ln(-2*c
*x^2+(-4*a*c+b^2)^(1/2)-b)*A*b-2/a^2/(32*a*c-8*b^2)*ln(-2*c*x^2+(-4*a*c+b^2
)^(1/2)-b)*A*b^3+4/a*c/(32*a*c-8*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-4
*a*c+b^2)^(1/2)*A-2/a^2/(32*a*c-8*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-
4*a*c+b^2)^(1/2)*A*b^2+2/a/(32*a*c-8*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)
*C*b*(-4*a*c+b^2)^(1/2)-8*c/(32*a*c-8*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b
)*C+2/a/(32*a*c-8*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*C*b^2-4/a*c/(32*a*
c-8*b^2)*B*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((
-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*(-4*a*c+b^2)^(1/2)+16*c^2/(32*a*c-8*b^2)*B
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2))-4/a*c/(32*a*c-8*b^2)*B*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)
*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2+8/a*c/(
32*a*c-8*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*b-2/a^2/(32*a*c-8*b^2)*ln(
2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*b^3-4/a*c/(32*a*c-8*b^2)*ln(2*c*x^2+(-4*a*c
+b^2)^(1/2)+b)*(-4*a*c+b^2)^(1/2)*A+2/a^2/(32*a*c-8*b^2)*ln(2*c*x^2+(-4*a*c
+b^2)^(1/2)+b)*(-4*a*c+b^2)^(1/2)*A*b^2-2/a/(32*a*c-8*b^2)*ln(2*c*x^2+(-4*a
*c+b^2)^(1/2)+b)*C*b*(-4*a*c+b^2)^(1/2)-8*c/(32*a*c-8*b^2)*ln(2*c*x^2+(-4*a
*c+b^2)^(1/2)+b)*C+2/a/(32*a*c-8*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*C*b^
2-4/a*c/(32*a*c-8*b^2)*B*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*(-4*a*c+b^2)^(1/2)-16*c^2/(32
*a*c-8*b^2)*B*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+4/a*c/(32*a*c-8*b^2)*B*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(Ca-Ab)\log(x)}{a^2}+\frac{-\int\frac{Bacx^2+(Ca-Ab)cx^3+Bab+\left(Cab-Ab^2+Aac\right)x}{cx^4+bx^2+a}\,dx}{a^2}-\frac{2\,Bx+A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] (C*a - A*b)*log(x)/a^2 + integrate(-(B*a*c*x^2 + (C*a - A*b)*c*x^3 + B*a*b
+ (C*a*b - A*b^2 + A*a*c)*x)/(c*x^4 + b*x^2 + a), x)/a^2 - 1/2*(2*B*x + A)/
(a*x^2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.29 $\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=412

$$\frac{\left(-\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\right)\tan^{-1}}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] ((2*A*c - b*C)*x)/(2*c*(b^2 - 4*a*c)) + (B*x^2*(2*a + b*x^2))/(2*(b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)) - (x^3*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) + ((A*b*c + (b^2 - 6*a*c)*C - (A*c*(b^2 + 4*a
*c) + b*(b^2 - 8*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + ((A*b*c + (b^2 - 6*a*c)*C + (A*c*(b^2 + 4*a*c) + b*(b^2 - 8*
a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2
*a*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 1.33379, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1662, 1275, 1279, 1166, 205, 12, 1114, 722, 618, 206}

$$\frac{\left(-\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\right)\tan^{-1}}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] ((2*A*c - b*C)*x)/(2*c*(b^2 - 4*a*c)) + (B*x^2*(2*a + b*x^2))/(2*(b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)) - (x^3*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) + ((A*b*c + (b^2 - 6*a*c)*C - (A*c*(b^2 + 4*a
*c) + b*(b^2 - 8*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + ((A*b*c + (b^2 - 6*a*c)*C + (A*c*(b^2 + 4*a*c) + b*(b^2 - 8*
a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2
*a*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```


Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
```

```
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
```

```
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx=\int \frac{Bx^5}{\left(a+bx^2+cx^4\right)^2}dx+\int \frac{x^4\left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$=-\frac{x^3\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+B\int \frac{x^5}{\left(a+bx^2+cx^4\right)^2}dx+\frac{\int \frac{x^2\left(3(Ab-2aC)+(2Ac-bC)x^2\right)}{a+bx^2+cx^4}d}{2\left(b^2-4ac\right)}$$

$$=\frac{(2Ac-bC)x}{2c\left(b^2-4ac\right)}-\frac{x^3\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^2}dx,x,x^2\right.$$

$$=\frac{(2Ac-bC)x}{2c\left(b^2-4ac\right)}+\frac{Bx^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{x^3\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(aB)\,\text{Sul}}{}$$

$$=\frac{(2Ac-bC)x}{2c\left(b^2-4ac\right)}+\frac{Bx^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{x^3\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(Abc+\right.}{}$$

$$=\frac{(2Ac-bC)x}{2c\left(b^2-4ac\right)}+\frac{Bx^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{x^3\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(Abc+\right.}{}$$

Mathematica [A] time = 1.56148, size = 444, normalized size = 1.08

$$\frac{1}{4}\left(\frac{2\left(a(b(B+Cx)-2cx(A+x(B+Cx)))+bx^2(b(B+Cx)-Acx)\right)}{c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(C\left(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}+8abc-b\right.\right.}{c^{3/2}\left(b^2-4a\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(b*x^2*(-(A*c*x) + b*(B + C*x)) + a*(b*(B + C*x) - 2*c*x*(A + x*(B + C*
x)))))/(c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(A*c*(b^2 + 4*a*
c - b*Sqrt[b^2 - 4*a*c])) + (-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c
*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(
A*c*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c]) + (b^3 - 8*a*b*c + b^2*Sqrt[b^2 - 4
*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
 - (4*a*B*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (4*a
*B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.042, size = 1429, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2*(A*b*c+2*C*a*c-C*b^2)/(4*a*c-b^2)/c*x^3-1/2*B*(2*a*c-b^2)/(4*a*c-b^2)
/c*x^2-1/2*a*(2*A*c-C*b)/(4*a*c-b^2)/c*x+1/2*B*a*b/c/(4*a*c-b^2))/(c*x^4+b*
x^2+a)-1/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^(1/2)*a*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2
)-b)+1/(4*a*c-b^2)^2*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*
c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4
*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b^2+1/(4*a*c-b^2)^2*c*2^(
1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1
/2)-b)*c)^(1/2))*A*a*b-1/4/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)
^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*b^3-2/(4*a*c
-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*a*b+1/4/(4*a*c-b^2)^2/c*2^
(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b^3-6/(4*a*c-b^2)^2*c*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2))*C*a^2+5/2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arct
anh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*a*b^2-1/4/(4*a*c-b^2)^2
/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b
^2)^(1/2)-b)*c)^(1/2))*C*b^4+1/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^(1/2)*a*ln(2*c*
x^2+(-4*a*c+b^2)^(1/2)+b)+1/(4*a*c-b^2)^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-4*a*c+b^
2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b^2-1/
(4*a*c-b^2)^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*a*b+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*A*b^3-2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*a*b+1/4/(4
*a*c-b^2)^2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b^3+6/(4*a*c-b^2)^2*c
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*C*a^2-5/2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*a*b^2+1/4/
(4*a*c-b^2)^2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(Cb^2-(2\,Ca+Ab)c\right)x^3+Bab+\left(Bb^2-2\,Bac\right)x^2+(Cab-2\,Aac)x}{2\left(\left(b^2c^2-4\,ac^3\right)x^4+ab^2c-4\,a^2c^2+\left(b^3c-4\,abc^2\right)x^2\right)}+\frac{-\int\frac{4\,Bacx-Cab+2\,Aac-\left(Cb^2-(6\,Ca-Ab)c\right)x^2}{cx^4+bx^2+a}\,dx}{2\left(b^2c-4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((C*b^2 - (2*C*a + A*b)*c)*x^3 + B*a*b + (B*b^2 - 2*B*a*c)*x^2 + (C*a*
b - 2*A*a*c)*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4
*a*b*c^2)*x^2) + 1/2*integrate(-(4*B*a*c*x - C*a*b + 2*A*a*c - (C*b^2 - (6*
C*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.30 $\int \frac{x^3\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=347

$$\frac{x^2\left(2acC+Abc+b^2(-C)\right)+a(2Ac-bC)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{Bx\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\left(b-\frac{4ac+b^{\cdots}}{\sqrt{b^2-4\cdots}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-\cdots\right.}$$

```
[Out] (B*x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (a*(2*A*c - b*C
) + (A*b*c - b^2*C + 2*a*c*C)*x^2)/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
+ (B*(b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + (B*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*S
qrt[b + Sqrt[b^2 - 4*a*c]]) - ((A*b - 2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.617503, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1662, 1251, 777, 618, 206, 12, 1120, 1166, 205}

$$\frac{x^2\left(2acC+Abc+b^2(-C)\right)+a(2Ac-bC)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{Bx\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\left(b-\frac{4ac+b^{\cdots}}{\sqrt{b^2-4\cdots}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (a*(2*A*c - b*C
) + (A*b*c - b^2*C + 2*a*c*C)*x^2)/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
+ (B*(b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + (B*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*S
qrt[b + Sqrt[b^2 - 4*a*c]]) - ((A*b - 2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
```

```
gerQ[(m - 1)/2]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx = \int \frac{Bx^4}{\left(a+bx^2+cx^4\right)^2}dx + \int \frac{x^3\left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Cx)}{\left(a+bx+cx^2\right)^2}dx, x, x^2\right) + B\int \frac{x^4}{\left(a+bx^2+cx^4\right)^2}dx$$

$$= \frac{Bx\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{a(2Ac-bC)+\left(Abc-b^2C+2acC\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{B\int \frac{2a-bx^2}{a+bx^2+cx^4}dx}{2\left(b^2-4ac\right)} + \cdots$$

$$= \frac{Bx\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{a(2Ac-bC)+\left(Abc-b^2C+2acC\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(B\left(b^2+4ac-b\sqrt{b^2\cdots\right.\right.}{4\left(\cdots\right.}$$

$$= \frac{Bx\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{a(2Ac-bC)+\left(Abc-b^2C+2acC\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{B\left(b^2+4ac-b\sqrt{b^2\cdots\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4a\cdots\right.}$$

Mathematica [A] time = 0.995371, size = 358, normalized size = 1.03

$$\frac{1}{4}\left(-\frac{2\left(a(2Ac-bC+2cx(B+Cx))+bx^2(Ac-bC+Bcx)\right)}{c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)} + \frac{2(Ab-2aC)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} - \frac{2(Ab-2aC)\log\cdots}{\left(\cdots\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*(b*x^2*(A*c - b*C + B*c*x) + a*(2*A*c - b*C + 2*c*x*(B + C*x))))/(c*(-
b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*B*(-b^2 - 4*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*
(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*B*(b^2 + 4*a*c
+ b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*(A*b -
2*a*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*(A*b
 - 2*a*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [F] time = 180., size = 0, normalized size = 0.

hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] int(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.31 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=356

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2}}}$$

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.901846, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1662, 1275, 1166, 205, 12, 1114, 638, 618, 206}

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```


Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
```

```
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx = \int \frac{Bx^3}{\left(a+bx^2+cx^4\right)^2}dx + \int \frac{x^2\left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + B\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}dx + \frac{\int \frac{Ab-2aC+(-2Ac+bC)x^2}{a+bx^2+cx^4}dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\,\mathrm{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}dx, x, x^2\right) - \frac{\left(2Ac-bC-\right.}{}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b}}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b}}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b}}$$

Mathematica [A] time = 1.16127, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x\left(bx(B+Cx)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-4ac-b^2\right)-2Ac\left(\sqrt{b^2-4ac}-2b\right)\right)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{b-}}\right.}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.036, size = 1119, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*B*b/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(4
*a*c-b^2)*x-B*a/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*B*(-4*a*c+b^
2)^(1/2)*b*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-c/(4*a*c-b^2)^2*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2))*A*(-4*a*c+b^2)^(1/2)*b-2*c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/
2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*a+1/
2*c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*b^2+c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a
*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1
/2))*C*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*(-4*a*c
+b^2)^(1/2)*b^2+c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*ar
ctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*a*b-1/4/(4*a*c-b^2)^2
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2))*C*b^3-1/2/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^(1/2)*b*ln(2*c*
x^2+(-4*a*c+b^2)^(1/2)+b)-c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-4*a*c+b^2)
^(1/2)*b+2*c^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*a-1/2*c/(4*a*c-b^2)^2*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*A*b^2+c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/
2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b^2-c/(4*a*c
-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*C*a*b+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bbx^2 + (Cb - 2\,Ac)x^3 + 2\,Ba + (2\,Ca - Ab)x}{2\left(\left(b^2c - 4\,ac^2\right)x^4 + ab^2 - 4\,a^2c + \left(b^3 - 4\,abc\right)x^2\right)} - \frac{-\int \frac{2\,Bbx + (Cb - 2\,Ac)x^2 - 2\,Ca + Ab}{cx^4 + bx^2 + a}\,dx}{2\left(b^2 - 4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.32 $\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=317

$$-\frac{-2aC+x^2(2Ac-bC)+Ab}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{(2Ac-bC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{Bx(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{B\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)}{\sqrt{2}(b^2-4ac)^{3/2}}$$

```
[Out] -(B*x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (A*b - 2*a*C +
 (2*A*c - b*C)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (B*Sqrt[c]*(2*b
 - Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (B*Sqrt[c]*
(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*A*c
 - b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.41535, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1662, 1247, 638, 618, 206, 12, 1119, 1166, 205}

$$-\frac{-2aC+x^2(2Ac-bC)+Ab}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{(2Ac-bC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{Bx(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{B\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)}{\sqrt{2}(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(B*x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (A*b - 2*a*C +
 (2*A*c - b*C)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (B*Sqrt[c]*(2*b
 - Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (B*Sqrt[c]*
(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*A*c
 - b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{Bx^2}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{x\left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{A+Cx}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) + B\int \frac{x^2}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= -\frac{Bx\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{Ab-2aC+(2Ac-bC)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{B\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)} - \frac{(2Ac-}{}$$

$$= -\frac{Bx\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{Ab-2aC+(2Ac-bC)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(Bc\left(2b-\sqrt{b^2-4ac}\right)\right)\int \frac{}{\frac{b}{2}}}{2\left(b^2-4ac\right)^3}$$

$$= -\frac{Bx\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{Ab-2aC+(2Ac-bC)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{B\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-}}$$

Mathematica [A] time = 1.41088, size = 335, normalized size = 1.06

$$\frac{1}{2}\left(\frac{2aC-A\left(b+2cx^2\right)+x\left(-bB+bCx-2Bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{(bC-2Ac)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{(2Ac-bC)\log\left(\sqrt{b^2-}\right.}{\left(b^2-4ac\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*a*C - A*(b + 2*c*x^2) + x*(-(b*B) + b*C*x - 2*B*c*x^2))/((b^2 - 4*a*c)*
(a + b*x^2 + c*x^4)) - (Sqrt[2]*B*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)
*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((-2*A*c + b*C)*Log[-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((2*A*c - b*C)*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/2
```

Maple [B] time = 0.117, size = 1344, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/2/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*A*b^2-1/2/(4*a*c-
b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*A*b^2+2*c^2/(4*a*c-b^2)^2*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*B*a-2*c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^
(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*B*a+1/2*c/(4*a*
c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4
```

```
*a*c+b^2)^(1/2)-b)*c)^(1/2))*B*b^2-1/2*c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B
*b^2+2*c/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*B*a*x+1/4/c/(
4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*C*(-4*a*c+b^2)^(1/2)*b^
2+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*B*a*x-1/4/c/(4*a
*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*C*(-4*a*c+b^2)^(1/2)*b^2-c
/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*B*b-c/(4*a*c-b^2)^2*2^
(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*B*b-1/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b
^2)^(1/2)/c+1/2*b/c)*C*(-4*a*c+b^2)^(1/2)*a-1/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*
c+b^2)^(1/2)/c+1/2*b/c)*C*a*b-1/2/(4*a*c-b^2)^2*ln(2*c*x^2+(-4*a*c+b^2)^(1/
2)+b)*C*(-4*a*c+b^2)^(1/2)*b-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2
)^(1/2)/c)*B*x*b^2+1/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*C
*(-4*a*c+b^2)^(1/2)*a-1/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c
)*C*a*b+1/2/(4*a*c-b^2)^2*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*C*(-4*a*c+b^2)^
(1/2)*b+2*c/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*A*a+1/4/c/
(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*C*b^3+c/(4*a*c-b^2)^2*
ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*(-4*a*c+b^2)^(1/2)+2*c/(4*a*c-b^2)^2/(x^
2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*A*a+1/4/c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/
2*(-4*a*c+b^2)^(1/2)/c)*C*b^3-c/(4*a*c-b^2)^2*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2
)-b)*A*(-4*a*c+b^2)^(1/2)-1/2/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1
/2*b/c)*B*x*b^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.33 $\int \frac{A+Bx+Cx^2}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=368

$$\frac{x\left(cx^2(Ab-2aC)-2aAc-abC+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{A\left(b^2-12ac\right)+4abC}{\sqrt{b^2-4ac}}-2aC+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4abC+Ab^2}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}a\left(b^2-\right.}$$

```
[Out] -(B*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (x*(A*b^2 - 2*a*
A*c - a*b*C + c*(A*b - 2*a*C)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
 + (Sqrt[c]*(A*b - 2*a*C + (A*(b^2 - 12*a*c) + 4*a*b*C)/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2
- 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a*C - (A*b^2 - 12
*a*A*c + 4*a*b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
+ (2*B*c*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.866742, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {1673, 1178, 1166, 205, 12, 1107, 614, 618, 206}

$$\frac{x\left(cx^2(Ab-2aC)-2aAc-abC+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{A\left(b^2-12ac\right)+4abC}{\sqrt{b^2-4ac}}-2aC+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4abC+Ab^2}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}a\left(b^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(B*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (x*(A*b^2 - 2*a*
A*c - a*b*C + c*(A*b - 2*a*C)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
 + (Sqrt[c]*(A*b - 2*a*C + (A*(b^2 - 12*a*c) + 4*a*b*C)/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2
- 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a*C - (A*b^2 - 12
*a*A*c + 4*a*b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
+ (2*B*c*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
```

```
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{Bx}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{A+Cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{x\left(Ab^2-2aAc-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + B\int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx - \frac{\int \frac{-Ab^2+6aAc-abC-c(Ab-2aC}{a+bx^2+cx^4}}{2a\left(b^2-4ac\right)}$$

$$= \frac{x\left(Ab^2-2aAc-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) + \frac{\left(c\left(A\left(b^2\right.\right.\right.}{}$$

$$= -\frac{B\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x\left(Ab^2-2aAc-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(A\left(b^2-12ac+\right.\right.}{}$$

$$= -\frac{B\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x\left(Ab^2-2aAc-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(A\left(b^2-12ac+\right.\right.}{}$$

$$= -\frac{B\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x\left(Ab^2-2aAc-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(A\left(b^2-12ac+\right.\right.}{}$$

Mathematica [A] time = 1.45573, size = 393, normalized size = 1.07

$$\frac{1}{4}\left(\frac{4acx(A+x(B+Cx))+2ab(B+Cx)-2Abx\left(b+cx^2\right)}{a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(A\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-2aC\left(\sqrt{b^2-4ac}-2b\right)\right.}{a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*a*b*(B + C*x) - 2*A*b*x*(b + c*x^2) + 4*a*c*x*(A + x*(B + C*x)))/(a*(-b
^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(A*(b^2 - 12*a*c + b*Sq
rt[b^2 - 4*a*c]) - 2*a*(-2*b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*(A*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c]) + 2*
a*(2*b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (4*B*c*L
og[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (4*B*c*Log[b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.105, size = 1813, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/4/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)/a*x*A*b^3+1/4/(4*a
*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)/a*x*A*b^3+2*c/(4*a*c-b^2)^
2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*a*C*x+c/(4*a*c-b^2)^2/(x^2+1/2*(-4
*a*c+b^2)^(1/2)/c+1/2*b/c)*x*A*(-4*a*c+b^2)^(1/2)-c/(4*a*c-b^2)^2/(x^2+1/2*
(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*A*b*x-c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*
c+b^2)^(1/2)/c)*x*A*(-4*a*c+b^2)^(1/2)-c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4
*a*c+b^2)^(1/2)/c)*A*b*x+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1
/2)/c)*a*C*x-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-4*a*c+b^2)^(1/2)*
b^2-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(
c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b^2-1/2/
(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*x*C*b^2-1/2/(4*a*c-b^2
)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*x*C*b^2+2*c/(4*a*c-b^2)^2/(x^2+1
/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*B*a+c/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^(1/2)*l
n(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+
b^2)^(1/2)/c)*B*a-c/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^(1/2)*ln(-2*c*x^2+(-4*a*c+
b^2)^(1/2)-b)-1/2/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*B*b^
2-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*B*b^2+3*c^2/(4*a
*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-4*a*c+b^2)^(1/2)-c^2/(4*a*c-b^2)^2*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))*A*b+2*c^2/(4*a*c-b^2)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C-1/2*c/(4*a*c-b^2
)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*C*b^2+1/4/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2
)^(1/2)/c)/a*x*A*(-4*a*c+b^2)^(1/2)*b^2-1/4/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+
b^2)^(1/2)/c+1/2*b/c)/a*x*A*(-4*a*c+b^2)^(1/2)*b^2+3*c^2/(4*a*c-b^2)^2*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/
2)-b)*c)^(1/2))*A*(-4*a*c+b^2)^(1/2)+c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^
2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*
A*b-2*c^2/(4*a*c-b^2)^2*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(
c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C+1/2*c/(4*a*c-b^2)^2*2^(1/2)
/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b)*c)^(1/2))*C*b^2-c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*
b-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*
x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*b^3-c/(4*a*c-b^2)^2*2^(1/2)/(
((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)
*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b+1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
A*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,Bacx^2+(2\,Ca-Ab)cx^3+Bab+\left(Cab-Ab^2+2\,Aac\right)x}{2\left(\left(ab^2c-4\,a^2c^2\right)x^4+a^2b^2-4\,a^3c+\left(ab^3-4\,a^2bc\right)x^2\right)}+\frac{-\int\frac{4\,Bacx+(2\,Ca-Ab)cx^2-Cab-Ab^2+6\,Aac}{cx^4+bx^2+a}\,dx}{2\left(ab^2-4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*B*a*c*x^2 + (2*C*a - A*b)*c*x^3 + B*a*b + (C*a*b - A*b^2 + 2*A*a*c)
*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^
2) + 1/2*integrate(-(4*B*a*c*x + (2*C*a - A*b)*c*x^2 - C*a*b - A*b^2 + 6*A*
a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.34 $\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=403

$$\frac{\left(4a^2cC+A\left(b^3-6abc\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{A\log(x)}{a^2}+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-ab}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] (B*x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (A*
(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2
+ c*x^4)) + (B*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (B*Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(
b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*(b^3 - 6*a*b*c) + 4*a
^2*c*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2
)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rubi [A] time = 0.931997, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1662, 1251, 822, 800, 634, 618, 206, 628, 12, 1092, 1166, 205}

$$\frac{\left(4a^2cC+A\left(b^3-6abc\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{A\log(x)}{a^2}+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-ab}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (B*x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (A*
(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2
+ c*x^4)) + (B*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (B*Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(
b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*(b^3 - 6*a*b*c) + 4*a
^2*c*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2
)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
```

```
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{B}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{A+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Cx}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) + B\int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{Bx\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-A\left(b^2-4a\cdots\right.}{x(a\cdots}\right.}{2a\left(b\cdots\right.}$$

$$= \frac{Bx\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{A\left(-b^2+\cdots\right.}{ax}\right.\right.}{\cdots}$$

$$= \frac{Bx\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{B\sqrt{c}\left(b^2-12ac+\cdots\right.}{2\sqrt{2}a\left(b^2\cdots\right.}$$

$$= \frac{Bx\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{B\sqrt{c}\left(b^2-12ac+\cdots\right.}{2\sqrt{2}a\left(b^2\cdots\right.}$$

$$= \frac{Bx\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{B\sqrt{c}\left(b^2-12ac+\cdots\right.}{2\sqrt{2}a\left(b^2\cdots\right.}$$

$$= \frac{Bx\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{B\sqrt{c}\left(b^2-12ac+\cdots\right.}{2\sqrt{2}a\left(b^2\cdots\right.}$$

Mathematica [A] time = 1.67619, size = 458, normalized size = 1.14

$$-\frac{\left(4a^2cC+A\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} - \frac{\left(A\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}+6abc-b^3\right)-4a^2cC\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} - \frac{2a\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*(a*b*C + 2*a*c*x*(B + C*x) - b*B*x*(b + c*x^2) - A*(b^2 - 2*a*c + b*
c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*a*B*Sqrt[c]*(b^2 -
12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*a*
B*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]]) + 4*A*Log[x] - ((A*(b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt
[b^2 - 4*a*c]) + 4*a^2*c*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4
*a*c)^(3/2) - ((A*(-b^3 + 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2
- 4*a*c]) - 4*a^2*c*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^
(3/2))/(4*a^2)
```

Maple [B] time = 0.042, size = 1603, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*b^3-1/a*c/(4*a*c-b^2)
/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2
)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*B*b^3-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)
*A*b^2-16*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*
A-1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*B*(-4*a*c+b^2)^(1/2)
*b^2-1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*(-4*a*c+b^2)^(1/
2)*b^2-6/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*
(-4*a*c+b^2)^(1/2)*b-16*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b
^2)^(1/2)+b)*A+1/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*C+1/(c*x^4+b*x^2+a)*B/(4
*a*c-b^2)*x*c-1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/
2)-b)*A*(-4*a*c+b^2)^(1/2)*b^3+8/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2
+(-4*a*c+b^2)^(1/2)-b)*A*b^2+1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+(-
4*a*c+b^2)^(1/2)+b)*A*(-4*a*c+b^2)^(1/2)*b^3+8/a*c/(4*a*c-b^2)/(16*a*c-4*b^
2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*b^2-1/2/a/(c*x^4+b*x^2+a)*B*b*c/(4*a*
c-b^2)*x^3-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*A*b+4*c/(4*a*c-b^2)/(16*
a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*C*(-4*a*c+b^2)^(1/2)-1/a^2/(4*a
*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*b^4-1/a^2/(4*a*c
-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*b^4-1/2/a/(c*x^4+b*
x^2+a)*B/(4*a*c-b^2)*x*b^2-4*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a
*c+b^2)^(1/2)-b)*C*(-4*a*c+b^2)^(1/2)+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A*c+1/2
/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*C+12*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*B*(-4*a*c+b^2)^(1/2)-4*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*B*b+12*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)
-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*B*(-4*a*
c+b^2)^(1/2)+4*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*B*b+6/a*c
/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*(-4*a*c+b^2
```

```
)^(1/2)*b+A*ln(x)/a^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.35 $\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=514

$$-\frac{-10aAc-abC+3Ab^2}{2a^2x\left(b^2-4ac\right)}-\frac{\sqrt{c}\left(A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-aC\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\right)\tan^{-1}}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] -(3*A*b^2 - 10*a*A*c - a*b*C)/(2*a^2*(b^2 - 4*a*c)*x) + (B*(b^2 - 2*a*c + b
*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (A*(b^2 - 2*a*c) - a*b*C
 + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt[c
]*(A*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]
) - a*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) - (Sqrt[c]*(3*A*b^2 - 10*a*A*c - a*b*C - (A*(3*b^3 - 16*a*
b*c) - a*(b^2 - 12*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2
- 4*a*c]]) + (b*B*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(
2*a^2*(b^2 - 4*a*c)^(3/2)) + (B*Log[x])/a^2 - (B*Log[a + b*x^2 + c*x^4])/(4
*a^2)
```

Rubi [A] time = 1.48571, antiderivative size = 514, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {1662, 1277, 1281, 1166, 205, 12, 1114, 740, 800, 634, 618, 206, 628}

$$-\frac{-10aAc-abC+3Ab^2}{2a^2x\left(b^2-4ac\right)}-\frac{\sqrt{c}\left(A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-aC\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\right)\tan^{-1}}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```


```
[Out] -(3*A*b^2 - 10*a*A*c - a*b*C)/(2*a^2*(b^2 - 4*a*c)*x) + (B*(b^2 - 2*a*c + b
*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (A*(b^2 - 2*a*c) - a*b*C
 + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt[c
]*(A*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]
) - a*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) - (Sqrt[c]*(3*A*b^2 - 10*a*A*c - a*b*C - (A*(3*b^3 - 16*a*
b*c) - a*(b^2 - 12*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2
- 4*a*c]]) + (b*B*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(
2*a^2*(b^2 - 4*a*c)^(3/2)) + (B*Log[x])/a^2 - (B*Log[a + b*x^2 + c*x^4])/(4
*a^2)
```


Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
```

```
lyQ[Pq, x^2]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{B}{x\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{A+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + B\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx - \frac{\int \frac{-3Ab^2+10aAc+abC-3}{x^2\left(a+bx^2+cx\right.}}{2a\left(b^2-4\right.}$$

$$= -\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\,\mathrm{Subst}\left(\int \frac{1}{x\left(a+bx\right.}\right.$$

$$= -\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x} + \frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2}{2a\left(b^2-4ac\right)x\left(a+bx^2+c\right.}$$

$$= -\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x} + \frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2}{2a\left(b^2-4ac\right)x\left(a+bx^2+c\right.}$$

$$= -\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x} + \frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2}{2a\left(b^2-4ac\right)x\left(a+bx^2+c\right.}$$

$$= -\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x} + \frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2}{2a\left(b^2-4ac\right)x\left(a+bx^2+c\right.}$$

$$= -\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x} + \frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2}{2a\left(b^2-4ac\right)x\left(a+bx^2+c\right.}$$

$$= -\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x} + \frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2}{2a\left(b^2-4ac\right)x\left(a+bx^2+c\right.}$$

Mathematica [A] time = 2.27809, size = 559, normalized size = 1.09

$$\frac{\frac{-4a^2c(B+Cx)+2a\left(bcx(3A+x(B+Cx))+2Ac^2x^3+b^2(B+Cx)\right)-2Ab^2x\left(b+cx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(A\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)+aC\left(b\sqrt{b^2-4ac}-12ac+b^2\right.\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*A)/x + (-4*a^2*c*(B + C*x) - 2*A*b^2*x*(b + c*x^2) + 2*a*(2*A*c^2*x^3
+ b^2*(B + C*x) + b*c*x*(3*A + x*(B + C*x))))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)) + (Sqrt[2]*Sqrt[c]*(A*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] +
 10*a*c*Sqrt[b^2 - 4*a*c]) + a*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(A*(3*b^3 - 16*a*b*c - 3*b^2
*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) + a*(-b^2 + 12*a*c + b*Sqrt[
b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
```

```
(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 4*B*Log[x] - (B*(b^3 - 6
*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2
 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (B*(-b^3 + 6*a*b*c + b^2*Sqrt[b
^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2]
)/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

Maple [B] time = 0.057, size = 2398, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b
^2+3/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-4*a*c+b^2)^(1/
2)*b^3+3/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c
)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*(-4*a*c+b^2
)^(1/2)*b^3-16/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2
)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*(-4*a
*c+b^2)^(1/2)*b-16/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-
4*a*c+b^2)^(1/2)*b-1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^
(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*(
-4*a*c+b^2)^(1/2)*b^2+40*c^3/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b
^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))
*A-40*c^3/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A-1/2/a/(c*x^4+b*x^
2+a)*c/(4*a*c-b^2)*x^3*b*C-1/2/a/(c*x^4+b*x^2+a)*B*b*c/(4*a*c-b^2)*x^2-3/2/
a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*A*b*c+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)
*x^3*A*b^2+8/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-
b)*B*b^2+8/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*
B*b^2-1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*B*
(-4*a*c+b^2)^(1/2)*b^3+1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+
b^2)^(1/2)+b)*B*(-4*a*c+b^2)^(1/2)*b^3+3/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2
^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^
(1/2)-b)*c)^(1/2))*A*b^4-1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c
+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2
))*C*b^3+1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*b^3-22/a*c^2
/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctan
h(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*b^2+22/a*c^2/(4*a*c-b^2)/
(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2-3/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*A*b^4-16*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a
*c+b^2)^(1/2)+b)*B+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*c*C-1/2/a/(c*x^4+b*x^2+a
)*B/(4*a*c-b^2)*b^2-16*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b
^2)^(1/2)-b)*B-A/a^2/x+1/(c*x^4+b*x^2+a)*B/(4*a*c-b^2)*c+1/2/a^2/(c*x^4+b*x
^2+a)/(4*a*c-b^2)*x*A*b^3-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*A-1/2/a/(
c*x^4+b*x^2+a)/(4*a*c-b^2)*x*C*b^2-1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c
*x^2+(-4*a*c+b^2)^(1/2)-b)*B*b^4-1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^
2+(-4*a*c+b^2)^(1/2)+b)*B*b^4+B*ln(x)/a^2+6/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*
ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*B*(-4*a*c+b^2)^(1/2)*b-6/a*c/(4*a*c-b^2)/
(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*B*(-4*a*c+b^2)^(1/2)*b+12*c
```

```
^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arct
anh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)+4*c^
2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arcta
nh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*b+12*c^2/(4*a*c-b^2)/(16
*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)-4*c^2/(4*a*c-b^2)/(16*a
*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*C*b
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.36 $\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=534

$$-\frac{\left(2A\left(6a^2c^2-6ab^2c+b^4\right)-abC\left(b^2-6ac\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}}-\frac{-6aAc-abC+2Ab^2}{2a^2x^2\left(b^2-4ac\right)}+\frac{(2Ab-aC)\log\left(a+bx^2+cx^4\right)}{4a^3}$$

```
[Out] -(2*A*b^2 - 6*a*A*c - a*b*C)/(2*a^2*(b^2 - 4*a*c)*x^2) - (B*(3*b^2 - 10*a*c
))/(2*a^2*(b^2 - 4*a*c)*x) + (B*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)
*x*(a + b*x^2 + c*x^4)) + (A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)/(
2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - (B*Sqrt[c]*(3*b^3 - 16*a*b*c +
 (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (B*Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(
b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((2*A*(b^4 - 6*a*b^2*c +
6*a^2*c^2) - a*b*(b^2 - 6*a*c)*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])
/(2*a^3*(b^2 - 4*a*c)^(3/2)) - ((2*A*b - a*C)*Log[x])/a^3 + ((2*A*b - a*C)*
Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rubi [A] time = 1.99236, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {1662, 1251, 822, 800, 634, 618, 206, 628, 12, 1121, 1281, 1166, 205}

$$-\frac{\left(2A\left(6a^2c^2-6ab^2c+b^4\right)-abC\left(b^2-6ac\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}}-\frac{-6aAc-abC+2Ab^2}{2a^2x^2\left(b^2-4ac\right)}+\frac{(2Ab-aC)\log\left(a+bx^2+cx^4\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```


```
[Out] -(2*A*b^2 - 6*a*A*c - a*b*C)/(2*a^2*(b^2 - 4*a*c)*x^2) - (B*(3*b^2 - 10*a*c
))/(2*a^2*(b^2 - 4*a*c)*x) + (B*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)
*x*(a + b*x^2 + c*x^4)) + (A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)/(
2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - (B*Sqrt[c]*(3*b^3 - 16*a*b*c +
 (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (B*Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(
b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((2*A*(b^4 - 6*a*b^2*c +
6*a^2*c^2) - a*b*(b^2 - 6*a*c)*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])
/(2*a^3*(b^2 - 4*a*c)^(3/2)) - ((2*A*b - a*C)*Log[x])/a^3 + ((2*A*b - a*C)*
Log[a + b*x^2 + c*x^4])/(4*a^3)
```


Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
```

```
lyQ[Pq, x^2]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{B}{x^2\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{A+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Cx}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) + B\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-2Ab^2 \cdots}{}\right.}{}$$

$$= -\frac{B\left(3b^2-10ac\right)}{2a^2\left(b^2-4ac\right)x} + \frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{2Ab^2-6aAc-abC}{2a^2\left(b^2-4ac\right)x^2} - \frac{B\left(3b^2-10ac\right)}{2a^2\left(b^2-4ac\right)x} + \frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right.}{2a\left(b^2-\right.}$$

$$= -\frac{2Ab^2-6aAc-abC}{2a^2\left(b^2-4ac\right)x^2} - \frac{B\left(3b^2-10ac\right)}{2a^2\left(b^2-4ac\right)x} + \frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right.}{2a\left(b^2-\right.}$$

$$= -\frac{2Ab^2-6aAc-abC}{2a^2\left(b^2-4ac\right)x^2} - \frac{B\left(3b^2-10ac\right)}{2a^2\left(b^2-4ac\right)x} + \frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right.}{2a\left(b^2-\right.}$$

$$= -\frac{2Ab^2-6aAc-abC}{2a^2\left(b^2-4ac\right)x^2} - \frac{B\left(3b^2-10ac\right)}{2a^2\left(b^2-4ac\right)x} + \frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right.}{2a\left(b^2-\right.}$$

Mathematica [A] time = 2.76088, size = 655, normalized size = 1.23

$$-\frac{2a\left(2a^2cC+A\left(-3abc-2ac^2x^2+b^2cx^2+b^3\right)-a\left(b^2C+bcx(3B+Cx)+2Bc^2x^3\right)+b^2Bx\left(b+cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(2A\left(6a^2c^2+b^3\sqrt{b^2-4ac}-6ab^2c-4abc\sqrt{b^2-4ac}+b^4\right)+aC\left(-b^2\sqrt{b^2 \cdots}\right.\right.}{\left(b^2-4ac\right)^{3/\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*A)/x^2 - (4*a*B)/x - (2*a*(2*a^2*c*C + b^2*B*x*(b + c*x^2) + A*(b^3
- 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2) - a*(b^2*C + 2*B*c^2*x^3 + b*c*x*(3*B
+ C*x))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*a*B*Sqrt[c]*(-3*b^
3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*a*B*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b^2*Sq
rt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
 + 4*(-2*A*b + a*C)*Log[x] + ((2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[
b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c]) + a*(-b^3 + 6*a*b*c - b^2*Sqrt[b^
2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x
^2])/(b^2 - 4*a*c)^(3/2) + ((2*A*(-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b
^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c]) + a*(b^3 - 6*a*b*c - b^2*Sqrt[b^2
- 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2]
```

```
)/(b^2 - 4*a*c)^(3/2))/(4*a^3)
```

Maple [B] time = 0.062, size = 2512, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -22/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/
2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*B*b^2+22/a*c^2/(4*
a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*b^2-3/a^2*c/(4*a*c-b^2)/(16*a*
c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*B*b^4+3/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2
)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)
-b)*c)^(1/2))*B*b^4-16/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b
^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))
*B*(-4*a*c+b^2)^(1/2)*b-16/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*B*(-4*a*c+b^2)^(1/2)*b+3/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/(((-4
*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^
(1/2))*B*(-4*a*c+b^2)^(1/2)*b^3+3/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*B*(-4*a*c+b^2)^(1/2)*b^3-1/a/(c*x^4+b*x^2+a)*B*c^2/(4*a*c-b^2)*x
^3-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^2*A+1/2/a^2/(c*x^4+b*x^2+a)*B*b^3/
(4*a*c-b^2)*x-3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A*b*c+2/a^3/(4*a*c-b^2)/(16
*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*b^5+2/a^3/(4*a*c-b^2)/(16*a
*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*b^5-1/a^2/(4*a*c-b^2)/(16*a*c-
4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*C*b^4-1/a^2/(4*a*c-b^2)/(16*a*c-4*
b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*C*b^4-2/a^3*ln(x)*A*b+1/(c*x^4+b*x^2+
a)/(4*a*c-b^2)*c*C+1/2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A*b^3-1/2/a/(c*x^4+b
*x^2+a)/(4*a*c-b^2)*C*b^2-16*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4
*a*c+b^2)^(1/2)-b)*C-16*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b
^2)^(1/2)+b)*C-B/a^2/x+1/a^2*ln(x)*C-1/2*A/a^2/x^2-40*c^3/(4*a*c-b^2)/(16*a
*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*B+12/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*
x^2+(-4*a*c+b^2)^(1/2)-b)*A*(-4*a*c+b^2)^(1/2)-3/2/a/(c*x^4+b*x^2+a)*B*b/(4
*a*c-b^2)*x*c+1/2/a^2/(c*x^4+b*x^2+a)*B*c/(4*a*c-b^2)*x^3*b^2+1/2/a^2/(c*x^
4+b*x^2+a)*c/(4*a*c-b^2)*x^2*A*b^2-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*
b*C-1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*C*(-
4*a*c+b^2)^(1/2)*b^3+1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^
2)^(1/2)+b)*C*(-4*a*c+b^2)^(1/2)*b^3+8/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*
c*x^2+(-4*a*c+b^2)^(1/2)+b)*C*b^2+2/a^3/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*
x^2+(-4*a*c+b^2)^(1/2)-b)*A*(-4*a*c+b^2)^(1/2)*b^4+32/a*c^2/(4*a*c-b^2)/(16
*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*b-12/a*c^2/(4*a*c-b^2)/(16*
a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*(-4*a*c+b^2)^(1/2)-2/a^3/(4*a
*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*(-4*a*c+b^2)^(1/2
)*b^4+32/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*
A*b-16/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A
*b^3+12/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A
*(-4*a*c+b^2)^(1/2)*b^2-6/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c
+b^2)^(1/2)+b)*C*(-4*a*c+b^2)^(1/2)*b-12/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*l
n(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*A*(-4*a*c+b^2)^(1/2)*b^2+6/a*c/(4*a*c-b^2)
/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*C*(-4*a*c+b^2)^(1/2)*b+8/
a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*C*b^2-16/a
```

```
^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*A*b^3+40*c
^3/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arct
anh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*B
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.37 $\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^3 dx$

Optimal. Leaf size=399

$$\frac{a^2(dx)^{m+3}(aC+3Ab)}{d^3(m+3)}+\frac{a^3A(dx)^{m+1}}{d(m+1)}+\frac{3a^2bB(dx)^{m+4}}{d^4(m+4)}+\frac{a^3B(dx)^{m+2}}{d^2(m+2)}+\frac{3a(dx)^{m+5}\left(A\left(ac+b^2\right)+abC\right)}{d^5(m+5)}+\frac{(dx)^{m+7}\left(A\left(6a\right.\right.}{}$$

```
[Out] (a^3*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^3*B*(d*x)^(2 + m))/(d^2*(2 + m)) + (
a^2*(3*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (3*a^2*b*B*(d*x)^(4 + m))/
(d^4*(4 + m)) + (3*a*(A*(b^2 + a*c) + a*b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) +
 (3*a*B*(b^2 + a*c)*(d*x)^(6 + m))/(d^6*(6 + m)) + ((A*(b^3 + 6*a*b*c) + 3*
a*(b^2 + a*c)*C)*(d*x)^(7 + m))/(d^7*(7 + m)) + (b*B*(b^2 + 6*a*c)*(d*x)^(8
 + m))/(d^8*(8 + m)) + ((3*A*c*(b^2 + a*c) + b*(b^2 + 6*a*c)*C)*(d*x)^(9 +
m))/(d^9*(9 + m)) + (3*B*c*(b^2 + a*c)*(d*x)^(10 + m))/(d^10*(10 + m)) + (3
*c*(A*b*c + (b^2 + a*c)*C)*(d*x)^(11 + m))/(d^11*(11 + m)) + (3*b*B*c^2*(d*
x)^(12 + m))/(d^12*(12 + m)) + (c^2*(A*c + 3*b*C)*(d*x)^(13 + m))/(d^13*(13
 + m)) + (B*c^3*(d*x)^(14 + m))/(d^14*(14 + m)) + (c^3*C*(d*x)^(15 + m))/(d
^15*(15 + m))
```

Rubi [A] time = 0.424928, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1628}

$$\frac{a^2(dx)^{m+3}(aC+3Ab)}{d^3(m+3)}+\frac{a^3A(dx)^{m+1}}{d(m+1)}+\frac{3a^2bB(dx)^{m+4}}{d^4(m+4)}+\frac{a^3B(dx)^{m+2}}{d^2(m+2)}+\frac{3a(dx)^{m+5}\left(A\left(ac+b^2\right)+abC\right)}{d^5(m+5)}+\frac{(dx)^{m+7}\left(A\left(6a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x]
```


```
[Out] (a^3*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^3*B*(d*x)^(2 + m))/(d^2*(2 + m)) + (
a^2*(3*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (3*a^2*b*B*(d*x)^(4 + m))/
(d^4*(4 + m)) + (3*a*(A*(b^2 + a*c) + a*b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) +
 (3*a*B*(b^2 + a*c)*(d*x)^(6 + m))/(d^6*(6 + m)) + ((A*(b^3 + 6*a*b*c) + 3*
a*(b^2 + a*c)*C)*(d*x)^(7 + m))/(d^7*(7 + m)) + (b*B*(b^2 + 6*a*c)*(d*x)^(8
 + m))/(d^8*(8 + m)) + ((3*A*c*(b^2 + a*c) + b*(b^2 + 6*a*c)*C)*(d*x)^(9 +
m))/(d^9*(9 + m)) + (3*B*c*(b^2 + a*c)*(d*x)^(10 + m))/(d^10*(10 + m)) + (3
*c*(A*b*c + (b^2 + a*c)*C)*(d*x)^(11 + m))/(d^11*(11 + m)) + (3*b*B*c^2*(d*
x)^(12 + m))/(d^12*(12 + m)) + (c^2*(A*c + 3*b*C)*(d*x)^(13 + m))/(d^13*(13
 + m)) + (B*c^3*(d*x)^(14 + m))/(d^14*(14 + m)) + (c^3*C*(d*x)^(15 + m))/(d
^15*(15 + m))
```


Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^3 dx = \int \left(a^3A(dx)^m+\frac{a^3B(dx)^{1+m}}{d}+\frac{a^2(3Ab+aC)(dx)^{2+m}}{d^2}+\frac{3a^2bB(dx)^{3+m}}{d^3}+\right.$$

$$=\frac{a^3A(dx)^{1+m}}{d(1+m)}+\frac{a^3B(dx)^{2+m}}{d^2(2+m)}+\frac{a^2(3Ab+aC)(dx)^{3+m}}{d^3(3+m)}+\frac{3a^2bB(dx)^{4+m}}{d^4(4+m)}+$$

Mathematica [A] time = 1.27055, size = 296, normalized size = 0.74

$$x(dx)^m\left(\frac{a^2x^2(aC+3Ab)}{m+3}+\frac{a^3A}{m+1}+\frac{3a^2bBx^3}{m+4}+\frac{a^3Bx}{m+2}+\frac{3cx^{10}\left(C\left(ac+b^2\right)+Abc\right)}{m+11}+\frac{x^8\left(3Ac\left(ac+b^2\right)+bC\left(6ac+\right.\right.}{m+9}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] x*(d*x)^m*((a^3*A)/(1 + m) + (a^3*B*x)/(2 + m) + (a^2*(3*A*b + a*C)*x^2)/(3
 + m) + (3*a^2*b*B*x^3)/(4 + m) + (3*a*(A*(b^2 + a*c) + a*b*C)*x^4)/(5 + m)
 + (3*a*B*(b^2 + a*c)*x^5)/(6 + m) + ((A*(b^3 + 6*a*b*c) + 3*a*(b^2 + a*c)*
C)*x^6)/(7 + m) + (b*B*(b^2 + 6*a*c)*x^7)/(8 + m) + ((3*A*c*(b^2 + a*c) + b
*(b^2 + 6*a*c)*C)*x^8)/(9 + m) + (3*B*c*(b^2 + a*c)*x^9)/(10 + m) + (3*c*(A
*b*c + (b^2 + a*c)*C)*x^10)/(11 + m) + (3*b*B*c^2*x^11)/(12 + m) + (c^2*(A*
c + 3*b*C)*x^12)/(13 + m) + (B*c^3*x^13)/(14 + m) + (c^3*C*x^14)/(15 + m))
```

Maple [B] time = 0.016, size = 5520, normalized size = 13.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.67241, size = 11187, normalized size = 28.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] ((C*c^3*m^14 + 105*C*c^3*m^13 + 5005*C*c^3*m^12 + 143325*C*c^3*m^11 + 27497
47*C*c^3*m^10 + 37312275*C*c^3*m^9 + 368411615*C*c^3*m^8 + 2681453775*C*c^3
*m^7 + 14409322928*C*c^3*m^6 + 56663366760*C*c^3*m^5 + 159721605680*C*c^3*m
^4 + 310989260400*C*c^3*m^3 + 392156797824*C*c^3*m^2 + 283465647360*C*c^3*m
 + 87178291200*C*c^3)*x^15 + (B*c^3*m^14 + 106*B*c^3*m^13 + 5096*B*c^3*m^12
 + 147056*B*c^3*m^11 + 2840838*B*c^3*m^10 + 38786748*B*c^3*m^9 + 385081268*
```

```
B*c^3*m^8 + 2816490248*B*c^3*m^7 + 15200266081*B*c^3*m^6 + 59999485546*B*c^
3*m^5 + 169679309436*B*c^3*m^4 + 331303013496*B*c^3*m^3 + 418753514880*B*c^
3*m^2 + 303268406400*B*c^3*m + 93405312000*B*c^3)*x^14 + ((3*C*b*c^2 + A*c^
3)*m^14 + 107*(3*C*b*c^2 + A*c^3)*m^13 + 5189*(3*C*b*c^2 + A*c^3)*m^12 + 15
0943*(3*C*b*c^2 + A*c^3)*m^11 + 2937363*(3*C*b*c^2 + A*c^3)*m^10 + 40372761
*(3*C*b*c^2 + A*c^3)*m^9 + 403249847*(3*C*b*c^2 + A*c^3)*m^8 + 2965379989*(
3*C*b*c^2 + A*c^3)*m^7 + 16081189696*(3*C*b*c^2 + A*c^3)*m^6 + 63747744632*
(3*C*b*c^2 + A*c^3)*m^5 + 180951426864*(3*C*b*c^2 + A*c^3)*m^4 + 3017710080
00*C*b*c^2 + 100590336000*A*c^3 + 354444796368*(3*C*b*c^2 + A*c^3)*m^3 + 44
9213351040*(3*C*b*c^2 + A*c^3)*m^2 + 326044051200*(3*C*b*c^2 + A*c^3)*m)*x^
13 + 3*(B*b*c^2*m^14 + 108*B*b*c^2*m^13 + 5284*B*b*c^2*m^12 + 154992*B*b*c^
2*m^11 + 3039718*B*b*c^2*m^10 + 42081864*B*b*c^2*m^9 + 423113372*B*b*c^2*m^
8 + 3130267536*B*b*c^2*m^7 + 17067919121*B*b*c^2*m^6 + 67988181228*B*b*c^2*
m^5 + 193813932344*B*b*c^2*m^4 + 381046157472*B*b*c^2*m^3 + 484441814160*B*
b*c^2*m^2 + 352515844800*B*b*c^2*m + 108972864000*B*b*c^2)*x^12 + 3*((C*b^2
*c + (C*a + A*b)*c^2)*m^14 + 109*(C*b^2*c + (C*a + A*b)*c^2)*m^13 + 5381*(C
*b^2*c + (C*a + A*b)*c^2)*m^12 + 159209*(C*b^2*c + (C*a + A*b)*c^2)*m^11 +
3148323*(C*b^2*c + (C*a + A*b)*c^2)*m^10 + 43926927*(C*b^2*c + (C*a + A*b)*
c^2)*m^9 + 444899543*(C*b^2*c + (C*a + A*b)*c^2)*m^8 + 3313733027*(C*b^2*c
+ (C*a + A*b)*c^2)*m^7 + 18180066256*(C*b^2*c + (C*a + A*b)*c^2)*m^6 + 7282
2481864*(C*b^2*c + (C*a + A*b)*c^2)*m^5 + 208624806576*(C*b^2*c + (C*a + A*
b)*c^2)*m^4 + 118879488000*C*b^2*c + 411940473264*(C*b^2*c + (C*a + A*b)*c^
2)*m^3 + 118879488000*(C*a + A*b)*c^2 + 525650497920*(C*b^2*c + (C*a + A*b)
*c^2)*m^2 + 383662137600*(C*b^2*c + (C*a + A*b)*c^2)*m)*x^11 + 3*((B*b^2*c
+ B*a*c^2)*m^14 + 110*(B*b^2*c + B*a*c^2)*m^13 + 5480*(B*b^2*c + B*a*c^2)*m
^12 + 163600*(B*b^2*c + B*a*c^2)*m^11 + 3263622*(B*b^2*c + B*a*c^2)*m^10 +
45922260*(B*b^2*c + B*a*c^2)*m^9 + 468873140*(B*b^2*c + B*a*c^2)*m^8 + 3518
896600*(B*b^2*c + B*a*c^2)*m^7 + 19442163553*(B*b^2*c + B*a*c^2)*m^6 + 7838
1575150*(B*b^2*c + B*a*c^2)*m^5 + 225856355580*(B*b^2*c + B*a*c^2)*m^4 + 13
0767436800*B*b^2*c + 130767436800*B*a*c^2 + 448249789800*(B*b^2*c + B*a*c^2
)*m^3 + 574497805824*(B*b^2*c + B*a*c^2)*m^2 + 420839556480*(B*b^2*c + B*a*
c^2)*m)*x^10 + ((C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^14 + 111*(C*b
^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^13 + 5581*(C*b^3 + 3*A*a*c^2 + 3*
(2*C*a*b + A*b^2)*c)*m^12 + 168171*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)
*c)*m^11 + 3386083*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^10 + 48083
733*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^9 + 495342143*(C*b^3 + 3*
A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^8 + 3749548713*(C*b^3 + 3*A*a*c^2 + 3*(2
*C*a*b + A*b^2)*c)*m^7 + 20885191136*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^
2)*c)*m^6 + 84836490456*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^5 + 2
46143692976*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^4 + 145297152000*
C*b^3 + 435891456000*A*a*c^2 + 491520108816*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b
 + A*b^2)*c)*m^3 + 633314724480*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)
*m^2 + 435891456000*(2*C*a*b + A*b^2)*c + 465985094400*(C*b^3 + 3*A*a*c^2 +
 3*(2*C*a*b + A*b^2)*c)*m)*x^9 + ((B*b^3 + 6*B*a*b*c)*m^14 + 112*(B*b^3 + 6
*B*a*b*c)*m^13 + 5684*(B*b^3 + 6*B*a*b*c)*m^12 + 172928*(B*b^3 + 6*B*a*b*c)
*m^11 + 3516198*(B*b^3 + 6*B*a*b*c)*m^10 + 50428896*(B*b^3 + 6*B*a*b*c)*m^9
 + 524664572*(B*b^3 + 6*B*a*b*c)*m^8 + 4010311424*(B*b^3 + 6*B*a*b*c)*m^7 +
 22548638161*(B*b^3 + 6*B*a*b*c)*m^6 + 92414105392*(B*b^3 + 6*B*a*b*c)*m^5
+ 270359263944*(B*b^3 + 6*B*a*b*c)*m^4 + 163459296000*B*b^3 + 980755776000*
B*a*b*c + 543939234048*(B*b^3 + 6*B*a*b*c)*m^3 + 705481831440*(B*b^3 + 6*B*
a*b*c)*m^2 + 521962963200*(B*b^3 + 6*B*a*b*c)*m)*x^8 + ((3*C*a*b^2 + A*b^3
+ 3*(C*a^2 + 2*A*a*b)*c)*m^14 + 113*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b
)*c)*m^13 + 5789*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^12 + 177877*
(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^11 + 3654483*(3*C*a*b^2 + A*b
^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^10 + 52977099*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 +
 2*A*a*b)*c)*m^9 + 557256047*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^
8 + 4306835671*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^7 + 2448327985
6*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^6 + 101420251688*(3*C*a*b^2
 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^5 + 299730345264*(3*C*a*b^2 + A*b^3 + 3
```

```
*(C*a^2 + 2*A*a*b)*c)*m^4 + 560431872000*C*a*b^2 + 186810624000*A*b^3 + 608
700928752*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^3 + 796089202560*(3
*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^2 + 560431872000*(C*a^2 + 2*A*a
*b)*c + 593193196800*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m)*x^7 + 3
*((B*a*b^2 + B*a^2*c)*m^14 + 114*(B*a*b^2 + B*a^2*c)*m^13 + 5896*(B*a*b^2 +
 B*a^2*c)*m^12 + 183024*(B*a*b^2 + B*a^2*c)*m^11 + 3801478*(B*a*b^2 + B*a^2
*c)*m^10 + 55749612*(B*a*b^2 + B*a^2*c)*m^9 + 593598068*(B*a*b^2 + B*a^2*c)
*m^8 + 4646039592*(B*a*b^2 + B*a^2*c)*m^7 + 26754892001*(B*a*b^2 + B*a^2*c)
*m^6 + 112273858674*(B*a*b^2 + B*a^2*c)*m^5 + 336028955036*(B*a*b^2 + B*a^2
*c)*m^4 + 217945728000*B*a*b^2 + 217945728000*B*a^2*c + 690639615384*(B*a*b
^2 + B*a^2*c)*m^3 + 913158011520*(B*a*b^2 + B*a^2*c)*m^2 + 686869545600*(B*
a*b^2 + B*a^2*c)*m)*x^6 + 3*((C*a^2*b + A*a*b^2 + A*a^2*c)*m^14 + 115*(C*a^
2*b + A*a*b^2 + A*a^2*c)*m^13 + 6005*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^12 + 1
88375*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^11 + 3957747*(C*a^2*b + A*a*b^2 + A*a
^2*c)*m^10 + 58769745*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^9 + 634247015*(C*a^2*
b + A*a*b^2 + A*a^2*c)*m^8 + 5036392925*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^7 +
 29449164928*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^6 + 125557386040*(C*a^2*b + A*
a*b^2 + A*a^2*c)*m^5 + 381885176880*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^4 + 261
534873600*C*a^2*b + 261534873600*A*a*b^2 + 261534873600*A*a^2*c + 797387461
200*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^3 + 1070058397824*(C*a^2*b + A*a*b^2 +
A*a^2*c)*m^2 + 815525625600*(C*a^2*b + A*a*b^2 + A*a^2*c)*m)*x^5 + 3*(B*a^2
*b*m^14 + 116*B*a^2*b*m^13 + 6116*B*a^2*b*m^12 + 193936*B*a^2*b*m^11 + 4123
878*B*a^2*b*m^10 + 62062968*B*a^2*b*m^9 + 679843868*B*a^2*b*m^8 + 548825252
8*B*a^2*b*m^7 + 32678119441*B*a^2*b*m^6 + 142090732916*B*a^2*b*m^5 + 441309
175416*B*a^2*b*m^4 + 941576643936*B*a^2*b*m^3 + 1290689128080*B*a^2*b*m^2 +
 1003061102400*B*a^2*b*m + 326918592000*B*a^2*b)*x^4 + ((C*a^3 + 3*A*a^2*b)
*m^14 + 117*(C*a^3 + 3*A*a^2*b)*m^13 + 6229*(C*a^3 + 3*A*a^2*b)*m^12 + 1997
13*(C*a^3 + 3*A*a^2*b)*m^11 + 4300483*(C*a^3 + 3*A*a^2*b)*m^10 + 65657031*(
C*a^3 + 3*A*a^2*b)*m^9 + 731124647*(C*a^3 + 3*A*a^2*b)*m^8 + 6014254059*(C*
a^3 + 3*A*a^2*b)*m^7 + 36588367376*(C*a^3 + 3*A*a^2*b)*m^6 + 163038108552*(
C*a^3 + 3*A*a^2*b)*m^5 + 520557781424*(C*a^3 + 3*A*a^2*b)*m^4 + 43589145600
0*C*a^3 + 1307674368000*A*a^2*b + 1145140001328*(C*a^3 + 3*A*a^2*b)*m^3 + 1
621575699840*(C*a^3 + 3*A*a^2*b)*m^2 + 1301090515200*(C*a^3 + 3*A*a^2*b)*m)
*x^3 + (B*a^3*m^14 + 118*B*a^3*m^13 + 6344*B*a^3*m^12 + 205712*B*a^3*m^11 +
 4488198*B*a^3*m^10 + 69582084*B*a^3*m^9 + 788931572*B*a^3*m^8 + 6629764856
*B*a^3*m^7 + 41371599841*B*a^3*m^6 + 190060010998*B*a^3*m^5 + 629552085084*
B*a^3*m^4 + 1447709175432*B*a^3*m^3 + 2161577352960*B*a^3*m^2 + 18426629088
00*B*a^3*m + 653837184000*B*a^3)*x^2 + (A*a^3*m^14 + 119*A*a^3*m^13 + 6461*
A*a^3*m^12 + 211939*A*a^3*m^11 + 4687683*A*a^3*m^10 + 73870797*A*a^3*m^9 +
854224943*A*a^3*m^8 + 7353403057*A*a^3*m^7 + 47277726496*A*a^3*m^6 + 225525
484184*A*a^3*m^5 + 784146622896*A*a^3*m^4 + 1922666722704*A*a^3*m^3 + 31343
28981120*A*a^3*m^2 + 3031488633600*A*a^3*m + 1307674368000*A*a^3)*x)*(d*x)^
m/(m^15 + 120*m^14 + 6580*m^13 + 218400*m^12 + 4899622*m^11 + 78558480*m^10
 + 928095740*m^9 + 8207628000*m^8 + 54631129553*m^7 + 272803210680*m^6 + 10
09672107080*m^5 + 2706813345600*m^4 + 5056995703824*m^3 + 6165817614720*m^2
 + 4339163001600*m + 1307674368000)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33832, size = 10541, normalized size = 26.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] ((d*x)^m*C*c^3*m^14*x^15 + (d*x)^m*B*c^3*m^14*x^14 + 105*(d*x)^m*C*c^3*m^13
*x^15 + 3*(d*x)^m*C*b*c^2*m^14*x^13 + (d*x)^m*A*c^3*m^14*x^13 + 106*(d*x)^m
*B*c^3*m^13*x^14 + 5005*(d*x)^m*C*c^3*m^12*x^15 + 3*(d*x)^m*B*b*c^2*m^14*x^
12 + 321*(d*x)^m*C*b*c^2*m^13*x^13 + 107*(d*x)^m*A*c^3*m^13*x^13 + 5096*(d*
x)^m*B*c^3*m^12*x^14 + 143325*(d*x)^m*C*c^3*m^11*x^15 + 3*(d*x)^m*C*b^2*c*m
^14*x^11 + 3*(d*x)^m*C*a*c^2*m^14*x^11 + 3*(d*x)^m*A*b*c^2*m^14*x^11 + 324*
(d*x)^m*B*b*c^2*m^13*x^12 + 15567*(d*x)^m*C*b*c^2*m^12*x^13 + 5189*(d*x)^m*
A*c^3*m^12*x^13 + 147056*(d*x)^m*B*c^3*m^11*x^14 + 2749747*(d*x)^m*C*c^3*m^
10*x^15 + 3*(d*x)^m*B*b^2*c*m^14*x^10 + 3*(d*x)^m*B*a*c^2*m^14*x^10 + 327*(
d*x)^m*C*b^2*c*m^13*x^11 + 327*(d*x)^m*C*a*c^2*m^13*x^11 + 327*(d*x)^m*A*b*
c^2*m^13*x^11 + 15852*(d*x)^m*B*b*c^2*m^12*x^12 + 452829*(d*x)^m*C*b*c^2*m^
11*x^13 + 150943*(d*x)^m*A*c^3*m^11*x^13 + 2840838*(d*x)^m*B*c^3*m^10*x^14
+ 37312275*(d*x)^m*C*c^3*m^9*x^15 + (d*x)^m*C*b^3*m^14*x^9 + 6*(d*x)^m*C*a*
b*c*m^14*x^9 + 3*(d*x)^m*A*b^2*c*m^14*x^9 + 3*(d*x)^m*A*a*c^2*m^14*x^9 + 33
0*(d*x)^m*B*b^2*c*m^13*x^10 + 330*(d*x)^m*B*a*c^2*m^13*x^10 + 16143*(d*x)^m
*C*b^2*c*m^12*x^11 + 16143*(d*x)^m*C*a*c^2*m^12*x^11 + 16143*(d*x)^m*A*b*c^
2*m^12*x^11 + 464976*(d*x)^m*B*b*c^2*m^11*x^12 + 8812089*(d*x)^m*C*b*c^2*m^
10*x^13 + 2937363*(d*x)^m*A*c^3*m^10*x^13 + 38786748*(d*x)^m*B*c^3*m^9*x^14
 + 368411615*(d*x)^m*C*c^3*m^8*x^15 + (d*x)^m*B*b^3*m^14*x^8 + 6*(d*x)^m*B*
a*b*c*m^14*x^8 + 111*(d*x)^m*C*b^3*m^13*x^9 + 666*(d*x)^m*C*a*b*c*m^13*x^9
+ 333*(d*x)^m*A*b^2*c*m^13*x^9 + 333*(d*x)^m*A*a*c^2*m^13*x^9 + 16440*(d*x)
^m*B*b^2*c*m^12*x^10 + 16440*(d*x)^m*B*a*c^2*m^12*x^10 + 477627*(d*x)^m*C*b
^2*c*m^11*x^11 + 477627*(d*x)^m*C*a*c^2*m^11*x^11 + 477627*(d*x)^m*A*b*c^2*
m^11*x^11 + 9119154*(d*x)^m*B*b*c^2*m^10*x^12 + 121118283*(d*x)^m*C*b*c^2*m
^9*x^13 + 40372761*(d*x)^m*A*c^3*m^9*x^13 + 385081268*(d*x)^m*B*c^3*m^8*x^1
4 + 2681453775*(d*x)^m*C*c^3*m^7*x^15 + 3*(d*x)^m*C*a*b^2*m^14*x^7 + (d*x)^
m*A*b^3*m^14*x^7 + 3*(d*x)^m*C*a^2*c*m^14*x^7 + 6*(d*x)^m*A*a*b*c*m^14*x^7
+ 112*(d*x)^m*B*b^3*m^13*x^8 + 672*(d*x)^m*B*a*b*c*m^13*x^8 + 5581*(d*x)^m*
C*b^3*m^12*x^9 + 33486*(d*x)^m*C*a*b*c*m^12*x^9 + 16743*(d*x)^m*A*b^2*c*m^1
2*x^9 + 16743*(d*x)^m*A*a*c^2*m^12*x^9 + 490800*(d*x)^m*B*b^2*c*m^11*x^10 +
 490800*(d*x)^m*B*a*c^2*m^11*x^10 + 9444969*(d*x)^m*C*b^2*c*m^10*x^11 + 944
4969*(d*x)^m*C*a*c^2*m^10*x^11 + 9444969*(d*x)^m*A*b*c^2*m^10*x^11 + 126245
592*(d*x)^m*B*b*c^2*m^9*x^12 + 1209749541*(d*x)^m*C*b*c^2*m^8*x^13 + 403249
847*(d*x)^m*A*c^3*m^8*x^13 + 2816490248*(d*x)^m*B*c^3*m^7*x^14 + 1440932292
8*(d*x)^m*C*c^3*m^6*x^15 + 3*(d*x)^m*B*a*b^2*m^14*x^6 + 3*(d*x)^m*B*a^2*c*m
^14*x^6 + 339*(d*x)^m*C*a*b^2*m^13*x^7 + 113*(d*x)^m*A*b^3*m^13*x^7 + 339*(
d*x)^m*C*a^2*c*m^13*x^7 + 678*(d*x)^m*A*a*b*c*m^13*x^7 + 5684*(d*x)^m*B*b^3
*m^12*x^8 + 34104*(d*x)^m*B*a*b*c*m^12*x^8 + 168171*(d*x)^m*C*b^3*m^11*x^9
+ 1009026*(d*x)^m*C*a*b*c*m^11*x^9 + 504513*(d*x)^m*A*b^2*c*m^11*x^9 + 5045
13*(d*x)^m*A*a*c^2*m^11*x^9 + 9790866*(d*x)^m*B*b^2*c*m^10*x^10 + 9790866*(
d*x)^m*B*a*c^2*m^10*x^10 + 131780781*(d*x)^m*C*b^2*c*m^9*x^11 + 131780781*(
d*x)^m*C*a*c^2*m^9*x^11 + 131780781*(d*x)^m*A*b*c^2*m^9*x^11 + 1269340116*(
d*x)^m*B*b*c^2*m^8*x^12 + 8896139967*(d*x)^m*C*b*c^2*m^7*x^13 + 2965379989*
(d*x)^m*A*c^3*m^7*x^13 + 15200266081*(d*x)^m*B*c^3*m^6*x^14 + 56663366760*(
d*x)^m*C*c^3*m^5*x^15 + 3*(d*x)^m*C*a^2*b*m^14*x^5 + 3*(d*x)^m*A*a*b^2*m^14
*x^5 + 3*(d*x)^m*A*a^2*c*m^14*x^5 + 342*(d*x)^m*B*a*b^2*m^13*x^6 + 342*(d*x
)^m*B*a^2*c*m^13*x^6 + 17367*(d*x)^m*C*a*b^2*m^12*x^7 + 5789*(d*x)^m*A*b^3*
m^12*x^7 + 17367*(d*x)^m*C*a^2*c*m^12*x^7 + 34734*(d*x)^m*A*a*b*c*m^12*x^7
+ 172928*(d*x)^m*B*b^3*m^11*x^8 + 1037568*(d*x)^m*B*a*b*c*m^11*x^8 + 338608
3*(d*x)^m*C*b^3*m^10*x^9 + 20316498*(d*x)^m*C*a*b*c*m^10*x^9 + 10158249*(d*
```

```
x)^m*A*b^2*c*m^10*x^9 + 10158249*(d*x)^m*A*a*c^2*m^10*x^9 + 137766780*(d*x)
^m*B*b^2*c*m^9*x^10 + 137766780*(d*x)^m*B*a*c^2*m^9*x^10 + 1334698629*(d*x)
^m*C*b^2*c*m^8*x^11 + 1334698629*(d*x)^m*C*a*c^2*m^8*x^11 + 1334698629*(d*x
)^m*A*b*c^2*m^8*x^11 + 9390802608*(d*x)^m*B*b*c^2*m^7*x^12 + 48243569088*(d
*x)^m*C*b*c^2*m^6*x^13 + 16081189696*(d*x)^m*A*c^3*m^6*x^13 + 59999485546*(
d*x)^m*B*c^3*m^5*x^14 + 159721605680*(d*x)^m*C*c^3*m^4*x^15 + 3*(d*x)^m*B*a
^2*b*m^14*x^4 + 345*(d*x)^m*C*a^2*b*m^13*x^5 + 345*(d*x)^m*A*a*b^2*m^13*x^5
 + 345*(d*x)^m*A*a^2*c*m^13*x^5 + 17688*(d*x)^m*B*a*b^2*m^12*x^6 + 17688*(d
*x)^m*B*a^2*c*m^12*x^6 + 533631*(d*x)^m*C*a*b^2*m^11*x^7 + 177877*(d*x)^m*A
*b^3*m^11*x^7 + 533631*(d*x)^m*C*a^2*c*m^11*x^7 + 1067262*(d*x)^m*A*a*b*c*m
^11*x^7 + 3516198*(d*x)^m*B*b^3*m^10*x^8 + 21097188*(d*x)^m*B*a*b*c*m^10*x^
8 + 48083733*(d*x)^m*C*b^3*m^9*x^9 + 288502398*(d*x)^m*C*a*b*c*m^9*x^9 + 14
4251199*(d*x)^m*A*b^2*c*m^9*x^9 + 144251199*(d*x)^m*A*a*c^2*m^9*x^9 + 14066
19420*(d*x)^m*B*b^2*c*m^8*x^10 + 1406619420*(d*x)^m*B*a*c^2*m^8*x^10 + 9941
199081*(d*x)^m*C*b^2*c*m^7*x^11 + 9941199081*(d*x)^m*C*a*c^2*m^7*x^11 + 994
1199081*(d*x)^m*A*b*c^2*m^7*x^11 + 51203757363*(d*x)^m*B*b*c^2*m^6*x^12 + 1
91243233896*(d*x)^m*C*b*c^2*m^5*x^13 + 63747744632*(d*x)^m*A*c^3*m^5*x^13 +
 169679309436*(d*x)^m*B*c^3*m^4*x^14 + 310989260400*(d*x)^m*C*c^3*m^3*x^15
+ (d*x)^m*C*a^3*m^14*x^3 + 3*(d*x)^m*A*a^2*b*m^14*x^3 + 348*(d*x)^m*B*a^2*b
*m^13*x^4 + 18015*(d*x)^m*C*a^2*b*m^12*x^5 + 18015*(d*x)^m*A*a*b^2*m^12*x^5
 + 18015*(d*x)^m*A*a^2*c*m^12*x^5 + 549072*(d*x)^m*B*a*b^2*m^11*x^6 + 54907
2*(d*x)^m*B*a^2*c*m^11*x^6 + 10963449*(d*x)^m*C*a*b^2*m^10*x^7 + 3654483*(d
*x)^m*A*b^3*m^10*x^7 + 10963449*(d*x)^m*C*a^2*c*m^10*x^7 + 21926898*(d*x)^m
*A*a*b*c*m^10*x^7 + 50428896*(d*x)^m*B*b^3*m^9*x^8 + 302573376*(d*x)^m*B*a*
b*c*m^9*x^8 + 495342143*(d*x)^m*C*b^3*m^8*x^9 + 2972052858*(d*x)^m*C*a*b*c*
m^8*x^9 + 1486026429*(d*x)^m*A*b^2*c*m^8*x^9 + 1486026429*(d*x)^m*A*a*c^2*m
^8*x^9 + 10556689800*(d*x)^m*B*b^2*c*m^7*x^10 + 10556689800*(d*x)^m*B*a*c^2
*m^7*x^10 + 54540198768*(d*x)^m*C*b^2*c*m^6*x^11 + 54540198768*(d*x)^m*C*a*
c^2*m^6*x^11 + 54540198768*(d*x)^m*A*b*c^2*m^6*x^11 + 203964543684*(d*x)^m*
B*b*c^2*m^5*x^12 + 542854280592*(d*x)^m*C*b*c^2*m^4*x^13 + 180951426864*(d*
x)^m*A*c^3*m^4*x^13 + 331303013496*(d*x)^m*B*c^3*m^3*x^14 + 392156797824*(d
*x)^m*C*c^3*m^2*x^15 + (d*x)^m*B*a^3*m^14*x^2 + 117*(d*x)^m*C*a^3*m^13*x^3
+ 351*(d*x)^m*A*a^2*b*m^13*x^3 + 18348*(d*x)^m*B*a^2*b*m^12*x^4 + 565125*(d
*x)^m*C*a^2*b*m^11*x^5 + 565125*(d*x)^m*A*a*b^2*m^11*x^5 + 565125*(d*x)^m*A
*a^2*c*m^11*x^5 + 11404434*(d*x)^m*B*a*b^2*m^10*x^6 + 11404434*(d*x)^m*B*a^
2*c*m^10*x^6 + 158931297*(d*x)^m*C*a*b^2*m^9*x^7 + 52977099*(d*x)^m*A*b^3*m
^9*x^7 + 158931297*(d*x)^m*C*a^2*c*m^9*x^7 + 317862594*(d*x)^m*A*a*b*c*m^9*
x^7 + 524664572*(d*x)^m*B*b^3*m^8*x^8 + 3147987432*(d*x)^m*B*a*b*c*m^8*x^8
+ 3749548713*(d*x)^m*C*b^3*m^7*x^9 + 22497292278*(d*x)^m*C*a*b*c*m^7*x^9 +
11248646139*(d*x)^m*A*b^2*c*m^7*x^9 + 11248646139*(d*x)^m*A*a*c^2*m^7*x^9 +
 58326490659*(d*x)^m*B*b^2*c*m^6*x^10 + 58326490659*(d*x)^m*B*a*c^2*m^6*x^1
0 + 218467445592*(d*x)^m*C*b^2*c*m^5*x^11 + 218467445592*(d*x)^m*C*a*c^2*m^
5*x^11 + 218467445592*(d*x)^m*A*b*c^2*m^5*x^11 + 581441797032*(d*x)^m*B*b*c
^2*m^4*x^12 + 1063334389104*(d*x)^m*C*b*c^2*m^3*x^13 + 354444796368*(d*x)^m
*A*c^3*m^3*x^13 + 418753514880*(d*x)^m*B*c^3*m^2*x^14 + 283465647360*(d*x)^
m*C*c^3*m*x^15 + (d*x)^m*A*a^3*m^14*x + 118*(d*x)^m*B*a^3*m^13*x^2 + 6229*(
d*x)^m*C*a^3*m^12*x^3 + 18687*(d*x)^m*A*a^2*b*m^12*x^3 + 581808*(d*x)^m*B*a
^2*b*m^11*x^4 + 11873241*(d*x)^m*C*a^2*b*m^10*x^5 + 11873241*(d*x)^m*A*a*b^
2*m^10*x^5 + 11873241*(d*x)^m*A*a^2*c*m^10*x^5 + 167248836*(d*x)^m*B*a*b^2*
m^9*x^6 + 167248836*(d*x)^m*B*a^2*c*m^9*x^6 + 1671768141*(d*x)^m*C*a*b^2*m^
8*x^7 + 557256047*(d*x)^m*A*b^3*m^8*x^7 + 1671768141*(d*x)^m*C*a^2*c*m^8*x^
7 + 3343536282*(d*x)^m*A*a*b*c*m^8*x^7 + 4010311424*(d*x)^m*B*b^3*m^7*x^8 +
 24061868544*(d*x)^m*B*a*b*c*m^7*x^8 + 20885191136*(d*x)^m*C*b^3*m^6*x^9 +
125311146816*(d*x)^m*C*a*b*c*m^6*x^9 + 62655573408*(d*x)^m*A*b^2*c*m^6*x^9
+ 62655573408*(d*x)^m*A*a*c^2*m^6*x^9 + 235144725450*(d*x)^m*B*b^2*c*m^5*x^
10 + 235144725450*(d*x)^m*B*a*c^2*m^5*x^10 + 625874419728*(d*x)^m*C*b^2*c*m
^4*x^11 + 625874419728*(d*x)^m*C*a*c^2*m^4*x^11 + 625874419728*(d*x)^m*A*b*
c^2*m^4*x^11 + 1143138472416*(d*x)^m*B*b*c^2*m^3*x^12 + 1347640053120*(d*x)
^m*C*b*c^2*m^2*x^13 + 449213351040*(d*x)^m*A*c^3*m^2*x^13 + 303268406400*(d
```

*x)^m*B*c^3*m*x^14 + 87178291200*(d*x)^m*C*c^3*x^15 + 119*(d*x)^m*A*a^3*m^1
3*x + 6344*(d*x)^m*B*a^3*m^12*x^2 + 199713*(d*x)^m*C*a^3*m^11*x^3 + 599139*
(d*x)^m*A*a^2*b*m^11*x^3 + 12371634*(d*x)^m*B*a^2*b*m^10*x^4 + 176309235*(d
*x)^m*C*a^2*b*m^9*x^5 + 176309235*(d*x)^m*A*a*b^2*m^9*x^5 + 176309235*(d*x)
^m*A*a^2*c*m^9*x^5 + 1780794204*(d*x)^m*B*a*b^2*m^8*x^6 + 1780794204*(d*x)^
m*B*a^2*c*m^8*x^6 + 12920507013*(d*x)^m*C*a*b^2*m^7*x^7 + 4306835671*(d*x)^
m*A*b^3*m^7*x^7 + 12920507013*(d*x)^m*C*a^2*c*m^7*x^7 + 25841014026*(d*x)^m
*A*a*b*c*m^7*x^7 + 22548638161*(d*x)^m*B*b^3*m^6*x^8 + 135291828966*(d*x)^m
*B*a*b*c*m^6*x^8 + 84836490456*(d*x)^m*C*b^3*m^5*x^9 + 509018942736*(d*x)^m
*C*a*b*c*m^5*x^9 + 254509471368*(d*x)^m*A*b^2*c*m^5*x^9 + 254509471368*(d*x
)^m*A*a*c^2*m^5*x^9 + 677569066740*(d*x)^m*B*b^2*c*m^4*x^10 + 677569066740*
(d*x)^m*B*a*c^2*m^4*x^10 + 1235821419792*(d*x)^m*C*b^2*c*m^3*x^11 + 1235821
419792*(d*x)^m*C*a*c^2*m^3*x^11 + 1235821419792*(d*x)^m*A*b*c^2*m^3*x^11 +
1453325442480*(d*x)^m*B*b*c^2*m^2*x^12 + 978132153600*(d*x)^m*C*b*c^2*m*x^1
3 + 326044051200*(d*x)^m*A*c^3*m*x^13 + 93405312000*(d*x)^m*B*c^3*x^14 + 64
61*(d*x)^m*A*a^3*m^12*x + 205712*(d*x)^m*B*a^3*m^11*x^2 + 4300483*(d*x)^m*C
*a^3*m^10*x^3 + 12901449*(d*x)^m*A*a^2*b*m^10*x^3 + 186188904*(d*x)^m*B*a^2
*b*m^9*x^4 + 1902741045*(d*x)^m*C*a^2*b*m^8*x^5 + 1902741045*(d*x)^m*A*a*b^
2*m^8*x^5 + 1902741045*(d*x)^m*A*a^2*c*m^8*x^5 + 13938118776*(d*x)^m*B*a*b^
2*m^7*x^6 + 13938118776*(d*x)^m*B*a^2*c*m^7*x^6 + 73449839568*(d*x)^m*C*a*b
^2*m^6*x^7 + 24483279856*(d*x)^m*A*b^3*m^6*x^7 + 73449839568*(d*x)^m*C*a^2*
c*m^6*x^7 + 146899679136*(d*x)^m*A*a*b*c*m^6*x^7 + 92414105392*(d*x)^m*B*b^
3*m^5*x^8 + 554484632352*(d*x)^m*B*a*b*c*m^5*x^8 + 246143692976*(d*x)^m*C*b
^3*m^4*x^9 + 1476862157856*(d*x)^m*C*a*b*c*m^4*x^9 + 738431078928*(d*x)^m*A
*b^2*c*m^4*x^9 + 738431078928*(d*x)^m*A*a*c^2*m^4*x^9 + 1344749369400*(d*x)
^m*B*b^2*c*m^3*x^10 + 1344749369400*(d*x)^m*B*a*c^2*m^3*x^10 + 157695149376
0*(d*x)^m*C*b^2*c*m^2*x^11 + 1576951493760*(d*x)^m*C*a*c^2*m^2*x^11 + 15769
51493760*(d*x)^m*A*b*c^2*m^2*x^11 + 1057547534400*(d*x)^m*B*b*c^2*m*x^12 +
301771008000*(d*x)^m*C*b*c^2*x^13 + 100590336000*(d*x)^m*A*c^3*x^13 + 21193
9*(d*x)^m*A*a^3*m^11*x + 4488198*(d*x)^m*B*a^3*m^10*x^2 + 65657031*(d*x)^m*
C*a^3*m^9*x^3 + 196971093*(d*x)^m*A*a^2*b*m^9*x^3 + 2039531604*(d*x)^m*B*a^
2*b*m^8*x^4 + 15109178775*(d*x)^m*C*a^2*b*m^7*x^5 + 15109178775*(d*x)^m*A*a
*b^2*m^7*x^5 + 15109178775*(d*x)^m*A*a^2*c*m^7*x^5 + 80264676003*(d*x)^m*B*
a*b^2*m^6*x^6 + 80264676003*(d*x)^m*B*a^2*c*m^6*x^6 + 304260755064*(d*x)^m*
C*a*b^2*m^5*x^7 + 101420251688*(d*x)^m*A*b^3*m^5*x^7 + 304260755064*(d*x)^m
*C*a^2*c*m^5*x^7 + 608521510128*(d*x)^m*A*a*b*c*m^5*x^7 + 270359263944*(d*x
)^m*B*b^3*m^4*x^8 + 1622155583664*(d*x)^m*B*a*b*c*m^4*x^8 + 491520108816*(d
*x)^m*C*b^3*m^3*x^9 + 2949120652896*(d*x)^m*C*a*b*c*m^3*x^9 + 1474560326448
*(d*x)^m*A*b^2*c*m^3*x^9 + 1474560326448*(d*x)^m*A*a*c^2*m^3*x^9 + 17234934
17472*(d*x)^m*B*b^2*c*m^2*x^10 + 1723493417472*(d*x)^m*B*a*c^2*m^2*x^10 + 1
150986412800*(d*x)^m*C*b^2*c*m*x^11 + 1150986412800*(d*x)^m*C*a*c^2*m*x^11
+ 1150986412800*(d*x)^m*A*b*c^2*m*x^11 + 326918592000*(d*x)^m*B*b*c^2*x^12
+ 4687683*(d*x)^m*A*a^3*m^10*x + 69582084*(d*x)^m*B*a^3*m^9*x^2 + 731124647
*(d*x)^m*C*a^3*m^8*x^3 + 2193373941*(d*x)^m*A*a^2*b*m^8*x^3 + 16464757584*(
d*x)^m*B*a^2*b*m^7*x^4 + 88347494784*(d*x)^m*C*a^2*b*m^6*x^5 + 88347494784*
(d*x)^m*A*a*b^2*m^6*x^5 + 88347494784*(d*x)^m*A*a^2*c*m^6*x^5 + 33682157602
2*(d*x)^m*B*a*b^2*m^5*x^6 + 336821576022*(d*x)^m*B*a^2*c*m^5*x^6 + 89919103
5792*(d*x)^m*C*a*b^2*m^4*x^7 + 299730345264*(d*x)^m*A*b^3*m^4*x^7 + 8991910
35792*(d*x)^m*C*a^2*c*m^4*x^7 + 1798382071584*(d*x)^m*A*a*b*c*m^4*x^7 + 543
939234048*(d*x)^m*B*b^3*m^3*x^8 + 3263635404288*(d*x)^m*B*a*b*c*m^3*x^8 + 6
33314724480*(d*x)^m*C*b^3*m^2*x^9 + 3799888346880*(d*x)^m*C*a*b*c*m^2*x^9 +
1899944173440*(d*x)^m*A*b^2*c*m^2*x^9 + 1899944173440*(d*x)^m*A*a*c^2*m^2*
x^9 + 1262518669440*(d*x)^m*B*b^2*c*m*x^10 + 1262518669440*(d*x)^m*B*a*c^2*
m*x^10 + 356638464000*(d*x)^m*C*b^2*c*x^11 + 356638464000*(d*x)^m*C*a*c^2*x
^11 + 356638464000*(d*x)^m*A*b*c^2*x^11 + 73870797*(d*x)^m*A*a^3*m^9*x + 78
8931572*(d*x)^m*B*a^3*m^8*x^2 + 6014254059*(d*x)^m*C*a^3*m^7*x^3 + 18042762
177*(d*x)^m*A*a^2*b*m^7*x^3 + 98034358323*(d*x)^m*B*a^2*b*m^6*x^4 + 3766721
58120*(d*x)^m*C*a^2*b*m^5*x^5 + 376672158120*(d*x)^m*A*a*b^2*m^5*x^5 + 3766
72158120*(d*x)^m*A*a^2*c*m^5*x^5 + 1008086865108*(d*x)^m*B*a*b^2*m^4*x^6 +

```
1008086865108*(d*x)^m*B*a^2*c*m^4*x^6 + 1826102786256*(d*x)^m*C*a*b^2*m^3*x
^7 + 608700928752*(d*x)^m*A*b^3*m^3*x^7 + 1826102786256*(d*x)^m*C*a^2*c*m^3
*x^7 + 3652205572512*(d*x)^m*A*a*b*c*m^3*x^7 + 705481831440*(d*x)^m*B*b^3*m
^2*x^8 + 4232890988640*(d*x)^m*B*a*b*c*m^2*x^8 + 465985094400*(d*x)^m*C*b^3
*m*x^9 + 2795910566400*(d*x)^m*C*a*b*c*m*x^9 + 1397955283200*(d*x)^m*A*b^2*
c*m*x^9 + 1397955283200*(d*x)^m*A*a*c^2*m*x^9 + 392302310400*(d*x)^m*B*b^2*
c*x^10 + 392302310400*(d*x)^m*B*a*c^2*x^10 + 854224943*(d*x)^m*A*a^3*m^8*x
+ 6629764856*(d*x)^m*B*a^3*m^7*x^2 + 36588367376*(d*x)^m*C*a^3*m^6*x^3 + 10
9765102128*(d*x)^m*A*a^2*b*m^6*x^3 + 426272198748*(d*x)^m*B*a^2*b*m^5*x^4 +
 1145655530640*(d*x)^m*C*a^2*b*m^4*x^5 + 1145655530640*(d*x)^m*A*a*b^2*m^4*
x^5 + 1145655530640*(d*x)^m*A*a^2*c*m^4*x^5 + 2071918846152*(d*x)^m*B*a*b^2
*m^3*x^6 + 2071918846152*(d*x)^m*B*a^2*c*m^3*x^6 + 2388267607680*(d*x)^m*C*
a*b^2*m^2*x^7 + 796089202560*(d*x)^m*A*b^3*m^2*x^7 + 2388267607680*(d*x)^m*
C*a^2*c*m^2*x^7 + 4776535215360*(d*x)^m*A*a*b*c*m^2*x^7 + 521962963200*(d*x
)^m*B*b^3*m*x^8 + 3131777779200*(d*x)^m*B*a*b*c*m*x^8 + 145297152000*(d*x)^
m*C*b^3*x^9 + 871782912000*(d*x)^m*C*a*b*c*x^9 + 435891456000*(d*x)^m*A*b^2
*c*x^9 + 435891456000*(d*x)^m*A*a*c^2*x^9 + 7353403057*(d*x)^m*A*a^3*m^7*x
+ 41371599841*(d*x)^m*B*a^3*m^6*x^2 + 163038108552*(d*x)^m*C*a^3*m^5*x^3 +
489114325656*(d*x)^m*A*a^2*b*m^5*x^3 + 1323927526248*(d*x)^m*B*a^2*b*m^4*x^
4 + 2392162383600*(d*x)^m*C*a^2*b*m^3*x^5 + 2392162383600*(d*x)^m*A*a*b^2*m
^3*x^5 + 2392162383600*(d*x)^m*A*a^2*c*m^3*x^5 + 2739474034560*(d*x)^m*B*a*
b^2*m^2*x^6 + 2739474034560*(d*x)^m*B*a^2*c*m^2*x^6 + 1779579590400*(d*x)^m
*C*a*b^2*m*x^7 + 593193196800*(d*x)^m*A*b^3*m*x^7 + 1779579590400*(d*x)^m*C
*a^2*c*m*x^7 + 3559159180800*(d*x)^m*A*a*b*c*m*x^7 + 163459296000*(d*x)^m*B
*b^3*x^8 + 980755776000*(d*x)^m*B*a*b*c*x^8 + 47277726496*(d*x)^m*A*a^3*m^6
*x + 190060010998*(d*x)^m*B*a^3*m^5*x^2 + 520557781424*(d*x)^m*C*a^3*m^4*x^
3 + 1561673344272*(d*x)^m*A*a^2*b*m^4*x^3 + 2824729931808*(d*x)^m*B*a^2*b*m
^3*x^4 + 3210175193472*(d*x)^m*C*a^2*b*m^2*x^5 + 3210175193472*(d*x)^m*A*a*
b^2*m^2*x^5 + 3210175193472*(d*x)^m*A*a^2*c*m^2*x^5 + 2060608636800*(d*x)^m
*B*a*b^2*m*x^6 + 2060608636800*(d*x)^m*B*a^2*c*m*x^6 + 560431872000*(d*x)^m
*C*a*b^2*x^7 + 186810624000*(d*x)^m*A*b^3*x^7 + 560431872000*(d*x)^m*C*a^2*
c*x^7 + 1120863744000*(d*x)^m*A*a*b*c*x^7 + 225525484184*(d*x)^m*A*a^3*m^5*
x + 629552085084*(d*x)^m*B*a^3*m^4*x^2 + 1145140001328*(d*x)^m*C*a^3*m^3*x^
3 + 3435420003984*(d*x)^m*A*a^2*b*m^3*x^3 + 3872067384240*(d*x)^m*B*a^2*b*m
^2*x^4 + 2446576876800*(d*x)^m*C*a^2*b*m*x^5 + 2446576876800*(d*x)^m*A*a*b^
2*m*x^5 + 2446576876800*(d*x)^m*A*a^2*c*m*x^5 + 653837184000*(d*x)^m*B*a*b^
2*x^6 + 653837184000*(d*x)^m*B*a^2*c*x^6 + 784146622896*(d*x)^m*A*a^3*m^4*x
 + 1447709175432*(d*x)^m*B*a^3*m^3*x^2 + 1621575699840*(d*x)^m*C*a^3*m^2*x^
3 + 4864727099520*(d*x)^m*A*a^2*b*m^2*x^3 + 3009183307200*(d*x)^m*B*a^2*b*m
*x^4 + 784604620800*(d*x)^m*C*a^2*b*x^5 + 784604620800*(d*x)^m*A*a*b^2*x^5
+ 784604620800*(d*x)^m*A*a^2*c*x^5 + 1922666722704*(d*x)^m*A*a^3*m^3*x + 21
61577352960*(d*x)^m*B*a^3*m^2*x^2 + 1301090515200*(d*x)^m*C*a^3*m*x^3 + 390
3271545600*(d*x)^m*A*a^2*b*m*x^3 + 980755776000*(d*x)^m*B*a^2*b*x^4 + 31343
28981120*(d*x)^m*A*a^3*m^2*x + 1842662908800*(d*x)^m*B*a^3*m*x^2 + 43589145
6000*(d*x)^m*C*a^3*x^3 + 1307674368000*(d*x)^m*A*a^2*b*x^3 + 3031488633600*
(d*x)^m*A*a^3*m*x + 653837184000*(d*x)^m*B*a^3*x^2 + 1307674368000*(d*x)^m*
A*a^3*x)/(m^15 + 120*m^14 + 6580*m^13 + 218400*m^12 + 4899622*m^11 + 785584
80*m^10 + 928095740*m^9 + 8207628000*m^8 + 54631129553*m^7 + 272803210680*m
^6 + 1009672107080*m^5 + 2706813345600*m^4 + 5056995703824*m^3 + 6165817614
720*m^2 + 4339163001600*m + 1307674368000)
```

3.38 $\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2 dx$

Optimal. Leaf size=260

$$\frac{a^2A(dx)^{m+1}}{d(m+1)}+\frac{a^2B(dx)^{m+2}}{d^2(m+2)}+\frac{(dx)^{m+5}\left(A\left(2ac+b^2\right)+2abC\right)}{d^5(m+5)}+\frac{(dx)^{m+7}\left(C\left(2ac+b^2\right)+2Abc\right)}{d^7(m+7)}+\frac{a(dx)^{m+3}(aC+2Ab)}{d^3(m+3)}+$$

```
[Out] (a^2*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^2*B*(d*x)^(2 + m))/(d^2*(2 + m)) + (
a*(2*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (2*a*b*B*(d*x)^(4 + m))/(d^4
*(4 + m)) + ((A*(b^2 + 2*a*c) + 2*a*b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) + (B*
(b^2 + 2*a*c)*(d*x)^(6 + m))/(d^6*(6 + m)) + ((2*A*b*c + (b^2 + 2*a*c)*C)*(
d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*B*c*(d*x)^(8 + m))/(d^8*(8 + m)) + (c*(A
*c + 2*b*C)*(d*x)^(9 + m))/(d^9*(9 + m)) + (B*c^2*(d*x)^(10 + m))/(d^10*(10
 + m)) + (c^2*C*(d*x)^(11 + m))/(d^11*(11 + m))
```

Rubi [A] time = 0.222501, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1628}

$$\frac{a^2A(dx)^{m+1}}{d(m+1)}+\frac{a^2B(dx)^{m+2}}{d^2(m+2)}+\frac{(dx)^{m+5}\left(A\left(2ac+b^2\right)+2abC\right)}{d^5(m+5)}+\frac{(dx)^{m+7}\left(C\left(2ac+b^2\right)+2Abc\right)}{d^7(m+7)}+\frac{a(dx)^{m+3}(aC+2Ab)}{d^3(m+3)}+$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^2*B*(d*x)^(2 + m))/(d^2*(2 + m)) + (
a*(2*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (2*a*b*B*(d*x)^(4 + m))/(d^4
*(4 + m)) + ((A*(b^2 + 2*a*c) + 2*a*b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) + (B*
(b^2 + 2*a*c)*(d*x)^(6 + m))/(d^6*(6 + m)) + ((2*A*b*c + (b^2 + 2*a*c)*C)*(
d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*B*c*(d*x)^(8 + m))/(d^8*(8 + m)) + (c*(A
*c + 2*b*C)*(d*x)^(9 + m))/(d^9*(9 + m)) + (B*c^2*(d*x)^(10 + m))/(d^10*(10
 + m)) + (c^2*C*(d*x)^(11 + m))/(d^11*(11 + m))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2 dx = \int \left(a^2A(dx)^m+\frac{a^2B(dx)^{1+m}}{d}+\frac{a(2Ab+aC)(dx)^{2+m}}{d^2}+\frac{2abB(dx)^{3+m}}{d^3}+\right.$$
$$=\frac{a^2A(dx)^{1+m}}{d(1+m)}+\frac{a^2B(dx)^{2+m}}{d^2(2+m)}+\frac{a(2Ab+aC)(dx)^{3+m}}{d^3(3+m)}+\frac{2abB(dx)^{4+m}}{d^4(4+m)}+$$

Mathematica [A] time = 0.38257, size = 185, normalized size = 0.71

$$x(dx)^m\left(\frac{a^2A}{m+1}+\frac{a^2Bx}{m+2}+\frac{x^6\left(C\left(2ac+b^2\right)+2Abc\right)}{m+7}+\frac{x^4\left(A\left(2ac+b^2\right)+2abC\right)}{m+5}+\frac{ax^2(aC+2Ab)}{m+3}+\frac{Bx^5\left(2ac+b^2\right)}{m+6}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] x*(d*x)^m*((a^2*A)/(1 + m) + (a^2*B*x)/(2 + m) + (a*(2*A*b + a*C)*x^2)/(3 +
 m) + (2*a*b*B*x^3)/(4 + m) + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^4)/(5 + m) + (
B*(b^2 + 2*a*c)*x^5)/(6 + m) + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^6)/(7 + m) +
(2*b*B*c*x^7)/(8 + m) + (c*(A*c + 2*b*C)*x^8)/(9 + m) + (B*c^2*x^9)/(10 + m
) + (c^2*C*x^10)/(11 + m))
```

Maple [B] time = 0.01, size = 2187, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] x*(C*c^2*m^10*x^10+B*c^2*m^10*x^9+55*C*c^2*m^9*x^10+A*c^2*m^10*x^8+56*B*c^2
*m^9*x^9+2*C*b*c*m^10*x^8+1320*C*c^2*m^8*x^10+57*A*c^2*m^9*x^8+2*B*b*c*m^10
*x^7+1365*B*c^2*m^8*x^9+114*C*b*c*m^9*x^8+18150*C*c^2*m^7*x^10+2*A*b*c*m^10
*x^6+1412*A*c^2*m^8*x^8+116*B*b*c*m^9*x^7+19020*B*c^2*m^7*x^9+2*C*a*c*m^10*
x^6+C*b^2*m^10*x^6+2824*C*b*c*m^8*x^8+157773*C*c^2*m^6*x^10+118*A*b*c*m^9*x
^6+19962*A*c^2*m^7*x^8+2*B*a*c*m^10*x^5+B*b^2*m^10*x^5+2922*B*b*c*m^8*x^7+1
67223*B*c^2*m^6*x^9+118*C*a*c*m^9*x^6+59*C*b^2*m^9*x^6+39924*C*b*c*m^7*x^8+
902055*C*c^2*m^5*x^10+2*A*a*c*m^10*x^4+A*b^2*m^10*x^4+3024*A*b*c*m^8*x^6+17
7765*A*c^2*m^6*x^8+120*B*a*c*m^9*x^5+60*B*b^2*m^9*x^5+41964*B*b*c*m^7*x^7+9
65328*B*c^2*m^5*x^9+2*C*a*b*m^10*x^4+3024*C*a*c*m^8*x^6+1512*C*b^2*m^8*x^6+
355530*C*b*c*m^6*x^8+3416930*C*c^2*m^4*x^10+122*A*a*c*m^9*x^4+61*A*b^2*m^9*
x^4+44172*A*b*c*m^7*x^6+1037673*A*c^2*m^5*x^8+2*B*a*b*m^10*x^3+3130*B*a*c*m
^8*x^5+1565*B*b^2*m^8*x^5+379134*B*b*c*m^6*x^7+3686255*B*c^2*m^4*x^9+122*C*
a*b*m^9*x^4+44172*C*a*c*m^7*x^6+22086*C*b^2*m^7*x^6+2075346*C*b*c*m^5*x^8+8
409500*C*c^2*m^3*x^10+2*A*a*b*m^10*x^2+3240*A*a*c*m^8*x^4+1620*A*b^2*m^8*x^
4+405642*A*b*c*m^6*x^6+4000478*A*c^2*m^4*x^8+124*B*a*b*m^9*x^3+46560*B*a*c*
m^7*x^5+23280*B*b^2*m^7*x^5+2242044*B*b*c*m^5*x^7+9133180*B*c^2*m^3*x^9+C*a
^2*m^10*x^2+3240*C*a*b*m^8*x^4+405642*C*a*c*m^6*x^6+202821*C*b^2*m^6*x^6+80
00956*C*b*c*m^4*x^8+12753576*C*c^2*m^2*x^10+126*A*a*b*m^9*x^2+49140*A*a*c*m
^7*x^4+24570*A*b^2*m^7*x^4+2435622*A*b*c*m^5*x^6+9991428*A*c^2*m^3*x^8+B*a^
2*m^10*x+3354*B*a*b*m^8*x^3+435486*B*a*c*m^6*x^5+217743*B*b^2*m^6*x^5+87427
18*B*b*c*m^4*x^7+13926276*B*c^2*m^2*x^9+63*C*a^2*m^9*x^2+49140*C*a*b*m^7*x^
4+2435622*C*a*c*m^5*x^6+1217811*C*b^2*m^5*x^6+19982856*C*b*c*m^3*x^8+106286
40*C*c^2*m*x^10+A*a^2*m^10+3472*A*a*b*m^8*x^2+469146*A*a*c*m^6*x^4+234573*A
*b^2*m^6*x^4+9629716*A*b*c*m^4*x^6+15335224*A*c^2*m^2*x^8+64*B*a^2*m^9*x+51
924*B*a*b*m^7*x^3+2662200*B*a*c*m^5*x^5+1331100*B*b^2*m^5*x^5+22049716*B*b*
c*m^3*x^7+11655216*B*c^2*m*x^9+1736*C*a^2*m^8*x^2+469146*C*a*b*m^6*x^4+9629
716*C*a*c*m^4*x^6+4814858*C*b^2*m^4*x^6+30670448*C*b*c*m^2*x^8+3628800*C*c^
2*x^10+65*A*a^2*m^9+54924*A*a*b*m^7*x^2+2929386*A*a*c*m^5*x^4+1464693*A*b^2
*m^5*x^4+24583448*A*b*c*m^3*x^6+12900960*A*c^2*m*x^8+1797*B*a^2*m^8*x+50715
0*B*a*b*m^6*x^3+10705870*B*a*c*m^4*x^5+5352935*B*b^2*m^4*x^5+34118424*B*b*c
*m^2*x^7+3991680*B*c^2*x^9+27462*C*a^2*m^7*x^2+2929386*C*a*b*m^5*x^4+245834
48*C*a*c*m^3*x^6+12291724*C*b^2*m^3*x^6+25801920*C*b*c*m*x^8+1860*A*a^2*m^8
+550074*A*a*b*m^6*x^2+12032140*A*a*c*m^4*x^4+6016070*A*b^2*m^4*x^4+38432016
*A*b*c*m^2*x^6+4435200*A*c^2*x^8+29076*B*a^2*m^7*x+3246516*B*a*b*m^5*x^3+27
756240*B*a*c*m^3*x^5+13878120*B*b^2*m^3*x^5+28888560*B*b*c*m*x^7+275037*C*a
^2*m^6*x^2+12032140*C*a*b*m^4*x^4+38432016*C*a*c*m^2*x^6+19216008*C*b^2*m^2
*x^6+8870400*C*b*c*x^8+30810*A*a^2*m^7+3624894*A*a*b*m^5*x^2+31830760*A*a*c
*m^3*x^4+15915380*A*b^2*m^3*x^4+32811840*A*b*c*m*x^6+299271*B*a^2*m^6*x+136
```

```
93006*B*a*b*m^4*x^3+43978712*B*a*c*m^2*x^5+21989356*B*b^2*m^2*x^5+9979200*B
*b*c*x^7+1812447*C*a^2*m^5*x^2+31830760*C*a*b*m^3*x^4+32811840*C*a*c*m*x^6+
16405920*C*b^2*m*x^6+326613*A*a^2*m^6+15804388*A*a*b*m^4*x^2+51362352*A*a*c
*m^2*x^4+25681176*A*b^2*m^2*x^4+11404800*A*b*c*x^6+2039016*B*a^2*m^5*x+3721
9436*B*a*b*m^3*x^3+37963680*B*a*c*m*x^5+18981840*B*b^2*m*x^5+7902194*C*a^2*
m^4*x^2+51362352*C*a*b*m^2*x^4+11404800*C*a*c*x^6+5702400*C*b^2*x^6+2310945
*A*a^2*m^5+44578296*A*a*b*m^3*x^2+45024192*A*a*c*m*x^4+22512096*A*b^2*m*x^4
+9261503*B*a^2*m^4*x+61638408*B*a*b*m^2*x^3+13305600*B*a*c*x^5+6652800*B*b^
2*x^5+22289148*C*a^2*m^3*x^2+45024192*C*a*b*m*x^4+11028590*A*a^2*m^4+767812
64*A*a*b*m^2*x^2+15966720*A*a*c*x^4+7983360*A*b^2*x^4+27472724*B*a^2*m^3*x+
55282320*B*a*b*m*x^3+38390632*C*a^2*m^2*x^2+15966720*C*a*b*x^4+34967140*A*a
^2*m^3+71492160*A*a*b*m*x^2+50312628*B*a^2*m^2*x+19958400*B*a*b*x^3+3574608
0*C*a^2*m*x^2+70290936*A*a^2*m^2+26611200*A*a*b*x^2+50292720*B*a^2*m*x+1330
5600*C*a^2*x^2+80627040*A*a^2*m+19958400*B*a^2*x+39916800*A*a^2)*(d*x)^m/(1
1+m)/(10+m)/(9+m)/(8+m)/(7+m)/(6+m)/(5+m)/(4+m)/(3+m)/(2+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.82465, size = 4469, normalized size = 17.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((C*c^2*m^10 + 55*C*c^2*m^9 + 1320*C*c^2*m^8 + 18150*C*c^2*m^7 + 157773*C*c
^2*m^6 + 902055*C*c^2*m^5 + 3416930*C*c^2*m^4 + 8409500*C*c^2*m^3 + 1275357
6*C*c^2*m^2 + 10628640*C*c^2*m + 3628800*C*c^2)*x^11 + (B*c^2*m^10 + 56*B*c
^2*m^9 + 1365*B*c^2*m^8 + 19020*B*c^2*m^7 + 167223*B*c^2*m^6 + 965328*B*c^2
*m^5 + 3686255*B*c^2*m^4 + 9133180*B*c^2*m^3 + 13926276*B*c^2*m^2 + 1165521
6*B*c^2*m + 3991680*B*c^2)*x^10 + ((2*C*b*c + A*c^2)*m^10 + 57*(2*C*b*c + A
*c^2)*m^9 + 1412*(2*C*b*c + A*c^2)*m^8 + 19962*(2*C*b*c + A*c^2)*m^7 + 1777
65*(2*C*b*c + A*c^2)*m^6 + 1037673*(2*C*b*c + A*c^2)*m^5 + 4000478*(2*C*b*c
 + A*c^2)*m^4 + 9991428*(2*C*b*c + A*c^2)*m^3 + 8870400*C*b*c + 4435200*A*c
^2 + 15335224*(2*C*b*c + A*c^2)*m^2 + 12900960*(2*C*b*c + A*c^2)*m)*x^9 + 2
*(B*b*c*m^10 + 58*B*b*c*m^9 + 1461*B*b*c*m^8 + 20982*B*b*c*m^7 + 189567*B*b
*c*m^6 + 1121022*B*b*c*m^5 + 4371359*B*b*c*m^4 + 11024858*B*b*c*m^3 + 17059
212*B*b*c*m^2 + 14444280*B*b*c*m + 4989600*B*b*c)*x^8 + ((C*b^2 + 2*(C*a +
A*b)*c)*m^10 + 59*(C*b^2 + 2*(C*a + A*b)*c)*m^9 + 1512*(C*b^2 + 2*(C*a + A*
b)*c)*m^8 + 22086*(C*b^2 + 2*(C*a + A*b)*c)*m^7 + 202821*(C*b^2 + 2*(C*a +
A*b)*c)*m^6 + 1217811*(C*b^2 + 2*(C*a + A*b)*c)*m^5 + 4814858*(C*b^2 + 2*(C
*a + A*b)*c)*m^4 + 12291724*(C*b^2 + 2*(C*a + A*b)*c)*m^3 + 5702400*C*b^2 +
 19216008*(C*b^2 + 2*(C*a + A*b)*c)*m^2 + 11404800*(C*a + A*b)*c + 16405920
*(C*b^2 + 2*(C*a + A*b)*c)*m)*x^7 + ((B*b^2 + 2*B*a*c)*m^10 + 60*(B*b^2 + 2
*B*a*c)*m^9 + 1565*(B*b^2 + 2*B*a*c)*m^8 + 23280*(B*b^2 + 2*B*a*c)*m^7 + 21
7743*(B*b^2 + 2*B*a*c)*m^6 + 1331100*(B*b^2 + 2*B*a*c)*m^5 + 5352935*(B*b^2
```

```
 + 2*B*a*c)*m^4 + 13878120*(B*b^2 + 2*B*a*c)*m^3 + 6652800*B*b^2 + 13305600
*B*a*c + 21989356*(B*b^2 + 2*B*a*c)*m^2 + 18981840*(B*b^2 + 2*B*a*c)*m)*x^6
 + ((2*C*a*b + A*b^2 + 2*A*a*c)*m^10 + 61*(2*C*a*b + A*b^2 + 2*A*a*c)*m^9 +
 1620*(2*C*a*b + A*b^2 + 2*A*a*c)*m^8 + 24570*(2*C*a*b + A*b^2 + 2*A*a*c)*m
^7 + 234573*(2*C*a*b + A*b^2 + 2*A*a*c)*m^6 + 1464693*(2*C*a*b + A*b^2 + 2*
A*a*c)*m^5 + 6016070*(2*C*a*b + A*b^2 + 2*A*a*c)*m^4 + 15915380*(2*C*a*b +
A*b^2 + 2*A*a*c)*m^3 + 15966720*C*a*b + 7983360*A*b^2 + 15966720*A*a*c + 25
681176*(2*C*a*b + A*b^2 + 2*A*a*c)*m^2 + 22512096*(2*C*a*b + A*b^2 + 2*A*a*
c)*m)*x^5 + 2*(B*a*b*m^10 + 62*B*a*b*m^9 + 1677*B*a*b*m^8 + 25962*B*a*b*m^7
 + 253575*B*a*b*m^6 + 1623258*B*a*b*m^5 + 6846503*B*a*b*m^4 + 18609718*B*a*
b*m^3 + 30819204*B*a*b*m^2 + 27641160*B*a*b*m + 9979200*B*a*b)*x^4 + ((C*a^
2 + 2*A*a*b)*m^10 + 63*(C*a^2 + 2*A*a*b)*m^9 + 1736*(C*a^2 + 2*A*a*b)*m^8 +
 27462*(C*a^2 + 2*A*a*b)*m^7 + 275037*(C*a^2 + 2*A*a*b)*m^6 + 1812447*(C*a^
2 + 2*A*a*b)*m^5 + 7902194*(C*a^2 + 2*A*a*b)*m^4 + 22289148*(C*a^2 + 2*A*a*
b)*m^3 + 13305600*C*a^2 + 26611200*A*a*b + 38390632*(C*a^2 + 2*A*a*b)*m^2 +
 35746080*(C*a^2 + 2*A*a*b)*m)*x^3 + (B*a^2*m^10 + 64*B*a^2*m^9 + 1797*B*a^
2*m^8 + 29076*B*a^2*m^7 + 299271*B*a^2*m^6 + 2039016*B*a^2*m^5 + 9261503*B*
a^2*m^4 + 27472724*B*a^2*m^3 + 50312628*B*a^2*m^2 + 50292720*B*a^2*m + 1995
8400*B*a^2)*x^2 + (A*a^2*m^10 + 65*A*a^2*m^9 + 1860*A*a^2*m^8 + 30810*A*a^2
*m^7 + 326613*A*a^2*m^6 + 2310945*A*a^2*m^5 + 11028590*A*a^2*m^4 + 34967140
*A*a^2*m^3 + 70290936*A*a^2*m^2 + 80627040*A*a^2*m + 39916800*A*a^2)*x)*(d*
x)^m/(m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 + 2637558*m^6 + 13
339535*m^5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2 + 120543840*m + 3
9916800)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

Giac [B] time = 1.19524, size = 4324, normalized size = 16.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] ((d*x)^m*C*c^2*m^10*x^11 + (d*x)^m*B*c^2*m^10*x^10 + 55*(d*x)^m*C*c^2*m^9*x
^11 + 2*(d*x)^m*C*b*c*m^10*x^9 + (d*x)^m*A*c^2*m^10*x^9 + 56*(d*x)^m*B*c^2*
m^9*x^10 + 1320*(d*x)^m*C*c^2*m^8*x^11 + 2*(d*x)^m*B*b*c*m^10*x^8 + 114*(d*
x)^m*C*b*c*m^9*x^9 + 57*(d*x)^m*A*c^2*m^9*x^9 + 1365*(d*x)^m*B*c^2*m^8*x^10
 + 18150*(d*x)^m*C*c^2*m^7*x^11 + (d*x)^m*C*b^2*m^10*x^7 + 2*(d*x)^m*C*a*c*
m^10*x^7 + 2*(d*x)^m*A*b*c*m^10*x^7 + 116*(d*x)^m*B*b*c*m^9*x^8 + 2824*(d*x
)^m*C*b*c*m^8*x^9 + 1412*(d*x)^m*A*c^2*m^8*x^9 + 19020*(d*x)^m*B*c^2*m^7*x^
10 + 157773*(d*x)^m*C*c^2*m^6*x^11 + (d*x)^m*B*b^2*m^10*x^6 + 2*(d*x)^m*B*a
*c*m^10*x^6 + 59*(d*x)^m*C*b^2*m^9*x^7 + 118*(d*x)^m*C*a*c*m^9*x^7 + 118*(d
*x)^m*A*b*c*m^9*x^7 + 2922*(d*x)^m*B*b*c*m^8*x^8 + 39924*(d*x)^m*C*b*c*m^7*
x^9 + 19962*(d*x)^m*A*c^2*m^7*x^9 + 167223*(d*x)^m*B*c^2*m^6*x^10 + 902055*
```

```
(d*x)^m*C*c^2*m^5*x^11 + 2*(d*x)^m*C*a*b*m^10*x^5 + (d*x)^m*A*b^2*m^10*x^5
+ 2*(d*x)^m*A*a*c*m^10*x^5 + 60*(d*x)^m*B*b^2*m^9*x^6 + 120*(d*x)^m*B*a*c*m
^9*x^6 + 1512*(d*x)^m*C*b^2*m^8*x^7 + 3024*(d*x)^m*C*a*c*m^8*x^7 + 3024*(d*
x)^m*A*b*c*m^8*x^7 + 41964*(d*x)^m*B*b*c*m^7*x^8 + 355530*(d*x)^m*C*b*c*m^6
*x^9 + 177765*(d*x)^m*A*c^2*m^6*x^9 + 965328*(d*x)^m*B*c^2*m^5*x^10 + 34169
30*(d*x)^m*C*c^2*m^4*x^11 + 2*(d*x)^m*B*a*b*m^10*x^4 + 122*(d*x)^m*C*a*b*m^
9*x^5 + 61*(d*x)^m*A*b^2*m^9*x^5 + 122*(d*x)^m*A*a*c*m^9*x^5 + 1565*(d*x)^m
*B*b^2*m^8*x^6 + 3130*(d*x)^m*B*a*c*m^8*x^6 + 22086*(d*x)^m*C*b^2*m^7*x^7 +
 44172*(d*x)^m*C*a*c*m^7*x^7 + 44172*(d*x)^m*A*b*c*m^7*x^7 + 379134*(d*x)^m
*B*b*c*m^6*x^8 + 2075346*(d*x)^m*C*b*c*m^5*x^9 + 1037673*(d*x)^m*A*c^2*m^5*
x^9 + 3686255*(d*x)^m*B*c^2*m^4*x^10 + 8409500*(d*x)^m*C*c^2*m^3*x^11 + (d*
x)^m*C*a^2*m^10*x^3 + 2*(d*x)^m*A*a*b*m^10*x^3 + 124*(d*x)^m*B*a*b*m^9*x^4
+ 3240*(d*x)^m*C*a*b*m^8*x^5 + 1620*(d*x)^m*A*b^2*m^8*x^5 + 3240*(d*x)^m*A*
a*c*m^8*x^5 + 23280*(d*x)^m*B*b^2*m^7*x^6 + 46560*(d*x)^m*B*a*c*m^7*x^6 + 2
02821*(d*x)^m*C*b^2*m^6*x^7 + 405642*(d*x)^m*C*a*c*m^6*x^7 + 405642*(d*x)^m
*A*b*c*m^6*x^7 + 2242044*(d*x)^m*B*b*c*m^5*x^8 + 8000956*(d*x)^m*C*b*c*m^4*
x^9 + 4000478*(d*x)^m*A*c^2*m^4*x^9 + 9133180*(d*x)^m*B*c^2*m^3*x^10 + 1275
3576*(d*x)^m*C*c^2*m^2*x^11 + (d*x)^m*B*a^2*m^10*x^2 + 63*(d*x)^m*C*a^2*m^9
*x^3 + 126*(d*x)^m*A*a*b*m^9*x^3 + 3354*(d*x)^m*B*a*b*m^8*x^4 + 49140*(d*x)
^m*C*a*b*m^7*x^5 + 24570*(d*x)^m*A*b^2*m^7*x^5 + 49140*(d*x)^m*A*a*c*m^7*x^
5 + 217743*(d*x)^m*B*b^2*m^6*x^6 + 435486*(d*x)^m*B*a*c*m^6*x^6 + 1217811*(
d*x)^m*C*b^2*m^5*x^7 + 2435622*(d*x)^m*C*a*c*m^5*x^7 + 2435622*(d*x)^m*A*b*
c*m^5*x^7 + 8742718*(d*x)^m*B*b*c*m^4*x^8 + 19982856*(d*x)^m*C*b*c*m^3*x^9
+ 9991428*(d*x)^m*A*c^2*m^3*x^9 + 13926276*(d*x)^m*B*c^2*m^2*x^10 + 1062864
0*(d*x)^m*C*c^2*m*x^11 + (d*x)^m*A*a^2*m^10*x + 64*(d*x)^m*B*a^2*m^9*x^2 +
1736*(d*x)^m*C*a^2*m^8*x^3 + 3472*(d*x)^m*A*a*b*m^8*x^3 + 51924*(d*x)^m*B*a
*b*m^7*x^4 + 469146*(d*x)^m*C*a*b*m^6*x^5 + 234573*(d*x)^m*A*b^2*m^6*x^5 +
469146*(d*x)^m*A*a*c*m^6*x^5 + 1331100*(d*x)^m*B*b^2*m^5*x^6 + 2662200*(d*x
)^m*B*a*c*m^5*x^6 + 4814858*(d*x)^m*C*b^2*m^4*x^7 + 9629716*(d*x)^m*C*a*c*m
^4*x^7 + 9629716*(d*x)^m*A*b*c*m^4*x^7 + 22049716*(d*x)^m*B*b*c*m^3*x^8 + 3
0670448*(d*x)^m*C*b*c*m^2*x^9 + 15335224*(d*x)^m*A*c^2*m^2*x^9 + 11655216*(
d*x)^m*B*c^2*m*x^10 + 3628800*(d*x)^m*C*c^2*x^11 + 65*(d*x)^m*A*a^2*m^9*x +
 1797*(d*x)^m*B*a^2*m^8*x^2 + 27462*(d*x)^m*C*a^2*m^7*x^3 + 54924*(d*x)^m*A
*a*b*m^7*x^3 + 507150*(d*x)^m*B*a*b*m^6*x^4 + 2929386*(d*x)^m*C*a*b*m^5*x^5
 + 1464693*(d*x)^m*A*b^2*m^5*x^5 + 2929386*(d*x)^m*A*a*c*m^5*x^5 + 5352935*
(d*x)^m*B*b^2*m^4*x^6 + 10705870*(d*x)^m*B*a*c*m^4*x^6 + 12291724*(d*x)^m*C
*b^2*m^3*x^7 + 24583448*(d*x)^m*C*a*c*m^3*x^7 + 24583448*(d*x)^m*A*b*c*m^3*
x^7 + 34118424*(d*x)^m*B*b*c*m^2*x^8 + 25801920*(d*x)^m*C*b*c*m*x^9 + 12900
960*(d*x)^m*A*c^2*m*x^9 + 3991680*(d*x)^m*B*c^2*x^10 + 1860*(d*x)^m*A*a^2*m
^8*x + 29076*(d*x)^m*B*a^2*m^7*x^2 + 275037*(d*x)^m*C*a^2*m^6*x^3 + 550074*
(d*x)^m*A*a*b*m^6*x^3 + 3246516*(d*x)^m*B*a*b*m^5*x^4 + 12032140*(d*x)^m*C*
a*b*m^4*x^5 + 6016070*(d*x)^m*A*b^2*m^4*x^5 + 12032140*(d*x)^m*A*a*c*m^4*x^
5 + 13878120*(d*x)^m*B*b^2*m^3*x^6 + 27756240*(d*x)^m*B*a*c*m^3*x^6 + 19216
008*(d*x)^m*C*b^2*m^2*x^7 + 38432016*(d*x)^m*C*a*c*m^2*x^7 + 38432016*(d*x)
^m*A*b*c*m^2*x^7 + 28888560*(d*x)^m*B*b*c*m*x^8 + 8870400*(d*x)^m*C*b*c*x^9
 + 4435200*(d*x)^m*A*c^2*x^9 + 30810*(d*x)^m*A*a^2*m^7*x + 299271*(d*x)^m*B
*a^2*m^6*x^2 + 1812447*(d*x)^m*C*a^2*m^5*x^3 + 3624894*(d*x)^m*A*a*b*m^5*x^
3 + 13693006*(d*x)^m*B*a*b*m^4*x^4 + 31830760*(d*x)^m*C*a*b*m^3*x^5 + 15915
380*(d*x)^m*A*b^2*m^3*x^5 + 31830760*(d*x)^m*A*a*c*m^3*x^5 + 21989356*(d*x)
^m*B*b^2*m^2*x^6 + 43978712*(d*x)^m*B*a*c*m^2*x^6 + 16405920*(d*x)^m*C*b^2*
m*x^7 + 32811840*(d*x)^m*C*a*c*m*x^7 + 32811840*(d*x)^m*A*b*c*m*x^7 + 99792
00*(d*x)^m*B*b*c*x^8 + 326613*(d*x)^m*A*a^2*m^6*x + 2039016*(d*x)^m*B*a^2*m
^5*x^2 + 7902194*(d*x)^m*C*a^2*m^4*x^3 + 15804388*(d*x)^m*A*a*b*m^4*x^3 + 3
7219436*(d*x)^m*B*a*b*m^3*x^4 + 51362352*(d*x)^m*C*a*b*m^2*x^5 + 25681176*(
d*x)^m*A*b^2*m^2*x^5 + 51362352*(d*x)^m*A*a*c*m^2*x^5 + 18981840*(d*x)^m*B*
b^2*m*x^6 + 37963680*(d*x)^m*B*a*c*m*x^6 + 5702400*(d*x)^m*C*b^2*x^7 + 1140
4800*(d*x)^m*C*a*c*x^7 + 11404800*(d*x)^m*A*b*c*x^7 + 2310945*(d*x)^m*A*a^2
*m^5*x + 9261503*(d*x)^m*B*a^2*m^4*x^2 + 22289148*(d*x)^m*C*a^2*m^3*x^3 + 4
4578296*(d*x)^m*A*a*b*m^3*x^3 + 61638408*(d*x)^m*B*a*b*m^2*x^4 + 45024192*(
```

```
d*x)^m*C*a*b*m*x^5 + 22512096*(d*x)^m*A*b^2*m*x^5 + 45024192*(d*x)^m*A*a*c*
m*x^5 + 6652800*(d*x)^m*B*b^2*x^6 + 13305600*(d*x)^m*B*a*c*x^6 + 11028590*(
d*x)^m*A*a^2*m^4*x + 27472724*(d*x)^m*B*a^2*m^3*x^2 + 38390632*(d*x)^m*C*a^
2*m^2*x^3 + 76781264*(d*x)^m*A*a*b*m^2*x^3 + 55282320*(d*x)^m*B*a*b*m*x^4 +
 15966720*(d*x)^m*C*a*b*x^5 + 7983360*(d*x)^m*A*b^2*x^5 + 15966720*(d*x)^m*
A*a*c*x^5 + 34967140*(d*x)^m*A*a^2*m^3*x + 50312628*(d*x)^m*B*a^2*m^2*x^2 +
 35746080*(d*x)^m*C*a^2*m*x^3 + 71492160*(d*x)^m*A*a*b*m*x^3 + 19958400*(d*
x)^m*B*a*b*x^4 + 70290936*(d*x)^m*A*a^2*m^2*x + 50292720*(d*x)^m*B*a^2*m*x^
2 + 13305600*(d*x)^m*C*a^2*x^3 + 26611200*(d*x)^m*A*a*b*x^3 + 80627040*(d*x
)^m*A*a^2*m*x + 19958400*(d*x)^m*B*a^2*x^2 + 39916800*(d*x)^m*A*a^2*x)/(m^1
1 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 + 2637558*m^6 + 13339535*m^
5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2 + 120543840*m + 39916800)
```

3.39 $\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=137

$$\frac{(dx)^{m+3}(aC+Ab)}{d^3(m+3)}+\frac{aA(dx)^{m+1}}{d(m+1)}+\frac{aB(dx)^{m+2}}{d^2(m+2)}+\frac{(dx)^{m+5}(Ac+bC)}{d^5(m+5)}+\frac{bB(dx)^{m+4}}{d^4(m+4)}+\frac{Bc(dx)^{m+6}}{d^6(m+6)}+\frac{cC(dx)^{m+7}}{d^7(m+7)}$$

```
[Out] (a*A*(d*x)^(1 + m))/(d*(1 + m)) + (a*B*(d*x)^(2 + m))/(d^2*(2 + m)) + ((A*b
 + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (b*B*(d*x)^(4 + m))/(d^4*(4 + m)) +
((A*c + b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) + (B*c*(d*x)^(6 + m))/(d^6*(6 + m
)) + (c*C*(d*x)^(7 + m))/(d^7*(7 + m))
```

Rubi [A] time = 0.0880793, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1628}

$$\frac{(dx)^{m+3}(aC+Ab)}{d^3(m+3)}+\frac{aA(dx)^{m+1}}{d(m+1)}+\frac{aB(dx)^{m+2}}{d^2(m+2)}+\frac{(dx)^{m+5}(Ac+bC)}{d^5(m+5)}+\frac{bB(dx)^{m+4}}{d^4(m+4)}+\frac{Bc(dx)^{m+6}}{d^6(m+6)}+\frac{cC(dx)^{m+7}}{d^7(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*(d*x)^(1 + m))/(d*(1 + m)) + (a*B*(d*x)^(2 + m))/(d^2*(2 + m)) + ((A*b
 + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (b*B*(d*x)^(4 + m))/(d^4*(4 + m)) +
((A*c + b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) + (B*c*(d*x)^(6 + m))/(d^6*(6 + m
)) + (c*C*(d*x)^(7 + m))/(d^7*(7 + m))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)dx=\int \left(aA(dx)^m+\frac{aB(dx)^{1+m}}{d}+\frac{(Ab+aC)(dx)^{2+m}}{d^2}+\frac{bB(dx)^{3+m}}{d^3}+\frac{(Ac+bC}{d}\right.$$

$$=\frac{aA(dx)^{1+m}}{d(1+m)}+\frac{aB(dx)^{2+m}}{d^2(2+m)}+\frac{(Ab+aC)(dx)^{3+m}}{d^3(3+m)}+\frac{bB(dx)^{4+m}}{d^4(4+m)}+\frac{(Ac+bC)}{d^5(5+}$$

Mathematica [A] time = 0.121717, size = 90, normalized size = 0.66

$$x(dx)^m\left(\frac{x^2(aC+Ab)}{m+3}+\frac{aA}{m+1}+\frac{aBx}{m+2}+\frac{x^4(Ac+bC)}{m+5}+\frac{bBx^3}{m+4}+\frac{Bcx^5}{m+6}+\frac{cCx^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] x*(d*x)^m*((a*A)/(1 + m) + (a*B*x)/(2 + m) + ((A*b + a*C)*x^2)/(3 + m) + (b
*B*x^3)/(4 + m) + ((A*c + b*C)*x^4)/(5 + m) + (B*c*x^5)/(6 + m) + (c*C*x^6)
```

```
/(7 + m))
```

Maple [B] time = 0.004, size = 585, normalized size = 4.3

$$\frac{\left(Ccm^6x^6 + Bcm^6x^5 + 21\,Ccm^5x^6 + Acm^6x^4 + 22\,Bcm^5x^5 + Cbm^6x^4 + 175\,Ccm^4x^6 + 23\,Acm^5x^4 + Bbm^6x^3 + 190\,Bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)
```

```
[Out] x*(C*c*m^6*x^6+B*c*m^6*x^5+21*C*c*m^5*x^6+A*c*m^6*x^4+22*B*c*m^5*x^5+C*b*m^
6*x^4+175*C*c*m^4*x^6+23*A*c*m^5*x^4+B*b*m^6*x^3+190*B*c*m^4*x^5+23*C*b*m^5
*x^4+735*C*c*m^3*x^6+A*b*m^6*x^2+207*A*c*m^4*x^4+24*B*b*m^5*x^3+820*B*c*m^3
*x^5+C*a*m^6*x^2+207*C*b*m^4*x^4+1624*C*c*m^2*x^6+25*A*b*m^5*x^2+925*A*c*m^
3*x^4+B*a*m^6*x+226*B*b*m^4*x^3+1849*B*c*m^2*x^5+25*C*a*m^5*x^2+925*C*b*m^3
*x^4+1764*C*c*m*x^6+A*a*m^6+247*A*b*m^4*x^2+2144*A*c*m^2*x^4+26*B*a*m^5*x+1
056*B*b*m^3*x^3+2038*B*c*m*x^5+247*C*a*m^4*x^2+2144*C*b*m^2*x^4+720*C*c*x^6
+27*A*a*m^5+1219*A*b*m^3*x^2+2412*A*c*m*x^4+270*B*a*m^4*x+2545*B*b*m^2*x^3+
840*B*c*x^5+1219*C*a*m^3*x^2+2412*C*b*m*x^4+295*A*a*m^4+3112*A*b*m^2*x^2+10
08*A*c*x^4+1420*B*a*m^3*x+2952*B*b*m*x^3+3112*C*a*m^2*x^2+1008*C*b*x^4+1665
*A*a*m^3+3796*A*b*m*x^2+3929*B*a*m^2*x+1260*B*b*x^3+3796*C*a*m*x^2+5104*A*a
*m^2+1680*A*b*x^2+5274*B*a*m*x+1680*C*a*x^2+8028*A*a*m+2520*B*a*x+5040*A*a)
*(d*x)^m/(7+m)/(6+m)/(5+m)/(4+m)/(3+m)/(2+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.5775, size = 1181, normalized size = 8.62

$$\frac{\left(\left(Ccm^6 + 21\,Ccm^5 + 175\,Ccm^4 + 735\,Ccm^3 + 1624\,Ccm^2 + 1764\,Ccm + 720\,Cc\right)x^7 + \left(Bcm^6 + 22\,Bcm^5 + 190\,Bcm^4 +\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] ((C*c*m^6 + 21*C*c*m^5 + 175*C*c*m^4 + 735*C*c*m^3 + 1624*C*c*m^2 + 1764*C*
c*m + 720*C*c)*x^7 + (B*c*m^6 + 22*B*c*m^5 + 190*B*c*m^4 + 820*B*c*m^3 + 18
49*B*c*m^2 + 2038*B*c*m + 840*B*c)*x^6 + ((C*b + A*c)*m^6 + 23*(C*b + A*c)*
m^5 + 207*(C*b + A*c)*m^4 + 925*(C*b + A*c)*m^3 + 2144*(C*b + A*c)*m^2 + 10
08*C*b + 1008*A*c + 2412*(C*b + A*c)*m)*x^5 + (B*b*m^6 + 24*B*b*m^5 + 226*B
*b*m^4 + 1056*B*b*m^3 + 2545*B*b*m^2 + 2952*B*b*m + 1260*B*b)*x^4 + ((C*a +
 A*b)*m^6 + 25*(C*a + A*b)*m^5 + 247*(C*a + A*b)*m^4 + 1219*(C*a + A*b)*m^3
 + 3112*(C*a + A*b)*m^2 + 1680*C*a + 1680*A*b + 3796*(C*a + A*b)*m)*x^3 + (
```

```
B*a*m^6 + 26*B*a*m^5 + 270*B*a*m^4 + 1420*B*a*m^3 + 3929*B*a*m^2 + 5274*B*a
*m + 2520*B*a)*x^2 + (A*a*m^6 + 27*A*a*m^5 + 295*A*a*m^4 + 1665*A*a*m^3 + 5
104*A*a*m^2 + 8028*A*a*m + 5040*A*a)*x)*(d*x)^m/(m^7 + 28*m^6 + 322*m^5 + 1
960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)
```

Sympy [A] time = 2.79044, size = 3735, normalized size = 27.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise(((-A*a/(6*x**6) - A*b/(4*x**4) - A*c/(2*x**2) - B*a/(5*x**5) - B*
b/(3*x**3) - B*c/x - C*a/(4*x**4) - C*b/(2*x**2) + C*c*log(x))/d**7, Eq(m,
-7)), ((-A*a/(5*x**5) - A*b/(3*x**3) - A*c/x - B*a/(4*x**4) - B*b/(2*x**2)
+ B*c*log(x) - C*a/(3*x**3) - C*b/x + C*c*x)/d**6, Eq(m, -6)), ((-A*a/(4*x*
*4) - A*b/(2*x**2) + A*c*log(x) - B*a/(3*x**3) - B*b/x + B*c*x - C*a/(2*x**
2) + C*b*log(x) + C*c*x**2/2)/d**5, Eq(m, -5)), ((-A*a/(3*x**3) - A*b/x + A
*c*x - B*a/(2*x**2) + B*b*log(x) + B*c*x**2/2 - C*a/x + C*b*x + C*c*x**3/3)
/d**4, Eq(m, -4)), ((-A*a/(2*x**2) + A*b*log(x) + A*c*x**2/2 - B*a/x + B*b*
x + B*c*x**3/3 + C*a*log(x) + C*b*x**2/2 + C*c*x**4/4)/d**3, Eq(m, -3)), ((
-A*a/x + A*b*x + A*c*x**3/3 + B*a*log(x) + B*b*x**2/2 + B*c*x**4/4 + C*a*x
+ C*b*x**3/3 + C*c*x**5/5)/d**2, Eq(m, -2)), ((A*a*log(x) + A*b*x**2/2 + A*
c*x**4/4 + B*a*x + B*b*x**3/3 + B*c*x**5/5 + C*a*x**2/2 + C*b*x**4/4 + C*c*
x**6/6)/d, Eq(m, -1)), (A*a*d**m*m**6*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1
960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 27*A*a*d**m*m**5*x*x*
*m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*
m + 5040) + 295*A*a*d**m*m**4*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4
 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1665*A*a*d**m*m**3*x*x**m/(m*
*7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 50
40) + 5104*A*a*d**m*m**2*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 67
69*m**3 + 13132*m**2 + 13068*m + 5040) + 8028*A*a*d**m*m*x*x**m/(m**7 + 28*
m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 50
40*A*a*d**m*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 131
32*m**2 + 13068*m + 5040) + A*b*d**m*m**6*x**3*x**m/(m**7 + 28*m**6 + 322*m
**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 25*A*b*d**m*m*
*5*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**
2 + 13068*m + 5040) + 247*A*b*d**m*m**4*x**3*x**m/(m**7 + 28*m**6 + 322*m**
5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1219*A*b*d**m*m*
*3*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**
2 + 13068*m + 5040) + 3112*A*b*d**m*m**2*x**3*x**m/(m**7 + 28*m**6 + 322*m*
*5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 3796*A*b*d**m*m
*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
+ 13068*m + 5040) + 1680*A*b*d**m*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 19
60*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + A*c*d**m*m**6*x**5*x**
m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m
 + 5040) + 23*A*c*d**m*m**5*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**
4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 207*A*c*d**m*m**4*x**5*x**m/
(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +
 5040) + 925*A*c*d**m*m**3*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4
 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2144*A*c*d**m*m**2*x**5*x**m/
(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +
 5040) + 2412*A*c*d**m*m*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1008*A*c*d**m*x**5*x**m/(m**7 +
 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040)
+ B*a*d**m*m**6*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**
```

```
3 + 13132*m**2 + 13068*m + 5040) + 26*B*a*d**m*m**5*x**2*x**m/(m**7 + 28*m*
*6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 270*
B*a*d**m*m**4*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3
 + 13132*m**2 + 13068*m + 5040) + 1420*B*a*d**m*m**3*x**2*x**m/(m**7 + 28*m*
*6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 3929
*B*a*d**m*m**2*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3
 + 13132*m**2 + 13068*m + 5040) + 5274*B*a*d**m*m*x**2*x**m/(m**7 + 28*m**6
 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2520*B
*a*d**m*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 1313
2*m**2 + 13068*m + 5040) + B*b*d**m*m**6*x**4*x**m/(m**7 + 28*m**6 + 322*m*
*5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 24*B*b*d**m*m**
5*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
 + 13068*m + 5040) + 226*B*b*d**m*m**4*x**4*x**m/(m**7 + 28*m**6 + 322*m**5
 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1056*B*b*d**m*m**
3*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
 + 13068*m + 5040) + 2545*B*b*d**m*m**2*x**4*x**m/(m**7 + 28*m**6 + 322*m**
5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2952*B*b*d**m*m*
x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 +
 13068*m + 5040) + 1260*B*b*d**m*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 196
0*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + B*c*d**m*m**6*x**6*x**m
/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m
+ 5040) + 22*B*c*d**m*m**5*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4
 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 190*B*c*d**m*m**4*x**6*x**m/(
m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +
5040) + 820*B*c*d**m*m**3*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4
+ 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1849*B*c*d**m*m**2*x**6*x**m/(
m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +
5040) + 2038*B*c*d**m*m*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
6769*m**3 + 13132*m**2 + 13068*m + 5040) + 840*B*c*d**m*x**6*x**m/(m**7 + 2
8*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) +
C*a*d**m*m**6*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3
+ 13132*m**2 + 13068*m + 5040) + 25*C*a*d**m*m**5*x**3*x**m/(m**7 + 28*m**6
 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 247*C*
a*d**m*m**4*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 +
13132*m**2 + 13068*m + 5040) + 1219*C*a*d**m*m**3*x**3*x**m/(m**7 + 28*m**6
 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 3112*C
*a*d**m*m**2*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 +
 13132*m**2 + 13068*m + 5040) + 3796*C*a*d**m*m*x**3*x**m/(m**7 + 28*m**6 +
 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1680*C*a
*d**m*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*
m**2 + 13068*m + 5040) + C*b*d**m*m**6*x**5*x**m/(m**7 + 28*m**6 + 322*m**5
 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 23*C*b*d**m*m**5*
x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 +
 13068*m + 5040) + 207*C*b*d**m*m**4*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 +
 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 925*C*b*d**m*m**3*x
**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 +
13068*m + 5040) + 2144*C*b*d**m*m**2*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 +
 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2412*C*b*d**m*m*x**
5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13
068*m + 5040) + 1008*C*b*d**m*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m
**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + C*c*d**m*m**6*x**7*x**m/(m
**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5
040) + 21*C*c*d**m*m**5*x**7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
6769*m**3 + 13132*m**2 + 13068*m + 5040) + 175*C*c*d**m*m**4*x**7*x**m/(m**
7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 504
0) + 735*C*c*d**m*m**3*x**7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6
769*m**3 + 13132*m**2 + 13068*m + 5040) + 1624*C*c*d**m*m**2*x**7*x**m/(m**
7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 504
0) + 1764*C*c*d**m*m*x**7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 676
```

```
9*m**3 + 13132*m**2 + 13068*m + 5040) + 720*C*c*d**m*x**7*x**m/(m**7 + 28*m
**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040), True
))
```

Giac [B] time = 1.12333, size = 1234, normalized size = 9.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] ((d*x)^m*C*c*m^6*x^7 + (d*x)^m*B*c*m^6*x^6 + 21*(d*x)^m*C*c*m^5*x^7 + (d*x)
^m*C*b*m^6*x^5 + (d*x)^m*A*c*m^6*x^5 + 22*(d*x)^m*B*c*m^5*x^6 + 175*(d*x)^m
*C*c*m^4*x^7 + (d*x)^m*B*b*m^6*x^4 + 23*(d*x)^m*C*b*m^5*x^5 + 23*(d*x)^m*A*
c*m^5*x^5 + 190*(d*x)^m*B*c*m^4*x^6 + 735*(d*x)^m*C*c*m^3*x^7 + (d*x)^m*C*a
*m^6*x^3 + (d*x)^m*A*b*m^6*x^3 + 24*(d*x)^m*B*b*m^5*x^4 + 207*(d*x)^m*C*b*m
^4*x^5 + 207*(d*x)^m*A*c*m^4*x^5 + 820*(d*x)^m*B*c*m^3*x^6 + 1624*(d*x)^m*C
*c*m^2*x^7 + (d*x)^m*B*a*m^6*x^2 + 25*(d*x)^m*C*a*m^5*x^3 + 25*(d*x)^m*A*b*
m^5*x^3 + 226*(d*x)^m*B*b*m^4*x^4 + 925*(d*x)^m*C*b*m^3*x^5 + 925*(d*x)^m*A
*c*m^3*x^5 + 1849*(d*x)^m*B*c*m^2*x^6 + 1764*(d*x)^m*C*c*m*x^7 + (d*x)^m*A*
a*m^6*x + 26*(d*x)^m*B*a*m^5*x^2 + 247*(d*x)^m*C*a*m^4*x^3 + 247*(d*x)^m*A*
b*m^4*x^3 + 1056*(d*x)^m*B*b*m^3*x^4 + 2144*(d*x)^m*C*b*m^2*x^5 + 2144*(d*x
)^m*A*c*m^2*x^5 + 2038*(d*x)^m*B*c*m*x^6 + 720*(d*x)^m*C*c*x^7 + 27*(d*x)^m
*A*a*m^5*x + 270*(d*x)^m*B*a*m^4*x^2 + 1219*(d*x)^m*C*a*m^3*x^3 + 1219*(d*x
)^m*A*b*m^3*x^3 + 2545*(d*x)^m*B*b*m^2*x^4 + 2412*(d*x)^m*C*b*m*x^5 + 2412*
(d*x)^m*A*c*m*x^5 + 840*(d*x)^m*B*c*x^6 + 295*(d*x)^m*A*a*m^4*x + 1420*(d*x
)^m*B*a*m^3*x^2 + 3112*(d*x)^m*C*a*m^2*x^3 + 3112*(d*x)^m*A*b*m^2*x^3 + 295
2*(d*x)^m*B*b*m*x^4 + 1008*(d*x)^m*C*b*x^5 + 1008*(d*x)^m*A*c*x^5 + 1665*(d
*x)^m*A*a*m^3*x + 3929*(d*x)^m*B*a*m^2*x^2 + 3796*(d*x)^m*C*a*m*x^3 + 3796*
(d*x)^m*A*b*m*x^3 + 1260*(d*x)^m*B*b*x^4 + 5104*(d*x)^m*A*a*m^2*x + 5274*(d
*x)^m*B*a*m*x^2 + 1680*(d*x)^m*C*a*x^3 + 1680*(d*x)^m*A*b*x^3 + 8028*(d*x)^
m*A*a*m*x + 2520*(d*x)^m*B*a*x^2 + 5040*(d*x)^m*A*a*x)/(m^7 + 28*m^6 + 322*
m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)
```

3.40 $\int \frac{(dx)^m \left(A+Bx+Cx^2\right)}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=368

$$\frac{(dx)^{m+1}\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(dx)^{m+1}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\left(\sqrt{b^2-4ac}+b\right)}+\frac{2Bc(dx}{d^2(m}$$

```
[Out] ((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*(d*x)^(1 + m)*Hypergeometric2F1[1, (
1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 -
4*a*c])*d*(1 + m)) + ((C - (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*(d*x)^(1 + m)*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)])/((b + Sqrt[b^2 - 4*a*c])*d*(1 + m)) + (2*B*c*(d*x)^(2 + m)*Hypergeometr
ic2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b
^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d^2*(2 + m)) - (2*B*c*(d*x)^(2 + m)*Hyp
ergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
)/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d^2*(2 + m))
```

Rubi [A] time = 0.621653, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1662, 1285, 364, 12, 1131}

$$\frac{(dx)^{m+1}\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(dx)^{m+1}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\left(\sqrt{b^2-4ac}+b\right)}+\frac{2Bc(dx}{d^2(m}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```


```
[Out] ((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*(d*x)^(1 + m)*Hypergeometric2F1[1, (
1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 -
4*a*c])*d*(1 + m)) + ((C - (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*(d*x)^(1 + m)*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)])/((b + Sqrt[b^2 - 4*a*c])*d*(1 + m)) + (2*B*c*(d*x)^(2 + m)*Hypergeometr
ic2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b
^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d^2*(2 + m)) - (2*B*c*(d*x)^(2 + m)*Hyp
ergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
)/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d^2*(2 + m))
```


Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rule 1285

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
```

```
f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1131

```
Int[((d_.)*(x_))^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*x^2), x]
, x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c
, d, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(dx)^m\left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx = \frac{\int \frac{B(dx)^{1+m}}{a+bx^2+cx^4}\,dx}{d}+\int \frac{(dx)^m\left(A+Cx^2\right)}{a+bx^2+cx^4}\,dx$$

$$= \frac{1}{2}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int \frac{(dx)^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx+\frac{1}{2}\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int \frac{(dx)^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}}$$

$$= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)d(1+m)}+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\right.}{\left(b+\sqrt{b^2-4ac}\right)d(1+}$$

$$= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)d(1+m)}+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\right.}{\left(b+\sqrt{b^2-4ac}\right)d(1+}$$

Mathematica [C] time = 0.22039, size = 168, normalized size = 0.46

$$\frac{1}{2}x(dx)^m\left(\frac{A\text{RootSum}\left[\#1^2b+\#1^4c+a\&,\frac{{}_2F_1\left(1,m+1;m+2;\frac{x}{\#1}\right)}{\#1^2b+2a}\&\right]}{m+1}+x\left(\frac{B\text{RootSum}\left[\#1^2b+\#1^4c+a\&,\frac{{}_2F_1\left(1,m+2;m+3;\frac{x}{\#1}\right)}{\#1^2b+2a}\&\right]}{m+2}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*(d*x)^m*((A*RootSum[a + b*#1^2 + c*#1^4 & , Hypergeometric2F1[1, 1 + m,
2 + m, x/#1]/(2*a + b*#1^2) & ])/(1 + m) + x*((B*RootSum[a + b*#1^2 + c*#1^
4 & , Hypergeometric2F1[1, 2 + m, 3 + m, x/#1]/(2*a + b*#1^2) & ])/(2 + m)
+ (C*x*RootSum[a + b*#1^2 + c*#1^4 & , Hypergeometric2F1[1, 3 + m, 4 + m, x
/#1]/(2*a + b*#1^2) & ])/(3 + m))))/2
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{(dx)^m \left(C x^2 + B x + A\right)}{c x^4 + b x^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C x^2 + B x + A\right)(dx)^m}{c x^4 + b x^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(C x^2 + B x + A\right)(dx)^m}{c x^4 + b x^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m \left(A + B x + C x^2\right)}{a + b x^2 + c x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((d*x)**m*(A + B*x + C*x**2)/(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(C x^2 + B x + A\right)(dx)^m}{c x^4 + b x^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

3.41 $\int \frac{(dx)^m(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=685

$$\frac{c(dx)^{m+1}\left(A\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)+2aC\left(2b-(1-m)\sqrt{b^2-4ac}\right)\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-}}\right.}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}$$

```
[Out] (B*(d*x)^(2 + m)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*d^2*(a + b*x^2
 + c*x^4)) + ((d*x)^(1 + m)*(A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)
)/(2*a*(b^2 - 4*a*c)*d*(a + b*x^2 + c*x^4)) + (c*(2*a*C*(2*b - Sqrt[b^2 - 4
*a*c]*(1 - m)) + A*(b^2*(1 - m) + b*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 -
m)))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*
(1 + m)) - (c*(2*a*C*(2*b + Sqrt[b^2 - 4*a*c]*(1 - m)) + A*(b^2*(1 - m) - b
*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 - m)))*(d*x)^(1 + m)*Hypergeometric2F
1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 -
 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m)) - (B*c*(4*a*c*(2 - m) + b*
(b + Sqrt[b^2 - 4*a*c])*m)*(d*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4
 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b -
 Sqrt[b^2 - 4*a*c])*d^2*(2 + m)) + (B*c*(4*a*c*(2 - m) + b*(b - Sqrt[b^2 -
4*a*c])*m)*(d*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*
c])*d^2*(2 + m))
```

Rubi [A] time = 2.37779, antiderivative size = 670, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1662, 1277, 1285, 364, 12, 1121}

$$\frac{c(dx)^{m+1}\left(A\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)+2aC\left(2b-(1-m)\sqrt{b^2-4ac}\right)\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-}}\right.}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(d*x)^(2 + m)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*d^2*(a + b*x^2
 + c*x^4)) + ((d*x)^(1 + m)*(A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)
)/(2*a*(b^2 - 4*a*c)*d*(a + b*x^2 + c*x^4)) + (c*(2*a*C*(2*b - Sqrt[b^2 - 4
*a*c]*(1 - m)) + A*(b^2*(1 - m) + b*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 -
m)))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*
(1 + m)) - (c*(4*a*b*C + A*b^2*(1 - m) - Sqrt[b^2 - 4*a*c]*(A*b - 2*a*C)*(1
 - m) - 4*a*A*c*(3 - m))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 +
 m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + S
qrt[b^2 - 4*a*c])*d*(1 + m)) - (B*c*(4*a*c*(2 - m) + b*(b + Sqrt[b^2 - 4*a*
c])*m)*(d*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*
d^2*(2 + m)) + (B*c*(4*a*c*(2 - m) + b*(b - Sqrt[b^2 - 4*a*c])*m)*(d*x)^(2
+ m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d^2*(2 + m))
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1285

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rubi steps

$$\int \frac{(dx)^m \left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx = \frac{\int \frac{B(dx)^{1+m}}{\left(a+bx^2+cx^4\right)^2}dx}{d} + \int \frac{(dx)^m \left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$= \frac{(dx)^{1+m}\left(A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)d\left(a+bx^2+cx^4\right)} - \frac{\int \frac{(dx)^m\left(-Ab^2(1-m)+2aAc(3-m)-abC(1+m)-\right.}{a+bx^2+cx^4}}{2a\left(b^2-4ac\right)}$$

$$= \frac{B(dx)^{2+m}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)d^2\left(a+bx^2+cx^4\right)} + \frac{(dx)^{1+m}\left(A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)d\left(a+bx^2+cx^4\right)} - \cdots$$

$$= \frac{B(dx)^{2+m}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)d^2\left(a+bx^2+cx^4\right)} + \frac{(dx)^{1+m}\left(A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)d\left(a+bx^2+cx^4\right)} + \cdots$$

$$= \frac{B(dx)^{2+m}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)d^2\left(a+bx^2+cx^4\right)} + \frac{(dx)^{1+m}\left(A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)d\left(a+bx^2+cx^4\right)} + \cdots$$

Mathematica [C] time = 0.344501, size = 242, normalized size = 0.35

$$\frac{x(dx)^m\left(A\left(m^2+5m+6\right)F_1\left(\frac{m+1}{2};2,2;\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+(m+1)x\left(B(m+3)F_1\left(\frac{m+2}{2};2,2;\frac{m+4}{2};-\frac{2cx^2}{b+\sqrt{b^2-4a}}\right.\right.\right.}{a^2(m+1)(m+2)(m+3)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(d*x)^m*(A*(6 + 5*m + m^2)*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + m)*x
*(B*(3 + m)*AppellF1[(2 + m)/2, 2, 2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + C*(2 + m)*x*AppellF1[(3 + m
)/2, 2, 2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])])))/(a^2*(1 + m)*(2 + m)*(3 + m))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int \frac{(dx)^m\left(Cx^2+Bx+A\right)}{\left(cx^4+bx^2+a\right)^2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2+Bx+A\right)(dx)^m}{\left(cx^4+bx^2+a\right)^2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)(dx)^m}{c^2x^8 + 2\,bcx^6 + \left(b^2 + 2\,ac\right)x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*(d*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^4
 + 2*a*b*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Cx^2 + Bx + A\right)(dx)^m}{\left(cx^4 + bx^2 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

3.42 $\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=356

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{}}}$$

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.923822, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1662, 1275, 1166, 205, 12, 1114, 638, 618, 206}

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
```

```
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2 \left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2} dx = \int \frac{Bx^3}{\left(a+bx^2+cx^4\right)^2} dx + \int \frac{x^2 \left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2} dx$$

$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + B\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2} dx + \frac{\int \frac{Ab-2aC+(-2Ac+bC)x^2}{a+bx^2+cx^4} dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2} dx, x, x^2\right) - \frac{\left(2Ac-bC-\right.}{}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)}$$

Mathematica [A] time = 1.18665, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x\left(bx(B+Cx)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-4ac-b^2\right)-2Ac\left(\sqrt{b^2-4ac}-2b\right)\right)\tan^{-1}\left(\right.}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0., size = 1119, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*B*b/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(4
*a*c-b^2)*x-B*a/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*B*(-4*a*c+b^
2)^(1/2)*b*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-c/(4*a*c-b^2)^2*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2))*A*(-4*a*c+b^2)^(1/2)*b-2*c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/
2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*a+1/
2*c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*b^2+c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a
*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1
/2))*C*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*(-4*a*c
+b^2)^(1/2)*b^2+c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*ar
ctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*a*b-1/4/(4*a*c-b^2)^2
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2))*C*b^3-1/2/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^(1/2)*b*ln(2*c*
x^2+(-4*a*c+b^2)^(1/2)+b)-c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-4*a*c+b^2)
^(1/2)*b+2*c^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*a-1/2*c/(4*a*c-b^2)^2*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*A*b^2+c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/
2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b^2-c/(4*a*c
-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*C*a*b+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bbx^2 + (Cb - 2\,Ac)x^3 + 2\,Ba + (2\,Ca - Ab)x}{2\left(\left(b^2c - 4\,ac^2\right)x^4 + ab^2 - 4\,a^2c + \left(b^3 - 4\,abc\right)x^2\right)} - \frac{-\int \frac{2\,Bbx + (Cb - 2\,Ac)x^2 - 2\,Ca + Ab}{cx^4 + bx^2 + a}\,dx}{2\left(b^2 - 4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.43 $\int \frac{x\left(Ax+Bx^2+Cx^3\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=356

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2}}}$$

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.370935, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1585, 1662, 1275, 1166, 205, 12, 1114, 638, 618, 206}

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
```

```
lyQ[Pq, x^2]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(Ax+Bx^2+Cx^3\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \int \frac{Bx^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{x^2\left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + B\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac+bC)x^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\,\mathrm{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) - \frac{\left(2Ac-bC-\right.}{}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{}}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{}}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{}}$$

Mathematica [A] time = 0.793289, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x\left(bx(B+Cx)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-4ac-b^2\right)-2Ac\left(\sqrt{b^2-4ac}-2b\right)\right)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{b-}}\right.}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.025, size = 1119, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*B*b/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(4
*a*c-b^2)*x-B*a/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*B*(-4*a*c+b^
2)^(1/2)*b*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-c/(4*a*c-b^2)^2*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2))*A*(-4*a*c+b^2)^(1/2)*b-2*c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/
2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*a+1/
2*c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*b^2+c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a
*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1
/2))*C*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*(-4*a*c
+b^2)^(1/2)*b^2+c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*ar
ctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*a*b-1/4/(4*a*c-b^2)^2
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2))*C*b^3-1/2/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^(1/2)*b*ln(2*c*
x^2+(-4*a*c+b^2)^(1/2)+b)-c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-4*a*c+b^2)
^(1/2)*b+2*c^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*a-1/2*c/(4*a*c-b^2)^2*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*A*b^2+c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/
2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b^2-c/(4*a*c
-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*C*a*b+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bbx^2 + (Cb - 2\,Ac)x^3 + 2\,Ba + (2\,Ca - Ab)x}{2\left(\left(b^2c - 4\,ac^2\right)x^4 + ab^2 - 4\,a^2c + \left(b^3 - 4\,abc\right)x^2\right)} - \frac{-\int \frac{2\,Bbx + (Cb - 2\,Ac)x^2 - 2\,Ca + Ab}{cx^4 + bx^2 + a}\,dx}{2\left(b^2 - 4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**3+B*x**2+A*x)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.44 $\int \frac{Ax^2+Bx^3+Cx^4}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=356

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{}}}$$

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.370477, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {1594, 1662, 1275, 1166, 205, 12, 1114, 638, 618, 206}

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```


Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{Ax^2+Bx^3+Cx^4}{\left(a+bx^2+cx^4\right)^2}dx = \int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$
$$= \int \frac{Bx^3}{\left(a+bx^2+cx^4\right)^2}dx + \int \frac{x^2\left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$
$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + B\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}dx + \frac{\int \frac{Ab-2aC+(-2Ac+bC)x^2}{a+bx^2+cx^4}dx}{2\left(b^2-4ac\right)}$$
$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\,\mathrm{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}dx, x, x^2\right) - \frac{\left(2Ac-bC-\right.}{}$$
$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{}}$$
$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{}}$$
$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{}}$$

Mathematica [A] time = 0.17052, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x\left(bx(B+Cx)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-4ac-b^2\right)-2Ac\left(\sqrt{b^2-4ac}-2b\right)\right)\tan^{-1}\left(\right.}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.022, size = 1119, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*B*b/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(4
*a*c-b^2)*x-B*a/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*B*(-4*a*c+b^
2)^(1/2)*b*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-c/(4*a*c-b^2)^2*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2))*A*(-4*a*c+b^2)^(1/2)*b-2*c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/
2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*a+1/
2*c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*b^2+c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a
*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1
/2))*C*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*(-4*a*c
+b^2)^(1/2)*b^2+c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*ar
ctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*a*b-1/4/(4*a*c-b^2)^2
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2))*C*b^3-1/2/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^(1/2)*b*ln(2*c*
x^2+(-4*a*c+b^2)^(1/2)+b)-c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-4*a*c+b^2)
^(1/2)*b+2*c^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*a-1/2*c/(4*a*c-b^2)^2*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*A*b^2+c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/
2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b^2-c/(4*a*c
-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*C*a*b+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bbx^2 + (Cb - 2\,Ac)x^3 + 2\,Ba + (2\,Ca - Ab)x}{2\left(\left(b^2c - 4\,ac^2\right)x^4 + ab^2 - 4\,a^2c + \left(b^3 - 4\,abc\right)x^2\right)} - \frac{-\int \frac{2\,Bbx + (Cb - 2\,Ac)x^2 - 2\,Ca + Ab}{cx^4 + bx^2 + a}\,dx}{2\left(b^2 - 4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**4+B*x**3+A*x**2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.45 $\int \frac{Ax^3+Bx^4+Cx^5}{x\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=356

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2}}}$$

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.356423, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1585, 1662, 1275, 1166, 205, 12, 1114, 638, 618, 206}

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
```

```
lyQ[Pq, x^2]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{Ax^3+Bx^4+Cx^5}{x\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \int \frac{Bx^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{x^2\left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + B\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac+bC)x^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\,\mathrm{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) - \frac{\left(2Ac-bC-\frac{4A}{}\right.}{}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right)\mathrm{t}}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-}}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right)\mathrm{t}}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-}}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right)C}{\sqrt{b^2-4ac}}\right)\mathrm{t}}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-}}$$

Mathematica [A] time = 0.165037, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x\left(bx(B+Cx)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-4ac-b^2\right)-2Ac\left(\sqrt{b^2-4ac}-2b\right)\right)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{b-}}\right.}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.021, size = 1119, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*B*b/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(4
*a*c-b^2)*x-B*a/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*B*(-4*a*c+b^
2)^(1/2)*b*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-c/(4*a*c-b^2)^2*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2))*A*(-4*a*c+b^2)^(1/2)*b-2*c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/
2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*a+1/
2*c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*b^2+c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a
*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1
/2))*C*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*(-4*a*c
+b^2)^(1/2)*b^2+c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*ar
ctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*a*b-1/4/(4*a*c-b^2)^2
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2))*C*b^3-1/2/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^(1/2)*b*ln(2*c*
x^2+(-4*a*c+b^2)^(1/2)+b)-c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-4*a*c+b^2)
^(1/2)*b+2*c^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*a-1/2*c/(4*a*c-b^2)^2*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*A*b^2+c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/
2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b^2-c/(4*a*c
-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*C*a*b+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bbx^2 + (Cb - 2\,Ac)x^3 + 2\,Ba + (2\,Ca - Ab)x}{2\left(\left(b^2c - 4\,ac^2\right)x^4 + ab^2 - 4\,a^2c + \left(b^3 - 4\,abc\right)x^2\right)} - \frac{-\int \frac{2\,Bbx + (Cb - 2\,Ac)x^2 - 2\,Ca + Ab}{cx^4 + bx^2 + a}\,dx}{2\left(b^2 - 4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**5+B*x**4+A*x**3)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.46 $\int \frac{Ax^4+Bx^5+Cx^6}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=356

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{}}}$$

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.358759, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1585, 1662, 1275, 1166, 205, 12, 1114, 638, 618, 206}

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
```

lyQ[Pq, x^2]

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{Ax^4+Bx^5+Cx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$
$$= \int \frac{Bx^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{x^2\left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$
$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + B\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac+bC)x^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$
$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) - \frac{\left(2Ac-bC \cdots\right.}{\cdots}$$
$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right.\cdots}{\sqrt{b^2-4ac}}\right.\cdots}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\cdots}$$
$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right.\cdots}{\sqrt{b^2-4ac}}\right.\cdots}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\cdots}$$
$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+4ac\right.\cdots}{\sqrt{b^2-4ac}}\right.\cdots}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\cdots}$$

Mathematica [A] time = 0.160147, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x\left(bx(B+Cx)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-4ac-b^2\right)-2Ac\left(\sqrt{b^2-4ac}-2b\right)\right)\tan^{-1}\left(\cdots\right.}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.\cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[(A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.024, size = 1119, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*B*b/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(4
*a*c-b^2)*x-B*a/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*B*(-4*a*c+b^
2)^(1/2)*b*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-c/(4*a*c-b^2)^2*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2))*A*(-4*a*c+b^2)^(1/2)*b-2*c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/
2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*a+1/
2*c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*A*b^2+c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a
*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1
/2))*C*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*(-4*a*c
+b^2)^(1/2)*b^2+c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*ar
ctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*C*a*b-1/4/(4*a*c-b^2)^2
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2))*C*b^3-1/2/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^(1/2)*b*ln(2*c*
x^2+(-4*a*c+b^2)^(1/2)+b)-c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*(-4*a*c+b^2)
^(1/2)*b+2*c^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*a-1/2*c/(4*a*c-b^2)^2*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*A*b^2+c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/
2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b^2-c/(4*a*c
-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*C*a*b+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*C*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bbx^2 + (Cb - 2\,Ac)x^3 + 2\,Ba + (2\,Ca - Ab)x}{2\left(\left(b^2c - 4\,ac^2\right)x^4 + ab^2 - 4\,a^2c + \left(b^3 - 4\,abc\right)x^2\right)} - \frac{-\int \frac{2\,Bbx + (Cb - 2\,Ac)x^2 - 2\,Ca + Ab}{cx^4 + bx^2 + a}\,dx}{2\left(b^2 - 4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**6+B*x**5+A*x**4)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.47 $\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=273

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2a^2c^3e-4ab^2c^2e-b^3c(cd-5af)+abc^2(3cd-5af)+b^4ce+b^5(-f)\right)}{2c^5\sqrt{b^2-4ac}}+\frac{x^4\left(-c(af+be)+b^2f+c^2d\right.}{4c^3}$$

```
[Out] ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f))*x^2)/(2*c^4) + ((c^2*d + b
^2*f - c*(b*e + a*f))*x^4)/(4*c^3) + ((c*e - b*f)*x^6)/(6*c^2) + (f*x^8)/(8
*c) - ((b^4*c*e - 4*a*b^2*c^2*e + 2*a^2*c^3*e - b^5*f - b^3*c*(c*d - 5*a*f)
 + a*b*c^2*(3*c*d - 5*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^
5*Sqrt[b^2 - 4*a*c]) - ((b^3*c*e - 2*a*b*c^2*e - b^4*f - b^2*c*(c*d - 3*a*f
) + a*c^2*(c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*c^5)
```

Rubi [A] time = 0.8541, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1663, 1628, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2a^2c^3e-4ab^2c^2e-b^3c(cd-5af)+abc^2(3cd-5af)+b^4ce+b^5(-f)\right)}{2c^5\sqrt{b^2-4ac}}+\frac{x^4\left(-c(af+be)+b^2f+c^2d\right.}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f))*x^2)/(2*c^4) + ((c^2*d + b
^2*f - c*(b*e + a*f))*x^4)/(4*c^3) + ((c*e - b*f)*x^6)/(6*c^2) + (f*x^8)/(8
*c) - ((b^4*c*e - 4*a*b^2*c^2*e + 2*a^2*c^3*e - b^5*f - b^3*c*(c*d - 5*a*f)
 + a*b*c^2*(3*c*d - 5*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^
5*Sqrt[b^2 - 4*a*c]) - ((b^3*c*e - 2*a*b*c^2*e - b^4*f - b^2*c*(c*d - 3*a*f
) + a*c^2*(c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*c^5)
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^7\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^3\left(d+ex+fx^2\right)}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{b^2ce-ac^2e-b^3f-bc(cd-2af)}{c^4} + \frac{\left(c^2d+b^2f-c(be+af)\right)x}{c^3} + \frac{(ce-bf)x^2}{c^2}\right.\right.$$

$$= \frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)\right)x^2}{2c^4} + \frac{\left(c^2d+b^2f-c(be+af)\right)x^4}{4c^3} + \frac{(ce-bf)x^6}{6c^2} + \frac{fx^8}{8c}$$

$$= \frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)\right)x^2}{2c^4} + \frac{\left(c^2d+b^2f-c(be+af)\right)x^4}{4c^3} + \frac{(ce-bf)x^6}{6c^2} + \frac{fx^8}{8c}$$

$$= \frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)\right)x^2}{2c^4} + \frac{\left(c^2d+b^2f-c(be+af)\right)x^4}{4c^3} + \frac{(ce-bf)x^6}{6c^2} + \frac{fx^8}{8c}$$

$$= \frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)\right)x^2}{2c^4} + \frac{\left(c^2d+b^2f-c(be+af)\right)x^4}{4c^3} + \frac{(ce-bf)x^6}{6c^2} + \frac{fx^8}{8c}$$

Mathematica [A] time = 0.208072, size = 260, normalized size = 0.95

$$-\frac{12\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(-2a^2c^3e+4ab^2c^2e+b^3c(cd-5af)+abc^2(5af-3cd)-b^4ce+b^5f\right)}{\sqrt{4ac-b^2}} + 6c^2x^4\left(-c(af+be)+b^2f+c^2d\right) - 12cx^2\left(bc(cd-2af\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-12*c*(-(b^2*c*e) + a*c^2*e + b^3*f + b*c*(c*d - 2*a*f))*x^2 + 6*c^2*(c^2*
d + b^2*f - c*(b*e + a*f))*x^4 + 4*c^3*(c*e - b*f)*x^6 + 3*c^4*f*x^8 - (12*
(-(b^4*c*e) + 4*a*b^2*c^2*e - 2*a^2*c^3*e + b^5*f + b^3*c*(c*d - 5*a*f) + a
*b*c^2*(-3*c*d + 5*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^
2 + 4*a*c] + 6*(-(b^3*c*e) + 2*a*b*c^2*e + b^4*f + b^2*c*(c*d - 3*a*f) + a*
c^2*(-(c*d) + a*f))*Log[a + b*x^2 + c*x^4])/(24*c^5)
```

Maple [B] time = 0.007, size = 622, normalized size = 2.3

$$-\frac{5a^2bf}{2c^3}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+\frac{5ab^3f}{2c^4}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-2\frac{ab^2e}{c^3\sqrt{4ac-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -5/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*b*f+5/
2/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^3*f-2/c^3
/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^2*e+3/2/c^2/(4
*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*d+1/4/c^3*ln(c*x^
4+b*x^2+a)*b^2*d+1/4/c^3*ln(c*x^4+b*x^2+a)*a^2*f-1/4/c^2*ln(c*x^4+b*x^2+a)*
a*d+1/4/c^5*ln(c*x^4+b*x^2+a)*b^4*f-1/4/c^4*ln(c*x^4+b*x^2+a)*b^3*e+1/2/c^3
*b^2*e*x^2-1/2/c^2*b*d*x^2-1/6/c^2*x^6*b*f-1/4/c^2*x^4*a*f+1/4/c^3*x^4*b^2*
f-1/4/c^2*x^4*b*e-1/2/c^2*x^2*a*e-1/2/c^4*b^3*f*x^2-3/4/c^4*ln(c*x^4+b*x^2+
a)*a*b^2*f+1/2/c^3*ln(c*x^4+b*x^2+a)*a*b*e+1/c^2/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*e-1/2/c^5/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^2+b)/(4*a*c-b^2)^(1/2))*b^5*f+1/2/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2))*b^4*e-1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a
*c-b^2)^(1/2))*b^3*d+1/c^3*a*b*f*x^2+1/4/c*x^4*d+1/6/c*x^6*e+1/8*f*x^8/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.16819, size = 1854, normalized size = 6.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(b^2*c^4 - 4*a*c^5)*f*x^8 + 4*((b^2*c^4 - 4*a*c^5)*e - (b^3*c^3 -
4*a*b*c^4)*f)*x^6 + 6*((b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e + (b
^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*f)*x^4 - 12*((b^3*c^3 - 4*a*b*c^4)*d - (b
^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e + (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*f
)*x^2 + 6*sqrt(b^2 - 4*a*c)*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2
 + 2*a^2*c^3)*e + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f)*log((2*c^2*x^4 + 2*b*c
*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))
+ 6*((b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b
*c^3)*e + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*f)*log(c*x^4 + b*x
^2 + a))/(b^2*c^5 - 4*a*c^6), 1/24*(3*(b^2*c^4 - 4*a*c^5)*f*x^8 + 4*((b^2*c
^4 - 4*a*c^5)*e - (b^3*c^3 - 4*a*b*c^4)*f)*x^6 + 6*((b^2*c^4 - 4*a*c^5)*d -
 (b^3*c^3 - 4*a*b*c^4)*e + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*f)*x^4 - 12*
((b^3*c^3 - 4*a*b*c^4)*d - (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e + (b^5*c -
```

```
 6*a*b^3*c^2 + 8*a^2*b*c^3)*f)*x^2 + 12*sqrt(-b^2 + 4*a*c)*((b^3*c^2 - 3*a*
b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e + (b^5 - 5*a*b^3*c + 5*a^2*b
*c^2)*f)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 6*((b^4*
c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e +
(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*f)*log(c*x^4 + b*x^2 + a))/(
b^2*c^5 - 4*a*c^6)]
```

Sympy [B] time = 49.7414, size = 1392, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*f - 2*a**2*c**3*e - 5*a*b**3*c*f + 4*a
*b**2*c**2*e - 3*a*b*c**3*d + b**5*f - b**4*c*e + b**3*c**2*d)/(4*c**5*(4*a
*c - b**2)) + (a**2*c**2*f - 3*a*b**2*c*f + 2*a*b*c**2*e - a*c**3*d + b**4*
f - b**3*c*e + b**2*c**2*d)/(4*c**5))*log(x**2 + (2*a**3*c**2*f - 4*a**2*b*
*2*c*f + 3*a**2*b*c**2*e - 2*a**2*c**3*d + a*b**4*f - a*b**3*c*e + a*b**2*c
**2*d - 8*a*c**5*(-sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*f - 2*a**2*c**3*e - 5
*a*b**3*c*f + 4*a*b**2*c**2*e - 3*a*b*c**3*d + b**5*f - b**4*c*e + b**3*c**
2*d)/(4*c**5*(4*a*c - b**2)) + (a**2*c**2*f - 3*a*b**2*c*f + 2*a*b*c**2*e -
 a*c**3*d + b**4*f - b**3*c*e + b**2*c**2*d)/(4*c**5)) + 2*b**2*c**4*(-sqrt
(-4*a*c + b**2)*(5*a**2*b*c**2*f - 2*a**2*c**3*e - 5*a*b**3*c*f + 4*a*b**2*
c**2*e - 3*a*b*c**3*d + b**5*f - b**4*c*e + b**3*c**2*d)/(4*c**5*(4*a*c - b
**2)) + (a**2*c**2*f - 3*a*b**2*c*f + 2*a*b*c**2*e - a*c**3*d + b**4*f - b*
*3*c*e + b**2*c**2*d)/(4*c**5)))/(5*a**2*b*c**2*f - 2*a**2*c**3*e - 5*a*b**
3*c*f + 4*a*b**2*c**2*e - 3*a*b*c**3*d + b**5*f - b**4*c*e + b**3*c**2*d))
+ (sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*f - 2*a**2*c**3*e - 5*a*b**3*c*f + 4*
a*b**2*c**2*e - 3*a*b*c**3*d + b**5*f - b**4*c*e + b**3*c**2*d)/(4*c**5*(4*
a*c - b**2)) + (a**2*c**2*f - 3*a*b**2*c*f + 2*a*b*c**2*e - a*c**3*d + b**4
*f - b**3*c*e + b**2*c**2*d)/(4*c**5))*log(x**2 + (2*a**3*c**2*f - 4*a**2*b
**2*c*f + 3*a**2*b*c**2*e - 2*a**2*c**3*d + a*b**4*f - a*b**3*c*e + a*b**2*
c**2*d - 8*a*c**5*(sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*f - 2*a**2*c**3*e - 5
*a*b**3*c*f + 4*a*b**2*c**2*e - 3*a*b*c**3*d + b**5*f - b**4*c*e + b**3*c**
2*d)/(4*c**5*(4*a*c - b**2)) + (a**2*c**2*f - 3*a*b**2*c*f + 2*a*b*c**2*e -
 a*c**3*d + b**4*f - b**3*c*e + b**2*c**2*d)/(4*c**5)) + 2*b**2*c**4*(sqrt(
-4*a*c + b**2)*(5*a**2*b*c**2*f - 2*a**2*c**3*e - 5*a*b**3*c*f + 4*a*b**2*c
**2*e - 3*a*b*c**3*d + b**5*f - b**4*c*e + b**3*c**2*d)/(4*c**5*(4*a*c - b*
*2)) + (a**2*c**2*f - 3*a*b**2*c*f + 2*a*b*c**2*e - a*c**3*d + b**4*f - b**
3*c*e + b**2*c**2*d)/(4*c**5)))/(5*a**2*b*c**2*f - 2*a**2*c**3*e - 5*a*b**3
*c*f + 4*a*b**2*c**2*e - 3*a*b*c**3*d + b**5*f - b**4*c*e + b**3*c**2*d)) +
 f*x**8/(8*c) - x**6*(b*f - c*e)/(6*c**2) - x**4*(a*c*f - b**2*f + b*c*e -
c**2*d)/(4*c**3) + x**2*(2*a*b*c*f - a*c**2*e - b**3*f + b**2*c*e - b*c**2*
d)/(2*c**4)
```

Giac [A] time = 1.15015, size = 413, normalized size = 1.51

$$\frac{3\,c^3 f x^8 - 4\,b c^2 f x^6 + 4\,c^3 x^6 e + 6\,c^3 d x^4 + 6\,b^2 c f x^4 - 6\,a c^2 f x^4 - 6\,b c^2 x^4 e - 12\,b c^2 d x^2 - 12\,b^3 f x^2 + 24\,a b c f x^2 + 12\,b^2 c x}{24\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/24*(3*c^3*f*x^8 - 4*b*c^2*f*x^6 + 4*c^3*x^6*e + 6*c^3*d*x^4 + 6*b^2*c*f*x
^4 - 6*a*c^2*f*x^4 - 6*b*c^2*x^4*e - 12*b*c^2*d*x^2 - 12*b^3*f*x^2 + 24*a*b
*c*f*x^2 + 12*b^2*c*x^2*e - 12*a*c^2*x^2*e)/c^4 + 1/4*(b^2*c^2*d - a*c^3*d
+ b^4*f - 3*a*b^2*c*f + a^2*c^2*f - b^3*c*e + 2*a*b*c^2*e)*log(c*x^4 + b*x^
2 + a)/c^5 - 1/2*(b^3*c^2*d - 3*a*b*c^3*d + b^5*f - 5*a*b^3*c*f + 5*a^2*b*c
^2*f - b^4*c*e + 4*a*b^2*c^2*e - 2*a^2*c^3*e)*arctan((2*c*x^2 + b)/sqrt(-b^
2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^5)
```

3.48 $\int \frac{x^5\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=203

$$\frac{x^2\left(-c(af+be)+b^2f+c^2d\right)}{2c^3}+\frac{\log\left(a+bx^2+cx^4\right)\left(-bc(cd-2af)-ac^2e+b^2ce+b^3(-f)\right)}{4c^4}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-b^2c\right.}{}$$

```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x^2)/(2*c^3) + ((c*e - b*f)*x^4)/(4*c^2) +
 (f*x^6)/(6*c) + ((b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*
a*c^2*(c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*Sqrt[b^
2 - 4*a*c]) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f))*Log[a + b*x^
2 + c*x^4])/(4*c^4)
```

Rubi [A] time = 0.423681, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1663, 1628, 634, 618, 206, 628}

$$\frac{x^2\left(-c(af+be)+b^2f+c^2d\right)}{2c^3}+\frac{\log\left(a+bx^2+cx^4\right)\left(-bc(cd-2af)-ac^2e+b^2ce+b^3(-f)\right)}{4c^4}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-b^2c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x^2)/(2*c^3) + ((c*e - b*f)*x^4)/(4*c^2) +
 (f*x^6)/(6*c) + ((b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*
a*c^2*(c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*Sqrt[b^
2 - 4*a*c]) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f))*Log[a + b*x^
2 + c*x^4])/(4*c^4)
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^5\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2\left(d+ex+fx^2\right)}{a+bx+cx^2}dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{c^2d+b^2f-c(be+af)}{c^3}+\frac{(ce-bf)x}{c^2}+\frac{fx^2}{c}-\frac{a\left(c^2d+b^2f-c(be+af)\right)-\left(b^2c\right.}{c^3\left(a+b\right.}\right.\right.$$
$$= \frac{\left(c^2d+b^2f-c(be+af)\right)x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c}-\frac{\text{Subst}\left(\int \frac{a\left(c^2d+b^2f-c(be+af)\right)-\left(b^2ce-ac^2e-b^3f-\right.}{a+bx+cx^2}\right.}{2c^3}$$
$$= \frac{\left(c^2d+b^2f-c(be+af)\right)x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c}+\frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)\right)\text{Subst}}{4c^4}$$
$$= \frac{\left(c^2d+b^2f-c(be+af)\right)x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c}+\frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)\right)\log\left(a\right.}{4c^4}$$
$$= \frac{\left(c^2d+b^2f-c(be+af)\right)x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c}+\frac{\left(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2\right.}{2c^4\sqrt{b^2-4a}}$$

Mathematica [A] time = 0.143298, size = 193, normalized size = 0.95

$$\frac{6cx^2\left(-c(af+be)+b^2f+c^2d\right)-3\log\left(a+bx^2+cx^4\right)\left(bc(cd-2af)+ac^2e-b^2ce+b^3f\right)+\frac{6\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(b^2c(cd-4af)+3abc\right.}{\sqrt{4ac-b^2}}}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (6*c*(c^2*d + b^2*f - c*(b*e + a*f))*x^2 + 3*c^2*(c*e - b*f)*x^4 + 2*c^3*f*
x^6 + (6*(-(b^3*c*e) + 3*a*b*c^2*e + b^4*f + b^2*c*(c*d - 4*a*f) + 2*a*c^2*
(-(c*d) + a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c
] - 3*(-(b^2*c*e) + a*c^2*e + b^3*f + b*c*(c*d - 2*a*f))*Log[a + b*x^2 + c*
x^4])/(12*c^4)
```

Maple [B] time = 0.006, size = 474, normalized size = 2.3

$$\frac{fx^6}{6c}-\frac{x^4bf}{4c^2}+\frac{x^4e}{4c}-\frac{x^2af}{2c^2}+\frac{b^2fx^2}{2c^3}-\frac{bex^2}{2c^2}+\frac{x^2d}{2c}+\frac{\ln\left(cx^4+bx^2+a\right)abf}{2c^3}-\frac{\ln\left(cx^4+bx^2+a\right)ae}{4c^2}-\frac{\ln\left(cx^4+bx^2+a\right)}{4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/6*f*x^6/c-1/4/c^2*x^4*b*f+1/4/c*x^4*e-1/2/c^2*x^2*a*f+1/2/c^3*b^2*f*x^2-1
/2/c^2*b*e*x^2+1/2/c*d*x^2+1/2/c^3*ln(c*x^4+b*x^2+a)*a*b*f-1/4/c^2*ln(c*x^4
+b*x^2+a)*a*e-1/4/c^4*ln(c*x^4+b*x^2+a)*b^3*f+1/4/c^3*ln(c*x^4+b*x^2+a)*b^2
*e-1/4/c^2*ln(c*x^4+b*x^2+a)*b*d+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2))*a^2*f-2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2))*a*b^2*f+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^
2)^(1/2))*a*b*e-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
*a*d+1/2/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*f-
1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*e+1/2/c
^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.82308, size = 1404, normalized size = 6.92

$$\left[\frac{2\left(b^{2}c^{3}-4\,ac^{4}\right)fx^{6}+3\left(\left(b^{2}c^{3}-4\,ac^{4}\right)e-\left(b^{3}c^{2}-4\,abc^{3}\right)f\right)x^{4}+6\left(\left(b^{2}c^{3}-4\,ac^{4}\right)d-\left(b^{3}c^{2}-4\,abc^{3}\right)e+\left(b^{4}c-5\,ab^{2}c^{2}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(2*(b^2*c^3 - 4*a*c^4)*f*x^6 + 3*((b^2*c^3 - 4*a*c^4)*e - (b^3*c^2 -
4*a*b*c^3)*f)*x^4 + 6*((b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*b*c^3)*e + (b
^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*f)*x^2 + 3*sqrt(b^2 - 4*a*c)*((b^2*c^2 - 2*
a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*f)*log((2*
c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4
 + b*x^2 + a)) - 3*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 5*a*b^2*c^2 + 4*a^2*
c^3)*e + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^
4 - 4*a*c^5), 1/12*(2*(b^2*c^3 - 4*a*c^4)*f*x^6 + 3*((b^2*c^3 - 4*a*c^4)*e
- (b^3*c^2 - 4*a*b*c^3)*f)*x^4 + 6*((b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*
b*c^3)*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*f)*x^2 - 6*sqrt(-b^2 + 4*a*c)*
((b^2*c^2 - 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*c + 2*a^2*c
^2)*f)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 3*((b^3*c^
2 - 4*a*b*c^3)*d - (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e + (b^5 - 6*a*b^3*c +
 8*a^2*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^4 - 4*a*c^5)]
```

Sympy [B] time = 35.7045, size = 1044, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a**2*c**2*f - 4*a*b**2*c*f + 3*a*b*c**2*e - 2*a*c*
*3*d + b**4*f - b**3*c*e + b**2*c**2*d)/(4*c**4*(4*a*c - b**2)) + (2*a*b*c*
f - a*c**2*e - b**3*f + b**2*c*e - b*c**2*d)/(4*c**4))*log(x**2 + (-3*a**2*
b*c*f + 2*a**2*c**2*e + a*b**3*f - a*b**2*c*e + a*b*c**2*d + 8*a*c**4*(-sqr
t(-4*a*c + b**2)*(2*a**2*c**2*f - 4*a*b**2*c*f + 3*a*b*c**2*e - 2*a*c**3*d
+ b**4*f - b**3*c*e + b**2*c**2*d)/(4*c**4*(4*a*c - b**2)) + (2*a*b*c*f - a
*c**2*e - b**3*f + b**2*c*e - b*c**2*d)/(4*c**4)) - 2*b**2*c**3*(-sqrt(-4*a
*c + b**2)*(2*a**2*c**2*f - 4*a*b**2*c*f + 3*a*b*c**2*e - 2*a*c**3*d + b**4
*f - b**3*c*e + b**2*c**2*d)/(4*c**4*(4*a*c - b**2)) + (2*a*b*c*f - a*c**2*
e - b**3*f + b**2*c*e - b*c**2*d)/(4*c**4)))/(2*a**2*c**2*f - 4*a*b**2*c*f
+ 3*a*b*c**2*e - 2*a*c**3*d + b**4*f - b**3*c*e + b**2*c**2*d)) + (sqrt(-4*
a*c + b**2)*(2*a**2*c**2*f - 4*a*b**2*c*f + 3*a*b*c**2*e - 2*a*c**3*d + b**
4*f - b**3*c*e + b**2*c**2*d)/(4*c**4*(4*a*c - b**2)) + (2*a*b*c*f - a*c**2
*e - b**3*f + b**2*c*e - b*c**2*d)/(4*c**4))*log(x**2 + (-3*a**2*b*c*f + 2*
a**2*c**2*e + a*b**3*f - a*b**2*c*e + a*b*c**2*d + 8*a*c**4*(sqrt(-4*a*c +
b**2)*(2*a**2*c**2*f - 4*a*b**2*c*f + 3*a*b*c**2*e - 2*a*c**3*d + b**4*f -
b**3*c*e + b**2*c**2*d)/(4*c**4*(4*a*c - b**2)) + (2*a*b*c*f - a*c**2*e - b
**3*f + b**2*c*e - b*c**2*d)/(4*c**4)) - 2*b**2*c**3*(sqrt(-4*a*c + b**2)*(
2*a**2*c**2*f - 4*a*b**2*c*f + 3*a*b*c**2*e - 2*a*c**3*d + b**4*f - b**3*c*
e + b**2*c**2*d)/(4*c**4*(4*a*c - b**2)) + (2*a*b*c*f - a*c**2*e - b**3*f +
 b**2*c*e - b*c**2*d)/(4*c**4)))/(2*a**2*c**2*f - 4*a*b**2*c*f + 3*a*b*c**2
*e - 2*a*c**3*d + b**4*f - b**3*c*e + b**2*c**2*d)) + f*x**6/(6*c) - x**4*(
b*f - c*e)/(4*c**2) - x**2*(a*c*f - b**2*f + b*c*e - c**2*d)/(2*c**3)
```

Giac [A] time = 1.1619, size = 289, normalized size = 1.42

$$\frac{2\,c^2fx^6 - 3\,bcfx^4 + 3\,c^2x^4e + 6\,c^2dx^2 + 6\,b^2fx^2 - 6\,acfx^2 - 6\,bcx^2e}{12\,c^3} - \frac{\left(bc^2d + b^3f - 2\,abcf - b^2ce + ac^2e\right)\log\left(cx^4 + bx^2\right.}{4\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*(2*c^2*f*x^6 - 3*b*c*f*x^4 + 3*c^2*x^4*e + 6*c^2*d*x^2 + 6*b^2*f*x^2 -
 6*a*c*f*x^2 - 6*b*c*x^2*e)/c^3 - 1/4*(b*c^2*d + b^3*f - 2*a*b*c*f - b^2*c*
e + a*c^2*e)*log(c*x^4 + b*x^2 + a)/c^4 + 1/2*(b^2*c^2*d - 2*a*c^3*d + b^4*
f - 4*a*b^2*c*f + 2*a^2*c^2*f - b^3*c*e + 3*a*b*c^2*e)*arctan((2*c*x^2 + b)
/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

3.49 $\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=144

$$\frac{\log\left(a+bx^2+cx^4\right)\left(-c(af+be)+b^2f+c^2d\right)}{4c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-bc(cd-3af)-2ac^2e+b^2ce+b^3(-f)\right)}{2c^3\sqrt{b^2-4ac}}+\frac{x^2(ce-}{2c^2}$$

```
[Out] ((c*e - b*f)*x^2)/(2*c^2) + (f*x^4)/(4*c) - ((b^2*c*e - 2*a*c^2*e - b^3*f -
 b*c*(c*d - 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b
^2 - 4*a*c]) + ((c^2*d + b^2*f - c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*
c^3)
```

Rubi [A] time = 0.271637, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1663, 1628, 634, 618, 206, 628}

$$\frac{\log\left(a+bx^2+cx^4\right)\left(-c(af+be)+b^2f+c^2d\right)}{4c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-bc(cd-3af)-2ac^2e+b^2ce+b^3(-f)\right)}{2c^3\sqrt{b^2-4ac}}+\frac{x^2(ce-}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```


```
[Out] ((c*e - b*f)*x^2)/(2*c^2) + (f*x^4)/(4*c) - ((b^2*c*e - 2*a*c^2*e - b^3*f -
 b*c*(c*d - 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b
^2 - 4*a*c]) + ((c^2*d + b^2*f - c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*
c^3)
```


Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x\left(d+ex+fx^2\right)}{a+bx+cx^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{ce-bf}{c^2}+\frac{fx}{c}-\frac{a(ce-bf)-\left(c^2d-bce+b^2f-acf\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx, x, x^2\right)\\ &= \frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c}-\frac{\operatorname{Subst}\left(\int \frac{a(ce-bf)-\left(c^2d-bce+b^2f-acf\right)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c^2}\\ &= \frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c}-\frac{\left(-c^2d+bce-b^2f+acf\right)\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^3}+\frac{\left(b^2ce-2ac^2e-\right.}{}\\ &= \frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c}+\frac{\left(c^2d-bce+b^2f-acf\right)\log\left(a+bx^2+cx^4\right)}{4c^3}-\frac{\left(b^2ce-2ac^2e-b^3f-bc(\right.}{}\\ &= \frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c}-\frac{\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{\left(c^2d-bce+b^2\right.}{}\end{aligned}$$

Mathematica [A] time = 0.106951, size = 136, normalized size = 0.94

$$\frac{\log\left(a+bx^2+cx^4\right)\left(-c(af+be)+b^2f+c^2d\right)-\frac{2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(bc(cd-3af)+2ac^2e-b^2ce+b^3f\right)}{\sqrt{4ac-b^2}}+2cx^2(ce-bf)+c^2fx^4}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(c*e - b*f)*x^2 + c^2*f*x^4 - (2*(-(b^2*c*e) + 2*a*c^2*e + b^3*f + b*c
*(c*d - 3*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c
] + (c^2*d + b^2*f - c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [B] time = 0.005, size = 321, normalized size = 2.2

$$\frac{fx^4}{4c}-\frac{bfx^2}{2c^2}+\frac{ex^2}{2c}-\frac{\ln\left(cx^4+bx^2+a\right)af}{4c^2}+\frac{\ln\left(cx^4+bx^2+a\right)b^2f}{4c^3}-\frac{\ln\left(cx^4+bx^2+a\right)be}{4c^2}+\frac{\ln\left(cx^4+bx^2+a\right)d}{4c}+\frac{3a}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*f*x^4/c-1/2/c^2*b*f*x^2+1/2/c*e*x^2-1/4/c^2*ln(c*x^4+b*x^2+a)*a*f+1/4/c
^3*ln(c*x^4+b*x^2+a)*b^2*f-1/4/c^2*ln(c*x^4+b*x^2+a)*b*e+1/4/c*ln(c*x^4+b*x
^2+a)*d+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b
*f-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*e*a-1/2/c^3/
(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*f+1/2/c^2/(4*a*
c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*e-1/2/c/(4*a*c-b^2)^
(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96605, size = 994, normalized size = 6.9

$$\left[\frac{\left(b^2c^2-4\,ac^3\right)fx^4+2\left(\left(b^2c^2-4\,ac^3\right)e-\left(b^3c-4\,abc^2\right)f\right)x^2-\left(bc^2d-\left(b^2c-2\,ac^2\right)e+\left(b^3-3\,abc\right)f\right)\sqrt{b^2-4\,ac}\log\left(\right.}{4\left(b^2c^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2 - 4*a*c^3)*f*x^4 + 2*((b^2*c^2 - 4*a*c^3)*e - (b^3*c - 4*a*b
*c^2)*f)*x^2 - (b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)*f)*sqrt(b^2
 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2
 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c
^2)*e + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 -
 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*f*x^4 + 2*((b^2*c^2 - 4*a*c^3)*e - (b^3
*c - 4*a*b*c^2)*f)*x^2 + 2*(b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)
*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*
c)) + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 5*a*b^2*c + 4
*a^2*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] time = 19.7194, size = 721, normalized size = 5.01

$$\left(-\frac{\sqrt{-4ac+b^2}\left(3abcf-2ac^2e-b^3f+b^2ce-bc^2d\right)}{4c^3\left(4ac-b^2\right)}-\frac{acf-b^2f+bce-c^2d}{4c^3}\right)\log\left(x^2+\frac{2a^2cf-ab^2f+abce+8ac^3\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(3*a*b*c*f - 2*a*c**2*e - b**3*f + b**2*c*e - b*c**2*
d)/(4*c**3*(4*a*c - b**2)) - (a*c*f - b**2*f + b*c*e - c**2*d)/(4*c**3))*lo
g(x**2 + (2*a**2*c*f - a*b**2*f + a*b*c*e + 8*a*c**3*(-sqrt(-4*a*c + b**2)*
(3*a*b*c*f - 2*a*c**2*e - b**3*f + b**2*c*e - b*c**2*d)/(4*c**3*(4*a*c - b*
*2)) - (a*c*f - b**2*f + b*c*e - c**2*d)/(4*c**3)) - 2*a*c**2*d - 2*b**2*c*
*2*(-sqrt(-4*a*c + b**2)*(3*a*b*c*f - 2*a*c**2*e - b**3*f + b**2*c*e - b*c*
*2*d)/(4*c**3*(4*a*c - b**2)) - (a*c*f - b**2*f + b*c*e - c**2*d)/(4*c**3))
)/(3*a*b*c*f - 2*a*c**2*e - b**3*f + b**2*c*e - b*c**2*d)) + (sqrt(-4*a*c +
 b**2)*(3*a*b*c*f - 2*a*c**2*e - b**3*f + b**2*c*e - b*c**2*d)/(4*c**3*(4*a
*c - b**2)) - (a*c*f - b**2*f + b*c*e - c**2*d)/(4*c**3))*log(x**2 + (2*a**
2*c*f - a*b**2*f + a*b*c*e + 8*a*c**3*(sqrt(-4*a*c + b**2)*(3*a*b*c*f - 2*a
*c**2*e - b**3*f + b**2*c*e - b*c**2*d)/(4*c**3*(4*a*c - b**2)) - (a*c*f -
b**2*f + b*c*e - c**2*d)/(4*c**3)) - 2*a*c**2*d - 2*b**2*c**2*(sqrt(-4*a*c
+ b**2)*(3*a*b*c*f - 2*a*c**2*e - b**3*f + b**2*c*e - b*c**2*d)/(4*c**3*(4*
a*c - b**2)) - (a*c*f - b**2*f + b*c*e - c**2*d)/(4*c**3)))/(3*a*b*c*f - 2*
a*c**2*e - b**3*f + b**2*c*e - b*c**2*d)) + f*x**4/(4*c) - x**2*(b*f - c*e)
/(2*c**2)
```

Giac [A] time = 1.14972, size = 190, normalized size = 1.32

$$\frac{cfx^4 - 2\,bfx^2 + 2\,cx^2e}{4\,c^2} + \frac{\left(c^2d + b^2f - acf - bce\right)\log\left(cx^4 + bx^2 + a\right)}{4\,c^3} - \frac{\left(bc^2d + b^3f - 3\,abcf - b^2ce + 2\,ac^2e\right)\arctan\left(\frac{2\,cx^2 + b}{\sqrt{-b^2 + 4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(c*f*x^4 - 2*b*f*x^2 + 2*c*x^2*e)/c^2 + 1/4*(c^2*d + b^2*f - a*c*f - b*
c*e)*log(c*x^4 + b*x^2 + a)/c^3 - 1/2*(b*c^2*d + b^3*f - 3*a*b*c*f - b^2*c*
e + 2*a*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)
*c^3)
```

3.50 $\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$

Optimal. Leaf size=103

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2acf+b^2f-bce+2c^2d\right)}{2c^2\sqrt{b^2-4ac}}+\frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{fx^2}{2c}$$

```
[Out] (f*x^2)/(2*c) - ((2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) + ((c*e - b*f)*Log[a + b*x^2
+ c*x^4])/(4*c^2)
```

Rubi [A] time = 0.178958, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1663, 1657, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2acf+b^2f-bce+2c^2d\right)}{2c^2\sqrt{b^2-4ac}}+\frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{fx^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (f*x^2)/(2*c) - ((2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) + ((c*e - b*f)*Log[a + b*x^2
+ c*x^4])/(4*c^2)
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{d+ex+fx^2}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{f}{c}+\frac{cd-af+(ce-bf)x}{c\left(a+bx+cx^2\right)}\right)dx, x, x^2\right)\\
&= \frac{fx^2}{2c}+\frac{\text{Subst}\left(\int \frac{cd-af+(ce-bf)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c}\\
&= \frac{fx^2}{2c}+\frac{(ce-bf)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2}+\frac{\left(2c^2d-bce+b^2f-2acf\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,d\right.}{4c^2}\\
&= \frac{fx^2}{2c}+\frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2}-\frac{\left(2c^2d-bce+b^2f-2acf\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+\right.}{2c^2}\\
&= \frac{fx^2}{2c}-\frac{\left(2c^2d-bce+b^2f-2acf\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2}
\end{aligned}$$

Mathematica [A] time = 0.068704, size = 100, normalized size = 0.97

$$\frac{\frac{2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(-c(2af+be)+b^2f+2c^2d\right)}{\sqrt{4ac-b^2}}+(ce-bf)\log\left(a+bx^2+cx^4\right)+2cfx^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*f*x^2 + (2*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*ArcTan[(b + 2*c*x^2)/Sq
rt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*e - b*f)*Log[a + b*x^2 + c*x^4])
/(4*c^2)
```

Maple [B] time = 0.005, size = 211, normalized size = 2.1

$$\frac{fx^2}{2c}-\frac{\ln\left(cx^4+bx^2+a\right)bf}{4c^2}+\frac{\ln\left(cx^4+bx^2+a\right)e}{4c}-\frac{af}{c}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+d\arctan\left((2cx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*f*x^2/c-1/4/c^2*ln(c*x^4+b*x^2+a)*b*f+1/4/c*ln(c*x^4+b*x^2+a)*e-1/c/(4*
a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*f+1/(4*a*c-b^2)^(1/2
)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*d+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*f-1/2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))*b*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74658, size = 691, normalized size = 6.71

$$\left[\frac{2\left(b^2c-4\,ac^2\right)fx^2-\left(2\,c^2d-bce+\left(b^2-2\,ac\right)f\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(\left(b^2c-4\,ac^2\right)e\right.}{4\left(b^2c^2-4\,ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*f*x^2 - (2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(
b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c - 4*a*c^2)*e - (b^3 - 4*a*b*c)
*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1/4*(2*(b^2*c - 4*a*c^2)*f
*x^2 - 2*(2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*e - (b^3
- 4*a*b*c)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] time = 11.0933, size = 498, normalized size = 4.83

$$\left(-\frac{\sqrt{-4ac+b^2}\left(2acf-b^2f+bce-2c^2d\right)}{4c^2\left(4ac-b^2\right)}-\frac{bf-ce}{4c^2}\right)\log\left(x^2+\frac{-abf-8ac^2\left(-\frac{\sqrt{-4ac+b^2}\left(2acf-b^2f+bce-2c^2d\right)}{4c^2\left(4ac-b^2\right)}-\frac{bf-ce}{4c^2}\right)+2a}{2acf-b^2f+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a*c*f - b**2*f + b*c*e - 2*c**2*d)/(4*c**2*(4*a*c
- b**2)) - (b*f - c*e)/(4*c**2))*log(x**2 + (-a*b*f - 8*a*c**2*(-sqrt(-4*a*
c + b**2)*(2*a*c*f - b**2*f + b*c*e - 2*c**2*d)/(4*c**2*(4*a*c - b**2)) - (
b*f - c*e)/(4*c**2)) + 2*a*c*e + 2*b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*f -
b**2*f + b*c*e - 2*c**2*d)/(4*c**2*(4*a*c - b**2)) - (b*f - c*e)/(4*c**2))
- b*c*d)/(2*a*c*f - b**2*f + b*c*e - 2*c**2*d)) + (sqrt(-4*a*c + b**2)*(2*a
```

```
*c*f - b**2*f + b*c*e - 2*c**2*d)/(4*c**2*(4*a*c - b**2)) - (b*f - c*e)/(4*
c**2))*log(x**2 + (-a*b*f - 8*a*c**2*(sqrt(-4*a*c + b**2)*(2*a*c*f - b**2*f
 + b*c*e - 2*c**2*d)/(4*c**2*(4*a*c - b**2)) - (b*f - c*e)/(4*c**2)) + 2*a*
c*e + 2*b**2*c*(sqrt(-4*a*c + b**2)*(2*a*c*f - b**2*f + b*c*e - 2*c**2*d)/(
4*c**2*(4*a*c - b**2)) - (b*f - c*e)/(4*c**2)) - b*c*d)/(2*a*c*f - b**2*f +
 b*c*e - 2*c**2*d)) + f*x**2/(2*c)
```

Giac [A] time = 1.14739, size = 134, normalized size = 1.3

$$\frac{fx^2}{2c} - \frac{(bf - ce)\log\left(cx^4 + bx^2 + a\right)}{4c^2} + \frac{\left(2c^2d + b^2f - 2acf - bce\right)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2 + 4ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/c - 1/4*(b*f - c*e)*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(2*c^2*d + b
^2*f - 2*a*c*f - b*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
 + 4*a*c)*c^2)
```

3.51 $\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=97

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(abf-2ace+bcd)}{2ac\sqrt{b^2-4ac}}-\frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac}+\frac{d\log(x)}{a}$$

```
[Out] ((b*c*d - 2*a*c*e + a*b*f)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*c
*Sqrt[b^2 - 4*a*c]) + (d*Log[x])/a - ((c*d - a*f)*Log[a + b*x^2 + c*x^4])/(
4*a*c)
```

Rubi [A] time = 0.200432, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1663, 1628, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(abf-2ace+bcd)}{2ac\sqrt{b^2-4ac}}-\frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac}+\frac{d\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*c*d - 2*a*c*e + a*b*f)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*c
*Sqrt[b^2 - 4*a*c]) + (d*Log[x])/a - ((c*d - a*f)*Log[a + b*x^2 + c*x^4])/(
4*a*c)
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{d+ex+fx^2}{x\left(a+bx+cx^2\right)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{d}{ax}+\frac{-bd+ae-(cd-af)x}{a\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)\\
&= \frac{d\log(x)}{a}+\frac{\text{Subst}\left(\int \frac{-bd+ae-(cd-af)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{d\log(x)}{a}-\frac{(cd-af)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4ac}-\frac{(bcd-2ace+abf)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,
\end{aligned}$$

$$\begin{aligned}
&= \frac{d\log(x)}{a}-\frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac}+\frac{(bcd-2ace+abf)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx
\end{aligned}$$

$$= \frac{(bcd-2ace+abf)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2ac\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac}$$

Mathematica [A] time = 0.143222, size = 178, normalized size = 1.84

$$\frac{-\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(cd\sqrt{b^2-4ac}-af\sqrt{b^2-4ac}+abf-2ace+bcd\right)+\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(-cd\sqrt{b^2\right.}{4ac\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*c*Sqrt[b^2 - 4*a*c]*d*Log[x] - (b*c*d + c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e
+ a*b*f - a*Sqrt[b^2 - 4*a*c]*f)*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2] + (b*
c*d - c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f + a*Sqrt[b^2 - 4*a*c]*f)*Log[
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*c*Sqrt[b^2 - 4*a*c])
```

Maple [A] time = 0.008, size = 165, normalized size = 1.7

$$\frac{d\ln(x)}{a}+\frac{\ln\left(cx^4+bx^2+a\right)f}{4c}-\frac{\ln\left(cx^4+bx^2+a\right)d}{4a}+e\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-\frac{bd}{2a}\arctan\left((2cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] d*ln(x)/a+1/4/c*ln(c*x^4+b*x^2+a)*f-1/4/a*ln(c*x^4+b*x^2+a)*d+1/(4*a*c-b^2)
^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*e-1/2/a/(4*a*c-b^2)^(1/2)*arct
an((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*d-1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))*b/c*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.63331, size = 683, normalized size = 7.04

$$\left[\frac{4\left(b^2c-4\,ac^2\right)d\log\left(x\right)+\left(bcd-2\,ace+abf\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(\left(b^2c-4\,ac^2\right)d-\left(\right.\right.}{4\left(ab^2c-4\,a^2c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(b^2*c - 4*a*c^2)*d*log(x) + (b*c*d - 2*a*c*e + a*b*f)*sqrt(b^2 - 4
*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4
*a*c))/(c*x^4 + b*x^2 + a)) - ((b^2*c - 4*a*c^2)*d - (a*b^2 - 4*a^2*c)*f)*l
og(c*x^4 + b*x^2 + a))/(a*b^2*c - 4*a^2*c^2), 1/4*(4*(b^2*c - 4*a*c^2)*d*lo
g(x) + 2*(b*c*d - 2*a*c*e + a*b*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)
*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^2*c - 4*a*c^2)*d - (a*b^2 - 4*a^2*
c)*f)*log(c*x^4 + b*x^2 + a))/(a*b^2*c - 4*a^2*c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17434, size = 131, normalized size = 1.35

$$\frac{d\log\left(x^2\right)}{2\,a}-\frac{\left(cd-af\right)\log\left(cx^4+bx^2+a\right)}{4\,ac}-\frac{\left(bcd+abf-2\,ace\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\sqrt{-b^2+4\,ac}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d*log(x^2)/a - 1/4*(c*d - a*f)*log(c*x^4 + b*x^2 + a)/(a*c) - 1/2*(b*c*
d + a*b*f - 2*a*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
4*a*c)*a*c)
```

3.52 $\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=118

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-abe-2a(cd-af)+b^2d\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(bd-ae)}{a^2}-\frac{d}{2ax^2}$$

```
[Out] -d/(2*a*x^2) - ((b^2*d - a*b*e - 2*a*(c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x])/a^2 + ((b
*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rubi [A] time = 0.285348, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1663, 1628, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-abe-2a(cd-af)+b^2d\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(bd-ae)}{a^2}-\frac{d}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x]
```


```
[Out] -d/(2*a*x^2) - ((b^2*d - a*b*e - 2*a*(c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x])/a^2 + ((b
*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*a^2)
```


Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{d+ex+fx^2}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{d}{ax^2}+\frac{-bd+ae}{a^2x}+\frac{b^2d-abe-a(cd-af)+c(bd-ae)x}{a^2\left(a+bx+cx^2\right)}\right)dx, x, x^2\right)$$

$$= -\frac{d}{2ax^2}-\frac{(bd-ae)\log(x)}{a^2}+\frac{\mathrm{Subst}\left(\int \frac{b^2d-abe-a(cd-af)+c(bd-ae)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2}$$

$$= -\frac{d}{2ax^2}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\,\mathrm{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2}+\frac{\left(b^2d-abe-2a(cd-af)\right. \cdots}{\cdots}$$

$$= -\frac{d}{2ax^2}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\left(b^2d-abe-2a(cd-af)\right)\mathrm{Subst}\left(\cdots\right.}{2a^2}$$

$$= -\frac{d}{2ax^2}-\frac{\left(b^2d-abe-2a(cd-af)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\log\left(a+b \cdots\right.}{4a^2}$$

Mathematica [A] time = 0.161855, size = 203, normalized size = 1.72

$$\frac{\frac{\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(a\left(-e\sqrt{b^2-4ac}+2af-2cd\right)+b\left(d\sqrt{b^2-4ac}-ae\right)+b^2d\right)}{\sqrt{b^2-4ac}}+\frac{\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(-a\left(e\sqrt{b^2-4ac}+2af-2cd\right)+b\left(d\sqrt{b^2-4ac}+ae\right)+b^2(-d)\right)}{\sqrt{b^2-4ac}}+ \cdots}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*d)/x^2 + 4*(-(b*d) + a*e)*Log[x] + ((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d
- a*e) + a*(-2*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f))*Log[b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-(b^2*d) + b*(Sqrt[b^2 - 4*a*c]*d + a*e
) - a*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Maple [B] time = 0.009, size = 227, normalized size = 1.9

$$-\frac{d}{2\,ax^2}+\frac{\ln(x)\,e}{a}-\frac{\ln(x)\,bd}{a^2}-\frac{\ln\left(cx^4+bx^2+a\right)e}{4\,a}+\frac{\ln\left(cx^4+bx^2+a\right)bd}{4\,a^2}+f\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac- \cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*d/a/x^2+1/a*ln(x)*e-1/a^2*ln(x)*b*d-1/4/a*ln(c*x^4+b*x^2+a)*e+1/4/a^2*
ln(c*x^4+b*x^2+a)*b*d+1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1
/2))*f-1/2/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*e-1/
a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*d+1/2/a^2/(4*a*
c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.99112, size = 873, normalized size = 7.4

$$\left[-\frac{\left(abe-2\,a^{2}f-\left(b^{2}-2\,ac\right)d\right)\sqrt{b^{2}-4\,ac}x^{2}\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac-\left(2\,cx^{2}+b\right)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)-\left(\left(b^{3}-4\,abc\right)d-\left(ab^{2}-4\,a^{2}c\right)e\right)x}{4\left(a^{2}b^{2}-4\,a^{3}c\right)x^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*e - 2*a^2*f - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2
*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 +
b*x^2 + a)) - ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(c*x^4 + b*x
^2 + a) + 4*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(x) + 2*(a*b^2
 - 4*a^2*c)*d)/((a^2*b^2 - 4*a^3*c)*x^2), 1/4*(2*(a*b*e - 2*a^2*f - (b^2 -
2*a*c)*d)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(
b^2 - 4*a*c)) + ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(c*x^4 + b
*x^2 + a) - 4*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(x) - 2*(a*b
^2 - 4*a^2*c)*d)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12911, size = 182, normalized size = 1.54

$$\frac{(bd-ae)\log\left(cx^{4}+bx^{2}+a\right)}{4\,a^{2}}-\frac{(bd-ae)\log\left(x^{2}\right)}{2\,a^{2}}+\frac{\left(b^{2}d-2\,acd+2\,a^{2}f-abe\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\,\sqrt{-b^{2}+4\,ac}a^{2}}+\frac{bdx^{2}-ax^{2}e-a}{2\,a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(b*d - a*e)*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*(b*d - a*e)*log(x^2)/a^2 +
 1/2*(b^2*d - 2*a*c*d + 2*a^2*f - a*b*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4
*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b*d*x^2 - a*x^2*e - a*d)/(a^2*x^2)
```

3.53 $\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=174

$$-\frac{\log\left(a+bx^2+cx^4\right)\left(-abe-a(cd-af)+b^2d\right)}{4a^3}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)}{2a^3\sqrt{b^2-4ac}}+\frac{\log(x)\left(-a\right.}{}$$

```
[Out] -d/(4*a*x^4) + (b*d - a*e)/(2*a^2*x^2) + ((b^3*d - a*b^2*e + 2*a^2*c*e - a*
b*(3*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*Sqrt[b^2
- 4*a*c]) + ((b^2*d - a*b*e - a*(c*d - a*f))*Log[x])/a^3 - ((b^2*d - a*b*e
- a*(c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rubi [A] time = 0.40734, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1663, 1628, 634, 618, 206, 628}

$$-\frac{\log\left(a+bx^2+cx^4\right)\left(-abe-a(cd-af)+b^2d\right)}{4a^3}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)}{2a^3\sqrt{b^2-4ac}}+\frac{\log(x)\left(-a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -d/(4*a*x^4) + (b*d - a*e)/(2*a^2*x^2) + ((b^3*d - a*b^2*e + 2*a^2*c*e - a*
b*(3*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*Sqrt[b^2
- 4*a*c]) + ((b^2*d - a*b*e - a*(c*d - a*f))*Log[x])/a^3 - ((b^2*d - a*b*e
- a*(c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{d+ex+fx^2}{x^3\left(a+bx+cx^2\right)}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{d}{ax^3}+\frac{-bd+ae}{a^2x^2}+\frac{b^2d-abe-a(cd-af)}{a^3x}+\frac{-b^3d+ab^2e-a^2ce+ab(2cd-af)}{a^3\left(a+bx\right.}\right.\right.$$

$$= -\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}+\frac{\left(b^2d-abe-a(cd-af)\right)\log(x)}{a^3}+\frac{\text{Subst}\left(\int \frac{-b^3d+ab^2e-a^2ce+ab(2cd-af)-c\left(b^2d-\right.}{a+bx+cx^2}\right.}{2a^3}$$

$$= -\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}+\frac{\left(b^2d-abe-a(cd-af)\right)\log(x)}{a^3}-\frac{\left(b^2d-abe-a(cd-af)\right)\text{Subst}\left(\int \frac{b+2}{a+bx}\right.}{4a^3}$$

$$= -\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}+\frac{\left(b^2d-abe-a(cd-af)\right)\log(x)}{a^3}-\frac{\left(b^2d-abe-a(cd-af)\right)\log\left(a+bx^2+\right.}{4a^3}$$

$$= -\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}+\frac{\left(b^3d-ab^2e+2a^2ce-ab(3cd-af)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}}+\frac{\left(b^2d-abe-a(c\right.}{a^3}$$

Mathematica [A] time = 0.35299, size = 314, normalized size = 1.8

$$-\frac{\frac{a^2d}{x^4}+\frac{\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(ab\left(-e\sqrt{b^2-4ac}+af-3cd\right)+a\left(-cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+2ace\right)+b^2\left(d\sqrt{b^2-4ac}-ae\right)+b^3d\right)}{\sqrt{b^2-4ac}}+\frac{\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(-ab\left(e\sqrt{b^2\right.\right.}{}}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((a^2*d)/x^4 + (2*a*(-(b*d) + a*e))/x^2 - 4*(b^2*d - a*b*e + a*(-(c*d) + a
*f))*Log[x] + ((b^3*d + b^2*(Sqrt[b^2 - 4*a*c]*d - a*e) + a*b*(-3*c*d - Sqr
t[b^2 - 4*a*c]*e + a*f) + a*(-(c*Sqrt[b^2 - 4*a*c]*d) + 2*a*c*e + a*Sqrt[b^
2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((
-(b^3*d) + b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*b*(-3*c*d + Sqrt[b^2 - 4*a*c
]*e + a*f) + a*(-(c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)) + a*Sqrt[b^2 - 4*a*c]*f)
)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^3)
```

Maple [B] time = 0.011, size = 356, normalized size = 2.1

$$-\frac{d}{4\,ax^4}-\frac{e}{2\,ax^2}+\frac{bd}{2\,a^2x^2}+\frac{\ln(x)\,f}{a}-\frac{\ln(x)\,be}{a^2}-\frac{\ln(x)\,cd}{a^2}+\frac{\ln(x)\,b^2d}{a^3}-\frac{\ln\left(cx^4+bx^2+a\right)f}{4\,a}+\frac{\ln\left(cx^4+bx^2+a\right)be}{4\,a^2}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*d/a/x^4-1/2/a/x^2*e+1/2/a^2/x^2*b*d+1/a*ln(x)*f-1/a^2*ln(x)*b*e-1/a^2*
ln(x)*c*d+1/a^3*ln(x)*b^2*d-1/4/a*ln(c*x^4+b*x^2+a)*f+1/4/a^2*ln(c*x^4+b*x^
2+a)*b*e+1/4/a^2*c*ln(c*x^4+b*x^2+a)*d-1/4/a^3*ln(c*x^4+b*x^2+a)*b^2*d-1/2/
a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*f-1/a/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*e+1/2/a^2/(4*a*c-b^2)^(1/2
)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*e+3/2/a^2/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c*d-1/2/a^3/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.26681, size = 1283, normalized size = 7.37

$$\left[\frac{\left(a^2bf + \left(b^3 - 3\,abc\right)d - \left(ab^2 - 2\,a^2c\right)e\right)\sqrt{b^2 - 4\,ac}x^4 \log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right) - \left(\left(b^4 - 5\,ab^2c + 4\,a^2c^2\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*b*f + (b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*sqrt(b^2 - 4*a*c)
*x^4*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*
a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x^4*log(c*x^4 + b*x^2 + a) + 4*((b^4 -
5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x
^4*log(x) + 2*((a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*e)*x^2 - (a^2*b^
2 - 4*a^3*c)*d)/((a^3*b^2 - 4*a^4*c)*x^4), 1/4*(2*(a^2*b*f + (b^3 - 3*a*b*c
)*d - (a*b^2 - 2*a^2*c)*e)*sqrt(-b^2 + 4*a*c)*x^4*arctan(-(2*c*x^2 + b)*sqr
t(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3
- 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x^4*log(c*x^4 + b*x^2 + a) + 4*((b^
4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*
f)*x^4*log(x) + 2*((a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*e)*x^2 - (a^
2*b^2 - 4*a^3*c)*d)/((a^3*b^2 - 4*a^4*c)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13726, size = 286, normalized size = 1.64

$$-\frac{\left(b^2d - acd + a^2f - abe\right)\log\left(cx^4 + bx^2 + a\right)}{4\,a^3} + \frac{\left(b^2d - acd + a^2f - abe\right)\log\left(x^2\right)}{2\,a^3} - \frac{\left(b^3d - 3\,abcd + a^2bf - ab^2e + 2\,a^2ce\right)}{2\,\sqrt{-b^2 + 4\,ac}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(b^2*d - a*c*d + a^2*f - a*b*e)*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*(b^2*
d - a*c*d + a^2*f - a*b*e)*log(x^2)/a^3 - 1/2*(b^3*d - 3*a*b*c*d + a^2*b*f
- a*b^2*e + 2*a^2*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
+ 4*a*c)*a^3) - 1/4*(3*b^2*d*x^4 - 3*a*c*d*x^4 + 3*a^2*f*x^4 - 3*a*b*x^4*e
- 2*a*b*d*x^2 + 2*a^2*x^2*e + a^2*d)/(a^3*x^4)
```

3.54 $\int \frac{d+ex^2+fx^4}{x^7(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=244

$$-\frac{-abe-a(cd-af)+b^2d}{2a^3x^2}+\frac{\log\left(a+bx^2+cx^4\right)\left(a^2ce-ab^2e-ab(2cd-af)+b^3d\right)}{4a^4}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(3a^2bce+2a^2\right.}{2a}$$

```
[Out] -d/(6*a*x^6) + (b*d - a*e)/(4*a^2*x^4) - (b^2*d - a*b*e - a*(c*d - a*f))/(2
*a^3*x^2) - ((b^4*d - a*b^3*e + 3*a^2*b*c*e + 2*a^2*c*(c*d - a*f) - a*b^2*(
4*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*Sqrt[b^2 - 4
*a*c]) - ((b^3*d - a*b^2*e + a^2*c*e - a*b*(2*c*d - a*f))*Log[x])/a^4 + ((b
^3*d - a*b^2*e + a^2*c*e - a*b*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^
4)
```

Rubi [A] time = 0.572902, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1663, 1628, 634, 618, 206, 628}

$$-\frac{-abe-a(cd-af)+b^2d}{2a^3x^2}+\frac{\log\left(a+bx^2+cx^4\right)\left(a^2ce-ab^2e-ab(2cd-af)+b^3d\right)}{4a^4}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(3a^2bce+2a^2\right.}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -d/(6*a*x^6) + (b*d - a*e)/(4*a^2*x^4) - (b^2*d - a*b*e - a*(c*d - a*f))/(2
*a^3*x^2) - ((b^4*d - a*b^3*e + 3*a^2*b*c*e + 2*a^2*c*(c*d - a*f) - a*b^2*(
4*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*Sqrt[b^2 - 4
*a*c]) - ((b^3*d - a*b^2*e + a^2*c*e - a*b*(2*c*d - a*f))*Log[x])/a^4 + ((b
^3*d - a*b^2*e + a^2*c*e - a*b*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^
4)
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^7\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{d+ex+fx^2}{x^4\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{d}{ax^4}+\frac{-bd+ae}{a^2x^3}+\frac{b^2d-abe-a(cd-af)}{a^3x^2}+\frac{-b^3d+ab^2e-a^2ce+ab(2cd-af)}{a^4x}\right.\right.$$

$$= -\frac{d}{6ax^6}+\frac{bd-ae}{4a^2x^4}-\frac{b^2d-abe-a(cd-af)}{2a^3x^2}-\frac{\left(b^3d-ab^2e+a^2ce-ab(2cd-af)\right)\log(x)}{a^4}+\frac{\mathrm{Su}}{}$$

$$= -\frac{d}{6ax^6}+\frac{bd-ae}{4a^2x^4}-\frac{b^2d-abe-a(cd-af)}{2a^3x^2}-\frac{\left(b^3d-ab^2e+a^2ce-ab(2cd-af)\right)\log(x)}{a^4}+\frac{\left(b^3\right.}{}$$

$$= -\frac{d}{6ax^6}+\frac{bd-ae}{4a^2x^4}-\frac{b^2d-abe-a(cd-af)}{2a^3x^2}-\frac{\left(b^3d-ab^2e+a^2ce-ab(2cd-af)\right)\log(x)}{a^4}+\frac{\left(b^3\right.}{}$$

$$= -\frac{d}{6ax^6}+\frac{bd-ae}{4a^2x^4}-\frac{b^2d-abe-a(cd-af)}{2a^3x^2}-\frac{\left(b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd\right.}{2a^4\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.351751, size = 416, normalized size = 1.7

$$\frac{\frac{3\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(a^2c\left(e\sqrt{b^2-4ac}-2af+2cd\right)+ab^2\left(-e\sqrt{b^2-4ac}+af-4cd\right)+ab\left(-2cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+3ace\right)+b^3\left(d\sqrt{b^2-4ac}-ae\right)+b^4d\right)}{\sqrt{b^2-4ac}}+\frac{3\log\left(\sqrt{b^2-\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a^3*d)/x^6 + (3*a^2*(b*d - a*e))/x^4 + (6*a*(-(b^2*d) + a*b*e + a*(c*d
 - a*f)))/x^2 - 12*(b^3*d - a*b^2*e + a^2*c*e + a*b*(-2*c*d + a*f))*Log[x]
+ (3*(b^4*d + b^3*(Sqrt[b^2 - 4*a*c]*d - a*e) + a^2*c*(2*c*d + Sqrt[b^2 - 4
*a*c]*e - 2*a*f) + a*b^2*(-4*c*d - Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(-2*c*S
qrt[b^2 - 4*a*c]*d + 3*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (3*(-(b^4*d) + b^3*(Sqrt[b^2 - 4*a*c]
*d + a*e) - a*b^2*(-4*c*d + Sqrt[b^2 - 4*a*c]*e + a*f) + a^2*c*(-2*c*d + Sq
rt[b^2 - 4*a*c]*e + 2*a*f) + a*b*(-2*c*Sqrt[b^2 - 4*a*c]*d - 3*a*c*e + a*Sq
rt[b^2 - 4*a*c]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c]
)/(12*a^4)
```

Maple [B] time = 0.014, size = 523, normalized size = 2.1

$$-\frac{d}{6\,a x^6}-\frac{e}{4\,a x^4}+\frac{bd}{4\,a^2 x^4}-\frac{f}{2\,a x^2}+\frac{be}{2\,a^2 x^2}+\frac{cd}{2\,a^2 x^2}-\frac{b^2 d}{2\,a^3 x^2}-\frac{\ln(x)\,bf}{a^2}-\frac{\ln(x)\,ce}{a^2}+\frac{\ln(x)\,b^2 e}{a^3}+2\,\frac{\ln(x)\,bcd}{a^3}-\frac{\ln(x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/6*d/a/x^6-1/4/a/x^4*e+1/4/a^2/x^4*b*d-1/2/a/x^2*f+1/2/a^2/x^2*b*e+1/2/a^
2/x^2*c*d-1/2/a^3/x^2*b^2*d-1/a^2*ln(x)*b*f-1/a^2*ln(x)*c*e+1/a^3*ln(x)*b^2
*e+2/a^3*ln(x)*b*c*d-1/a^4*ln(x)*b^3*d+1/4/a^2*ln(c*x^4+b*x^2+a)*b*f+1/4/a^
2*c*ln(c*x^4+b*x^2+a)*e-1/4/a^3*ln(c*x^4+b*x^2+a)*b^2*e-1/2/a^3*c*ln(c*x^4+
b*x^2+a)*b*d+1/4/a^4*ln(c*x^4+b*x^2+a)*b^3*d-1/a/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*f+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))*b^2*f+3/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(
4*a*c-b^2)^(1/2))*b*c*e+1/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b
^2)^(1/2))*c^2*d-1/2/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(
1/2))*b^3*e-2/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b
^2*c*d+1/2/a^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*
d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 13.7013, size = 1747, normalized size = 7.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*sqrt(b^2 - 4*a*c)*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a
^2*b*c)*e + (a^2*b^2 - 2*a^3*c)*f)*x^6*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 3*((b^5 - 6*
a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^2*b^3 -
 4*a^3*b*c)*f)*x^6*log(c*x^4 + b*x^2 + a) + 12*((b^5 - 6*a*b^3*c + 8*a^2*b*
c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^6
*log(x) + 6*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e
+ (a^3*b^2 - 4*a^4*c)*f)*x^4 - 3*((a^2*b^3 - 4*a^3*b*c)*d - (a^3*b^2 - 4*a^
4*c)*e)*x^2 + 2*(a^3*b^2 - 4*a^4*c)*d)/((a^4*b^2 - 4*a^5*c)*x^6), -1/12*(6*
sqrt(-b^2 + 4*a*c)*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e
 + (a^2*b^2 - 2*a^3*c)*f)*x^6*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
 - 4*a*c)) - 3*((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c +
```

```
4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^6*log(c*x^4 + b*x^2 + a) + 12*((b
^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^
2*b^3 - 4*a^3*b*c)*f)*x^6*log(x) + 6*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d -
 (a^2*b^3 - 4*a^3*b*c)*e + (a^3*b^2 - 4*a^4*c)*f)*x^4 - 3*((a^2*b^3 - 4*a^3
*b*c)*d - (a^3*b^2 - 4*a^4*c)*e)*x^2 + 2*(a^3*b^2 - 4*a^4*c)*d)/((a^4*b^2 -
 4*a^5*c)*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**7/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14577, size = 423, normalized size = 1.73

$$\frac{\left(b^3d - 2\,abcd + a^2bf - ab^2e + a^2ce\right)\log\left(cx^4 + bx^2 + a\right)}{4\,a^4} - \frac{\left(b^3d - 2\,abcd + a^2bf - ab^2e + a^2ce\right)\log\left(x^2\right)}{2\,a^4} + \frac{\left(b^4d - 4\,ab^2cd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(b^3*d - 2*a*b*c*d + a^2*b*f - a*b^2*e + a^2*c*e)*log(c*x^4 + b*x^2 + a
)/a^4 - 1/2*(b^3*d - 2*a*b*c*d + a^2*b*f - a*b^2*e + a^2*c*e)*log(x^2)/a^4
+ 1/2*(b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d + a^2*b^2*f - 2*a^3*c*f - a*b^3*e
+ 3*a^2*b*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)
*a^4) + 1/12*(11*b^3*d*x^6 - 22*a*b*c*d*x^6 + 11*a^2*b*f*x^6 - 11*a*b^2*x^6
*e + 11*a^2*c*x^6*e - 6*a*b^2*d*x^4 + 6*a^2*c*d*x^4 - 6*a^3*f*x^4 + 6*a^2*b
*x^4*e + 3*a^2*b*d*x^2 - 3*a^3*x^2*e - 2*a^3*d)/(a^4*x^6)
```

3.55 $\int \frac{x^4\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=369

$$\frac{x\left(-c(af+be)+b^2f+c^2d\right)}{c^3}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^3ce+b^4(-f)}{\sqrt{b^2-4ac}}-bc(cd-2af)-ac^2e+b\ldots\right.}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x)/c^3 + ((c*e - b*f)*x^3)/(3*c^2) + (f*x^
5)/(5*c) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f) - (b^3*c*e - 3*a
*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(c*d - a*f))/Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(
7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d
- 2*a*f) + (b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(
c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 4.57737, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1664, 1166, 205}

$$\frac{x\left(-c(af+be)+b^2f+c^2d\right)}{c^3}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^3ce+b^4(-f)}{\sqrt{b^2-4ac}}-bc(cd-2af)-ac^2e+b\ldots\right.}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```


```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x)/c^3 + ((c*e - b*f)*x^3)/(3*c^2) + (f*x^
5)/(5*c) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f) - (b^3*c*e - 3*a
*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(c*d - a*f))/Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(
7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d
- 2*a*f) + (b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(
c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```


Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx = \int \left(\frac{c^2d+b^2f-c(be+af)}{c^3}+\frac{(ce-bf)x^2}{c^2}+\frac{fx^4}{c}-\frac{a\left(c^2d+b^2f-c(be+af)\right)-\left(b^2ce-ac^2e\right.}{c^3\left(a+bx^2+cx^4\right)}\right.$$

$$=\frac{\left(c^2d+b^2f-c(be+af)\right)x}{c^3}+\frac{(ce-bf)x^3}{3c^2}+\frac{fx^5}{5c}-\frac{\int \frac{a\left(c^2d+b^2f-c(be+af)\right)+\left(-b^2ce+ac^2e+b^3f+bc(cd-2af\right.}{a+bx^2+cx^4}}{c^3}$$

$$=\frac{\left(c^2d+b^2f-c(be+af)\right)x}{c^3}+\frac{(ce-bf)x^3}{3c^2}+\frac{fx^5}{5c}+\frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)-\frac{b^3ce-3a}{}\right.}{}$$

$$=\frac{\left(c^2d+b^2f-c(be+af)\right)x}{c^3}+\frac{(ce-bf)x^3}{3c^2}+\frac{fx^5}{5c}+\frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)-\frac{b^3ce-3a}{}\right.}{\sqrt{2}c^{7/2}\sqrt{}}$$

Mathematica [A] time = 0.54579, size = 456, normalized size = 1.24

$$\frac{x\left(-c(af+be)+b^2f+c^2d\right)}{c^3}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(ac^2\left(e\sqrt{b^2-4ac}-2af+2cd\right)-b^2c\left(e\sqrt{b^2-4ac}-4af+cd\right)+bc\left(cd\right.\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x)/c^3 + ((c*e - b*f)*x^3)/(3*c^2) + (f*x^
5)/(5*c) - ((-(b^4*f) - b^2*c*(c*d + Sqrt[b^2 - 4*a*c]*e - 4*a*f) + a*c^2*(
2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f) + b^3*(c*e + Sqrt[b^2 - 4*a*c]*f) + b*
c*(c*Sqrt[b^2 - 4*a*c]*d - 3*a*c*e - 2*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*
a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^4*f + b^2*c*(c*d - Sqrt[b^2 - 4*a*c
]*e - 4*a*f) + a*c^2*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b^3*(-(c*e) +
 Sqrt[b^2 - 4*a*c]*f) + b*c*(c*Sqrt[b^2 - 4*a*c]*d + 3*a*c*e - 2*a*Sqrt[b^2
 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [B] time = 0.036, size = 1450, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d+1/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
```

```
*a*c+b^2)^(1/2))*c)^(1/2))*a*d+1/2/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/
2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*e+1/2/c^3*2^(1/2
)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)
-b)*c)^(1/2))*b^3*f-1/2/c^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctan
h(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*e+1/2/c*2^(1/2)/(((-4*a
*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1
/2))*b*d+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arct
anh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*d-1/2/c*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*a*e-1/2/c^3*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*f-1/3/c^2*x^3*b*f-1/c^2*a*f*x+1/c^3*
b^2*f*x-1/c^2*b*e*x+1/c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c
*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*f-1/c/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*a^2*f-1/2/c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*f
+1/2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*e-1/2/c/(-4*a*c+b^2)^(1/
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*b^2*d-1/c^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*
arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*b*f-1/c/(-4*a*c+b^2
)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2))*a^2*f-1/2/c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2))*b^4*f+1/2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*e-1/2/c/(-4*
a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)
/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*d+1/3/c*x^3*e+1/c*d*x+1/5*f*x^5/c+2/
c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*b^2*f-3/2/c/(-4*a*c+b^2)^(1/2)
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2))*a*b*e+2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^
2*f-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\,c^2fx^5+5\left(c^2e-bcf\right)x^3+15\left(c^2d-bce+\left(b^2-ac\right)f\right)x}{15\,c^3}+\frac{-\int\frac{ac^2d-abce+\left(bc^2d-\left(b^2c-ac^2\right)e+\left(b^3-2\,abc\right)f\right)x^2+\left(ab^2-a^2c\right)f}{cx^4+bx^2+a}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^2*f*x^5 + 5*(c^2*e - b*c*f)*x^3 + 15*(c^2*d - b*c*e + (b^2 - a*c)
*f)*x)/c^3 + integrate(-(a*c^2*d - a*b*c*e + (b*c^2*d - (b^2*c - a*c^2)*e +
 (b^3 - 2*a*b*c)*f)*x^2 + (a*b^2 - a^2*c)*f)/(c*x^4 + b*x^2 + a), x)/c^3
```

Fricas [B] time = 100.239, size = 31190, normalized size = 84.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/30*(6*c^2*f*x^5 - 15*sqrt(1/2)*c^3*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(
b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c
^4)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^
2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2
*a^3*c^4)*e)*f + (b^2*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^1
0)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*
a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 +
7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^
6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 6
2*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c
^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^
7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3
*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6
 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^
5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66
*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^
6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*
c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*
a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16
*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15))
)/(b^2*c^7 - 4*a*c^8))*log(-2*((a*b^2*c^6 - a^2*c^7)*d^4 - (3*a*b^3*c^5 - 5
*a^2*b*c^6)*d^3*e + 3*(a*b^4*c^4 - 2*a^2*b^2*c^5)*d^2*e^2 - (a*b^5*c^3 - a^
2*b^3*c^4 - 3*a^3*b*c^5)*d*e^3 + (a^2*b^4*c^3 - 3*a^3*b^2*c^4 + a^4*c^5)*e^
4 + (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*f^4 + ((a*b^8 - 7*a^2
*b^6*c + 18*a^3*b^4*c^2 - 19*a^4*b^2*c^3 + 4*a^5*c^4)*d - (a^2*b^7 - 3*a^3*
b^5*c - 2*a^4*b^3*c^2 + 5*a^5*b*c^3)*e)*f^3 + 3*((a*b^6*c^2 - 5*a^2*b^4*c^3
 + 7*a^3*b^2*c^4 - 2*a^4*c^5)*d^2 - (a*b^7*c - 5*a^2*b^5*c^2 + 8*a^3*b^3*c^
3 - 5*a^4*b*c^4)*d*e + (a^2*b^6*c - 4*a^3*b^4*c^2 + 3*a^4*b^2*c^3)*e^2)*f^2
 + ((3*a*b^4*c^4 - 9*a^2*b^2*c^5 + 4*a^3*c^6)*d^3 - 3*(2*a*b^5*c^3 - 7*a^2*
b^3*c^4 + 5*a^3*b*c^5)*d^2*e + 3*(a*b^6*c^2 - 3*a^2*b^4*c^3 + a^3*b^2*c^4)*
d*e^2 - (3*a^2*b^5*c^2 - 11*a^3*b^3*c^3 + 7*a^4*b*c^4)*e^3)*f)*x + sqrt(1/2
)*((b^4*c^6 - 5*a*b^2*c^7 + 4*a^2*c^8)*d^3 - (3*b^5*c^5 - 17*a*b^3*c^6 + 20
*a^2*b*c^7)*d^2*e + (3*b^6*c^4 - 19*a*b^4*c^5 + 29*a^2*b^2*c^6 - 4*a^3*c^7)
*d*e^2 - (b^7*c^3 - 7*a*b^5*c^4 + 13*a^2*b^3*c^5 - 4*a^3*b*c^6)*e^3 + (b^10
 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^
5)*f^3 + ((3*b^8*c^2 - 25*a*b^6*c^3 + 66*a^2*b^4*c^4 - 59*a^3*b^2*c^5 + 12*
a^4*c^6)*d - (3*b^9*c - 27*a*b^7*c^2 + 80*a^2*b^5*c^3 - 87*a^3*b^3*c^4 + 28
*a^4*b*c^5)*e)*f^2 + ((3*b^6*c^4 - 20*a*b^4*c^5 + 35*a^2*b^2*c^6 - 12*a^3*c
^7)*d^2 - 2*(3*b^7*c^3 - 22*a*b^5*c^4 + 46*a^2*b^3*c^5 - 24*a^3*b*c^6)*d*e
+ (3*b^8*c^2 - 24*a*b^6*c^3 + 58*a^2*b^4*c^4 - 41*a^3*b^2*c^5 + 4*a^4*c^6)*
e^2)*f - ((b^3*c^9 - 4*a*b*c^10)*d - (b^4*c^8 - 6*a*b^2*c^9 + 8*a^2*c^10)*e
 + (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*f)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 +
 a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c
^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^
5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^
2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8
*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*
((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6
- a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22
*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*
b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 4
8*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*
c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2
+ 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 1
5*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c
^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*
c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*
```

```
a*c^15)))*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a
^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*b^5*c
+ 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c
^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f + (b^2*c^7 -
 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b
^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8
- a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8
)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8
)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*
c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b
^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^
2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3
+ 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^
8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*
a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c
^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2
*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^
3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^
7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c
^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))) + 1
5*sqrt(1/2)*c^3*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4
 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*
b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3 + 5*a
^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f + (b^2
*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 -
 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^
2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3
*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a
^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^
4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22
*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*
b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e
)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*
a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^
6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3
*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 +
 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7
- 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*
b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3
*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)
)*log(-2*((a*b^2*c^6 - a^2*c^7)*d^4 - (3*a*b^3*c^5 - 5*a^2*b*c^6)*d^3*e + 3
*(a*b^4*c^4 - 2*a^2*b^2*c^5)*d^2*e^2 - (a*b^5*c^3 - a^2*b^3*c^4 - 3*a^3*b*c
^5)*d*e^3 + (a^2*b^4*c^3 - 3*a^3*b^2*c^4 + a^4*c^5)*e^4 + (a^3*b^6 - 5*a^4*
b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*f^4 + ((a*b^8 - 7*a^2*b^6*c + 18*a^3*b^4*c
^2 - 19*a^4*b^2*c^3 + 4*a^5*c^4)*d - (a^2*b^7 - 3*a^3*b^5*c - 2*a^4*b^3*c^2
 + 5*a^5*b*c^3)*e)*f^3 + 3*((a*b^6*c^2 - 5*a^2*b^4*c^3 + 7*a^3*b^2*c^4 - 2*
a^4*c^5)*d^2 - (a*b^7*c - 5*a^2*b^5*c^2 + 8*a^3*b^3*c^3 - 5*a^4*b*c^4)*d*e
+ (a^2*b^6*c - 4*a^3*b^4*c^2 + 3*a^4*b^2*c^3)*e^2)*f^2 + ((3*a*b^4*c^4 - 9*
a^2*b^2*c^5 + 4*a^3*c^6)*d^3 - 3*(2*a*b^5*c^3 - 7*a^2*b^3*c^4 + 5*a^3*b*c^5
)*d^2*e + 3*(a*b^6*c^2 - 3*a^2*b^4*c^3 + a^3*b^2*c^4)*d*e^2 - (3*a^2*b^5*c^
2 - 11*a^3*b^3*c^3 + 7*a^4*b*c^4)*e^3)*f)*x - sqrt(1/2)*((b^4*c^6 - 5*a*b^2
*c^7 + 4*a^2*c^8)*d^3 - (3*b^5*c^5 - 17*a*b^3*c^6 + 20*a^2*b*c^7)*d^2*e + (
3*b^6*c^4 - 19*a*b^4*c^5 + 29*a^2*b^2*c^6 - 4*a^3*c^7)*d*e^2 - (b^7*c^3 - 7
*a*b^5*c^4 + 13*a^2*b^3*c^5 - 4*a^3*b*c^6)*e^3 + (b^10 - 10*a*b^8*c + 35*a^
2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*f^3 + ((3*b^8*c^2
- 25*a*b^6*c^3 + 66*a^2*b^4*c^4 - 59*a^3*b^2*c^5 + 12*a^4*c^6)*d - (3*b^9*c
 - 27*a*b^7*c^2 + 80*a^2*b^5*c^3 - 87*a^3*b^3*c^4 + 28*a^4*b*c^5)*e)*f^2 +
((3*b^6*c^4 - 20*a*b^4*c^5 + 35*a^2*b^2*c^6 - 12*a^3*c^7)*d^2 - 2*(3*b^7*c^
3 - 22*a*b^5*c^4 + 46*a^2*b^3*c^5 - 24*a^3*b*c^6)*d*e + (3*b^8*c^2 - 24*a*b
```

^6*c^3 + 58*a^2*b^4*c^4 - 41*a^3*b^2*c^5 + 4*a^4*c^6)*e^2)*f - ((b^3*c^9 - 4*a*b*c^10)*d - (b^4*c^8 - 6*a*b^2*c^9 + 8*a^2*c^10)*e + (b^5*c^7 - 7*a*b^3 *c^8 + 12*a^2*b*c^9)*f)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b ^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 1 2*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2 *c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8* c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11* c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5* b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2* c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^ 3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b ^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2* b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))*sqrt(-((b^3 *c^4 - 3*a*b*c^5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^ 2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7* a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d - (b^6*c - 6*a* b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f + (b^2*c^7 - 4*a*c^8)*sqrt(((b^4* c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9) *d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a* b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c ^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18* a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b ^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d ^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b ^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))) - 15*sqrt(1/2)*c^3*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^ 2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f - (b^2*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2* b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2* e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8* c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^ 2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3 *b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7* c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b ^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2 *c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3* c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d* e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^ 7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(-2*((a*b^2*c^6 - a^2*c^7)*d^4 - (3*a*b^3*c^5 - 5*a^2*b*c^6)*d^3*e + 3*(a*b^4*c^4 - 2*a^2*b

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^2*c^5)*d^2*e^2 - (a*b^5*c^3 - a^2*b^3*c^4 - 3*a^3*b*c^5)*d*e^3 + (a^2*b^4*
c^3 - 3*a^3*b^2*c^4 + a^4*c^5)*e^4 + (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2
 - a^6*c^3)*f^4 + ((a*b^8 - 7*a^2*b^6*c + 18*a^3*b^4*c^2 - 19*a^4*b^2*c^3 +
 4*a^5*c^4)*d - (a^2*b^7 - 3*a^3*b^5*c - 2*a^4*b^3*c^2 + 5*a^5*b*c^3)*e)*f^
3 + 3*((a*b^6*c^2 - 5*a^2*b^4*c^3 + 7*a^3*b^2*c^4 - 2*a^4*c^5)*d^2 - (a*b^7
*c - 5*a^2*b^5*c^2 + 8*a^3*b^3*c^3 - 5*a^4*b*c^4)*d*e + (a^2*b^6*c - 4*a^3*
b^4*c^2 + 3*a^4*b^2*c^3)*e^2)*f^2 + ((3*a*b^4*c^4 - 9*a^2*b^2*c^5 + 4*a^3*c
^6)*d^3 - 3*(2*a*b^5*c^3 - 7*a^2*b^3*c^4 + 5*a^3*b*c^5)*d^2*e + 3*(a*b^6*c^
2 - 3*a^2*b^4*c^3 + a^3*b^2*c^4)*d*e^2 - (3*a^2*b^5*c^2 - 11*a^3*b^3*c^3 +
7*a^4*b*c^4)*e^3)*f)*x + sqrt(1/2)*((b^4*c^6 - 5*a*b^2*c^7 + 4*a^2*c^8)*d^3
 - (3*b^5*c^5 - 17*a*b^3*c^6 + 20*a^2*b*c^7)*d^2*e + (3*b^6*c^4 - 19*a*b^4*
c^5 + 29*a^2*b^2*c^6 - 4*a^3*c^7)*d*e^2 - (b^7*c^3 - 7*a*b^5*c^4 + 13*a^2*b
^3*c^5 - 4*a^3*b*c^6)*e^3 + (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^
4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*f^3 + ((3*b^8*c^2 - 25*a*b^6*c^3 + 66*a
^2*b^4*c^4 - 59*a^3*b^2*c^5 + 12*a^4*c^6)*d - (3*b^9*c - 27*a*b^7*c^2 + 80*
a^2*b^5*c^3 - 87*a^3*b^3*c^4 + 28*a^4*b*c^5)*e)*f^2 + ((3*b^6*c^4 - 20*a*b^
4*c^5 + 35*a^2*b^2*c^6 - 12*a^3*c^7)*d^2 - 2*(3*b^7*c^3 - 22*a*b^5*c^4 + 46
*a^2*b^3*c^5 - 24*a^3*b*c^6)*d*e + (3*b^8*c^2 - 24*a*b^6*c^3 + 58*a^2*b^4*c
^4 - 41*a^3*b^2*c^5 + 4*a^4*c^6)*e^2)*f + ((b^3*c^9 - 4*a*b*c^10)*d - (b^4*
c^8 - 6*a*b^2*c^9 + 8*a^2*c^10)*e + (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*
f)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8
+ 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c
^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3
 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 +
 (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 1
2*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4
- 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*
a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3
*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2
- 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c
^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27
*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^
8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8
)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4
*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*
a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2
 - 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a
^2*b*c^4)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((
b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c
^3 - 2*a^3*c^4)*e)*f - (b^2*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a
^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6
 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*
c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*
b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c
^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((
b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 -
a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a
^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^
4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*
a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^
3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 +
4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*
a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5
 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^
4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*
c^15)))/(b^2*c^7 - 4*a*c^8))) + 15*sqrt(1/2)*c^3*sqrt(-((b^3*c^4 - 3*a*b*c^
5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3
 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2
+ 2*((b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2
```

```
*b^2*c^3 - 2*a^3*c^4)*e)*f - (b^2*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c
^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b
^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*
a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 1
1*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2
*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4
+ 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*
c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4
+ 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*
a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4
 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*
b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*
f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5
 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b
^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*
b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14
- 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(-2*((a*b^2*c^6 - a^2*c^7)*d^4 - (3*a
*b^3*c^5 - 5*a^2*b*c^6)*d^3*e + 3*(a*b^4*c^4 - 2*a^2*b^2*c^5)*d^2*e^2 - (a*
b^5*c^3 - a^2*b^3*c^4 - 3*a^3*b*c^5)*d*e^3 + (a^2*b^4*c^3 - 3*a^3*b^2*c^4 +
 a^4*c^5)*e^4 + (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*f^4 + ((a
*b^8 - 7*a^2*b^6*c + 18*a^3*b^4*c^2 - 19*a^4*b^2*c^3 + 4*a^5*c^4)*d - (a^2*
b^7 - 3*a^3*b^5*c - 2*a^4*b^3*c^2 + 5*a^5*b*c^3)*e)*f^3 + 3*((a*b^6*c^2 - 5
*a^2*b^4*c^3 + 7*a^3*b^2*c^4 - 2*a^4*c^5)*d^2 - (a*b^7*c - 5*a^2*b^5*c^2 +
8*a^3*b^3*c^3 - 5*a^4*b*c^4)*d*e + (a^2*b^6*c - 4*a^3*b^4*c^2 + 3*a^4*b^2*c
^3)*e^2)*f^2 + ((3*a*b^4*c^4 - 9*a^2*b^2*c^5 + 4*a^3*c^6)*d^3 - 3*(2*a*b^5*
c^3 - 7*a^2*b^3*c^4 + 5*a^3*b*c^5)*d^2*e + 3*(a*b^6*c^2 - 3*a^2*b^4*c^3 + a
^3*b^2*c^4)*d*e^2 - (3*a^2*b^5*c^2 - 11*a^3*b^3*c^3 + 7*a^4*b*c^4)*e^3)*f)*
x - sqrt(1/2)*((b^4*c^6 - 5*a*b^2*c^7 + 4*a^2*c^8)*d^3 - (3*b^5*c^5 - 17*a*
b^3*c^6 + 20*a^2*b*c^7)*d^2*e + (3*b^6*c^4 - 19*a*b^4*c^5 + 29*a^2*b^2*c^6
- 4*a^3*c^7)*d*e^2 - (b^7*c^3 - 7*a*b^5*c^4 + 13*a^2*b^3*c^5 - 4*a^3*b*c^6)
*e^3 + (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^
4 - 4*a^5*c^5)*f^3 + ((3*b^8*c^2 - 25*a*b^6*c^3 + 66*a^2*b^4*c^4 - 59*a^3*b
^2*c^5 + 12*a^4*c^6)*d - (3*b^9*c - 27*a*b^7*c^2 + 80*a^2*b^5*c^3 - 87*a^3*
b^3*c^4 + 28*a^4*b*c^5)*e)*f^2 + ((3*b^6*c^4 - 20*a*b^4*c^5 + 35*a^2*b^2*c^
6 - 12*a^3*c^7)*d^2 - 2*(3*b^7*c^3 - 22*a*b^5*c^4 + 46*a^2*b^3*c^5 - 24*a^3
*b*c^6)*d*e + (3*b^8*c^2 - 24*a*b^6*c^3 + 58*a^2*b^4*c^4 - 41*a^3*b^2*c^5 +
 4*a^4*c^6)*e^2)*f + ((b^3*c^9 - 4*a*b*c^10)*d - (b^4*c^8 - 6*a*b^2*c^9 + 8
*a^2*c^10)*e + (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*f)*sqrt(((b^4*c^8 - 2
*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e
+ 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*
c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6
*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c
+ 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c
^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*
a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*
b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c
^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a
*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2
 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^
7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3
*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4
- 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^
3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b
^2*c^14 - 4*a*c^15)))*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(b^4*c^3 - 4*a*b
^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (b^7
- 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3
 + 5*a^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f
- (b^2*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5
*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*
```

```
a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 -
 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c
^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 +
 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^
3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c
- 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*
c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^
7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*
b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 -
72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4
*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^
3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 1
8*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 -
13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*
a*c^8))) + 10*(c^2*e - b*c*f)*x^3 + 30*(c^2*d - b*c*e + (b^2 - a*c)*f)*x)/c
^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.56 $\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=282

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-bc(cd-3af)-2ac^2e+b^2ce+b^3(-f)}{\sqrt{b^2-4ac}}-acf+b^2f-bce+c^2d\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-bc(cd-3af)-2ac^2e+b^2ce+b^3(-}{\sqrt{b^2-4ac}}\right.}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}-}}$$

```
[Out] ((c*e - b*f)*x)/c^2 + (f*x^3)/(3*c) + ((c^2*d - b*c*e + b^2*f - a*c*f + (b^
2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) + ((c^2*d - b*c*e + b^2*f - a*c*f - (b^2*c*e - 2*a*c^2*e
 - b^3*f - b*c*(c*d - 3*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
)
```

Rubi [A] time = 3.58969, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1664, 1166, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-bc(cd-3af)-2ac^2e+b^2ce+b^3(-f)}{\sqrt{b^2-4ac}}-acf+b^2f-bce+c^2d\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-bc(cd-3af)-2ac^2e+b^2ce+b^3(-}{\sqrt{b^2-4ac}}\right.}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}-}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*e - b*f)*x)/c^2 + (f*x^3)/(3*c) + ((c^2*d - b*c*e + b^2*f - a*c*f + (b^
2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) + ((c^2*d - b*c*e + b^2*f - a*c*f - (b^2*c*e - 2*a*c^2*e
 - b^3*f - b*c*(c*d - 3*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
)
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx &= \int \left(\frac{ce-bf}{c^2}+\frac{fx^2}{c}-\frac{a(ce-bf)-\left(c^2d-bce+b^2f-acf\right)x^2}{c^2\left(a+bx^2+cx^4\right)}\right)dx\\
&= \frac{(ce-bf)x}{c^2}+\frac{fx^3}{3c}-\frac{\int \frac{a(ce-bf)+\left(-c^2d+bce-b^2f+acf\right)x^2}{a+bx^2+cx^4}\,dx}{c^2}\\
&= \frac{(ce-bf)x}{c^2}+\frac{fx^3}{3c}+\frac{\left(c^2d-bce+b^2f-acf-\frac{b^2ce-2ac^2e-b^3f-bc(cd-3af)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2}\\
&= \frac{(ce-bf)x}{c^2}+\frac{fx^3}{3c}+\frac{\left(c^2d-bce+b^2f-acf+\frac{b^2ce-2ac^2e-b^3f-bc(cd-3af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] time = 0.537855, size = 365, normalized size = 1.29

$$
\frac{\frac{3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-bc\left(e\sqrt{b^2-4ac}-3af+cd\right)+c\left(cd\sqrt{b^2-4ac}-af\sqrt{b^2-4ac}-2ace\right)+b^2\left(f\sqrt{b^2-4ac}+ce\right)+b^3(-f)\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(bc\left(-e\sqrt{b^2-4\right.\right.}{}}{6c^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (6*Sqrt[c]*(c*e - b*f)*x + 2*c^(3/2)*f*x^3 + (3*Sqrt[2]*(-(b^3*f) - b*c*(c*
d + Sqrt[b^2 - 4*a*c]*e - 3*a*f) + b^2*(c*e + Sqrt[b^2 - 4*a*c]*f) + c*(c*S
qrt[b^2 - 4*a*c]*d - 2*a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) + (3*Sqrt[2]*(b^3*f + b*c*(c*d - Sqrt[b^2 - 4*a*c]*e - 3*a*f) + b
^2*(-(c*e) + Sqrt[b^2 - 4*a*c]*f) + c*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e - a*
Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*c^(5/2))
```

Maple [B] time = 0.031, size = 1035, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*f*x^3/c-1/c^2*b*f*x+1/c*e*x+1/2/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1
/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*f-1/2/c^2*2^(1/
2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2
)-b)*c)^(1/2))*b^2*f+1/2/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh
(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*e-1/2*d*2^(1/2)/(((-4*a*c+
b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
)-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh
(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*b*f+1/(-4*a*c+b^2)^(1/2)*2
^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^
(1/2)-b)*c)^(1/2))*a*e+1/2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1
```

```
/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*f
-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(
c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*e+1/2/(-4*a*c+b^2)^(1/2)*
2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)
^(1/2)-b)*c)^(1/2))*b*d-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*f+1/2/c^2*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*b^2*f-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e+1/2*d*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-3/2/c/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*f+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*a*e+1/2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*f-1/2/c/(-4*a*c+
b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*b*d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 22.0875, size = 18515, normalized size = 65.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c*f*x^3 + 3*sqrt(1/2)*c^2*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d
*e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((
b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f + (b^2*c^5
- 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 -
4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 -
 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^
3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2
- 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3
 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c
^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^
3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4
*a*c^6))*log(2*(c^6*d^4 - 3*b*c^5*d^3*e + 3*b^2*c^4*d^2*e^2 - (b^3*c^3 + a*
b*c^4)*d*e^3 + (a*b^2*c^3 - a^2*c^4)*e^4 + (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2
)*f^4 + ((b^6 - 5*a*b^4*c + 9*a^2*b^2*c^2 - 4*a^3*c^3)*d - (a*b^5 - a^2*b^3
*c - 3*a^3*b*c^2)*e)*f^3 + 3*((b^4*c^2 - 3*a*b^2*c^3 + 2*a^2*c^4)*d^2 - (b^
5*c - 3*a*b^3*c^2 + 3*a^2*b*c^3)*d*e + (a*b^4*c - 2*a^2*b^2*c^2)*e^2)*f^2 +
```

```
((3*b^2*c^4 - 4*a*c^5)*d^3 - 3*(2*b^3*c^3 - 3*a*b*c^4)*d^2*e + 3*(b^4*c^2
- a*b^2*c^3)*d*e^2 - (3*a*b^3*c^2 - 5*a^2*b*c^3)*e^3)*f)*x + sqrt(1/2)*((b^
2*c^5 - 4*a*c^6)*d^2*e - 2*(b^3*c^4 - 4*a*b*c^5)*d*e^2 + (b^4*c^3 - 5*a*b^2
*c^4 + 4*a^2*c^5)*e^3 - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*f^
3 - (2*(b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d - (3*b^6*c - 19*a*b^4*c^2 +
29*a^2*b^2*c^3 - 4*a^3*c^4)*e)*f^2 - ((b^3*c^4 - 4*a*b*c^5)*d^2 - 2*(2*b^4*
c^3 - 9*a*b^2*c^4 + 4*a^2*c^5)*d*e + (3*b^5*c^2 - 17*a*b^3*c^3 + 20*a^2*b*c
^4)*e^2)*f + (2*(b^2*c^7 - 4*a*c^8)*d - (b^3*c^6 - 4*a*b*c^7)*e + (b^4*c^5
- 6*a*b^2*c^6 + 8*a^2*c^7)*f)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6
- a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a
^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*
f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a
*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^
5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6
*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*
c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c
^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c
^11)))*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)
*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (
b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f + (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 -
 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^
3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c
^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^
3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c
^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^
5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3
*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*
c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(1/2)*c^
2*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*e^2
+ (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c
 - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f + (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*
c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (
b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 -
a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2
*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 +
5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^
2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*
c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*
e^3)*f)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(c^6*d^4 - 3*b*c
^5*d^3*e + 3*b^2*c^4*d^2*e^2 - (b^3*c^3 + a*b*c^4)*d*e^3 + (a*b^2*c^3 - a^2
*c^4)*e^4 + (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*f^4 + ((b^6 - 5*a*b^4*c + 9*a
^2*b^2*c^2 - 4*a^3*c^3)*d - (a*b^5 - a^2*b^3*c - 3*a^3*b*c^2)*e)*f^3 + 3*((
b^4*c^2 - 3*a*b^2*c^3 + 2*a^2*c^4)*d^2 - (b^5*c - 3*a*b^3*c^2 + 3*a^2*b*c^3
)*d*e + (a*b^4*c - 2*a^2*b^2*c^2)*e^2)*f^2 + ((3*b^2*c^4 - 4*a*c^5)*d^3 - 3
*(2*b^3*c^3 - 3*a*b*c^4)*d^2*e + 3*(b^4*c^2 - a*b^2*c^3)*d*e^2 - (3*a*b^3*c
^2 - 5*a^2*b*c^3)*e^3)*f)*x - sqrt(1/2)*((b^2*c^5 - 4*a*c^6)*d^2*e - 2*(b^3
*c^4 - 4*a*b*c^5)*d*e^2 + (b^4*c^3 - 5*a*b^2*c^4 + 4*a^2*c^5)*e^3 - (b^7 -
7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*f^3 - (2*(b^5*c^2 - 5*a*b^3*c^3 +
 4*a^2*b*c^4)*d - (3*b^6*c - 19*a*b^4*c^2 + 29*a^2*b^2*c^3 - 4*a^3*c^4)*e)*
f^2 - ((b^3*c^4 - 4*a*b*c^5)*d^2 - 2*(2*b^4*c^3 - 9*a*b^2*c^4 + 4*a^2*c^5)*
d*e + (3*b^5*c^2 - 17*a*b^3*c^3 + 20*a^2*b*c^4)*e^2)*f + (2*(b^2*c^7 - 4*a*
c^8)*d - (b^3*c^6 - 4*a*b*c^7)*e + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*f)*s
qrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 -
 a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3
 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^
3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c
```

```
^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^
2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c
^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*
c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b*c^4*d^2 - 2*(b^
2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2
*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3
)*e)*f + (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 -
 a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^
2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f
^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*
b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5
 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*
c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c
^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^
6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^
11)))/(b^2*c^5 - 4*a*c^6))) + 3*sqrt(1/2)*c^2*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3
 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^
2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*
f - (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^
7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6
)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 +
4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c
^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*
a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 -
 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d
^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*
e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))
/(b^2*c^5 - 4*a*c^6))*log(2*(c^6*d^4 - 3*b*c^5*d^3*e + 3*b^2*c^4*d^2*e^2 -
(b^3*c^3 + a*b*c^4)*d*e^3 + (a*b^2*c^3 - a^2*c^4)*e^4 + (a^2*b^4 - 3*a^3*b^
2*c + a^4*c^2)*f^4 + ((b^6 - 5*a*b^4*c + 9*a^2*b^2*c^2 - 4*a^3*c^3)*d - (a*
b^5 - a^2*b^3*c - 3*a^3*b*c^2)*e)*f^3 + 3*((b^4*c^2 - 3*a*b^2*c^3 + 2*a^2*c
^4)*d^2 - (b^5*c - 3*a*b^3*c^2 + 3*a^2*b*c^3)*d*e + (a*b^4*c - 2*a^2*b^2*c^
2)*e^2)*f^2 + ((3*b^2*c^4 - 4*a*c^5)*d^3 - 3*(2*b^3*c^3 - 3*a*b*c^4)*d^2*e
+ 3*(b^4*c^2 - a*b^2*c^3)*d*e^2 - (3*a*b^3*c^2 - 5*a^2*b*c^3)*e^3)*f)*x + s
qrt(1/2)*((b^2*c^5 - 4*a*c^6)*d^2*e - 2*(b^3*c^4 - 4*a*b*c^5)*d*e^2 + (b^4*
c^3 - 5*a*b^2*c^4 + 4*a^2*c^5)*e^3 - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*
a^3*b*c^3)*f^3 - (2*(b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d - (3*b^6*c - 19
*a*b^4*c^2 + 29*a^2*b^2*c^3 - 4*a^3*c^4)*e)*f^2 - ((b^3*c^4 - 4*a*b*c^5)*d^
2 - 2*(2*b^4*c^3 - 9*a*b^2*c^4 + 4*a^2*c^5)*d*e + (3*b^5*c^2 - 17*a*b^3*c^3
 + 20*a^2*b*c^4)*e^2)*f - (2*(b^2*c^7 - 4*a*c^8)*d - (b^3*c^6 - 4*a*b*c^7)*
e + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*f)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e +
2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*
a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^
3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d -
 (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4
 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)
*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*(
(b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^
2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2
*c^10 - 4*a*c^11)))*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2
 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a
*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f - (b^2*c^5 - 4*a*c^6)*sq
rt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 -
a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c +
 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3
+ 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3
*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^
3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2
*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^
```

```
6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c
^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3
*sqrt(1/2)*c^2*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*
a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^
3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f - (b^2*c^5 - 4*a*c^6)*sqrt((c
^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c
^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a
^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^
4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9
*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4
- a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^
2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 +
2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(c^
6*d^4 - 3*b*c^5*d^3*e + 3*b^2*c^4*d^2*e^2 - (b^3*c^3 + a*b*c^4)*d*e^3 + (a*
b^2*c^3 - a^2*c^4)*e^4 + (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*f^4 + ((b^6 - 5*
a*b^4*c + 9*a^2*b^2*c^2 - 4*a^3*c^3)*d - (a*b^5 - a^2*b^3*c - 3*a^3*b*c^2)*
e)*f^3 + 3*((b^4*c^2 - 3*a*b^2*c^3 + 2*a^2*c^4)*d^2 - (b^5*c - 3*a*b^3*c^2
+ 3*a^2*b*c^3)*d*e + (a*b^4*c - 2*a^2*b^2*c^2)*e^2)*f^2 + ((3*b^2*c^4 - 4*a
*c^5)*d^3 - 3*(2*b^3*c^3 - 3*a*b*c^4)*d^2*e + 3*(b^4*c^2 - a*b^2*c^3)*d*e^2
 - (3*a*b^3*c^2 - 5*a^2*b*c^3)*e^3)*f)*x - sqrt(1/2)*((b^2*c^5 - 4*a*c^6)*d
^2*e - 2*(b^3*c^4 - 4*a*b*c^5)*d*e^2 + (b^4*c^3 - 5*a*b^2*c^4 + 4*a^2*c^5)*
e^3 - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*f^3 - (2*(b^5*c^2 -
5*a*b^3*c^3 + 4*a^2*b*c^4)*d - (3*b^6*c - 19*a*b^4*c^2 + 29*a^2*b^2*c^3 - 4
*a^3*c^4)*e)*f^2 - ((b^3*c^4 - 4*a*b*c^5)*d^2 - 2*(2*b^4*c^3 - 9*a*b^2*c^4
+ 4*a^2*c^5)*d*e + (3*b^5*c^2 - 17*a*b^3*c^3 + 20*a^2*b*c^4)*e^2)*f - (2*(b
^2*c^7 - 4*a*c^8)*d - (b^3*c^6 - 4*a*b*c^7)*e + (b^4*c^5 - 6*a*b^2*c^6 + 8*
a^2*c^7)*f)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 -
 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8
 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2
- 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b
^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2
 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^
3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*
c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c
^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b*c^
4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*
b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^
2 + 2*a^2*c^3)*e)*f - (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2
*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a
*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d -
(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4
- 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*
d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((
b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2
*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*
c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 6*(c*e - b*f)*x)/c^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.57 $\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=219

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2af+be)+b^2f+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2acf+b^2f-bce+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{fx}{c}$$

```
[Out] (f*x)/c + ((c*e - b*f + (2*c^2*d + b^2*f - c*(b*e + 2*a*f))/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*e - b*f - (2*c^2*d - b*c*e + b^2*f -
2*a*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.637272, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1676, 1166, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2af+be)+b^2f+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2acf+b^2f-bce+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{fx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (f*x)/c + ((c*e - b*f + (2*c^2*d + b^2*f - c*(b*e + 2*a*f))/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*e - b*f - (2*c^2*d - b*c*e + b^2*f -
2*a*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \int \left(\frac{f}{c} + \frac{cd-af+(ce-bf)x^2}{c\left(a+bx^2+cx^4\right)}\right)dx$$
$$= \frac{fx}{c} + \frac{\int \frac{cd-af+(ce-bf)x^2}{a+bx^2+cx^4}\,dx}{c}$$
$$= \frac{fx}{c} + \frac{\left(ce-bf-\frac{2c^2d-bce+b^2f-2acf}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} + \frac{\left(ce-bf+\frac{2c^2d+b^2f-c(be+2af)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4}}$$
$$= \frac{fx}{c} + \frac{\left(ce-bf+\frac{2c^2d+b^2f-c(be+2af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(ce-bf-\frac{2c^2d-bce+b^2f-2acf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{b+}}\right.}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.346859, size = 258, normalized size = 1.18

$$\frac{\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(c\left(e\sqrt{b^2-4ac}-2af-be\right)+bf\left(b-\sqrt{b^2-4ac}\right)+2c^2d\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-c\left(e\sqrt{b^2-4ac}+2af+be\right)+bf\left(\sqrt{b^2-4ac}+b\right)+2c^2d\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + 2\sqrt{c}f}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*f*x + (Sqrt[2]*(2*c^2*d + b*(b - Sqrt[b^2 - 4*a*c])*f + c*(-(b*e
) + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*
(2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c])*f - c*(b*e + Sqrt[b^2 - 4*a*c]*e + 2*a
*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4
*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2))
```

Maple [B] time = 0.025, size = 676, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] f*x/c+1/2/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((
(-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*f-1/2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^
(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e+1/(-4*a*c+b^2
)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2))*a*f-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+
b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
)*b^2*f+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arc
tanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*e-c/(-4*a*c+b^2)^(1/2)
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2))*d-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*f+1/2*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
e+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*f-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1
```

```
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*b^2*f+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e-c/(-4*a*
c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{fx}{c} - \frac{-\int \frac{(ce-bf)x^2+cd-af}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] f*x/c - integrate(-((c*e - b*f)*x^2 + c*d - a*f)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [B] time = 11.1798, size = 11135, normalized size = 50.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3
*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f + (a*b^2*c^3 - 4*
a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b
^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2
)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^
2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*
b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c
^5*d^4 - b*c^4*d^3*e + a*b*c^3*d*e^3 - a^2*c^3*e^4 - (a^3*b^2 - a^4*c)*f^4
- ((a*b^4 - 3*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 + a^3*b*c)*e)*f^3 - 3*(a^
2*b^2*c*e^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^2 - (a*b^3*c - a^2*b*c^2)*d*e)*f^2
+ (3*a*b*c^3*d^2*e - 3*a*b^2*c^2*d*e^2 + 3*a^2*b*c^2*e^3 + (b^2*c^3 - 4*a*c
^4)*d^3)*f)*x + sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^3 - (a*b^2*c^3 - 4*a^2*c^4
)*d*e^2 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*f^3 - ((a*b^4*c - 7*a^2*b^2*c
^2 + 12*a^3*c^3)*d + 2*(a^2*b^3*c - 4*a^3*b*c^2)*e)*f^2 - (3*(a*b^2*c^3 - 4
*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d*e - (a^2*b^2*c^2 - 4*a^3*c^3)
*e^2)*f - ((a*b^3*c^4 - 4*a^2*b*c^5)*d - 2*(a^2*b^2*c^4 - 4*a^3*c^5)*e + (a
^2*b^3*c^3 - 4*a^3*b*c^4)*f)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4
+ (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a
^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)
*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e -
a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*
d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d -
(a*b^2*c - 2*a^2*c^2)*e)*f + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^
5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b
^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e +
 (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^
5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*
a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) - sqrt(1/2)*c*sqrt(-(b*c^3*d^2 - 4*a*c
^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c -
```

```
2*a^2*c^2)*e)*f + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 +
 a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^
3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3
 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b
*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/
(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c^5*d^4 - b*c^4*d^3*e + a*b*c^3*d*e^3 - a^2
*c^3*e^4 - (a^3*b^2 - a^4*c)*f^4 - ((a*b^4 - 3*a^2*b^2*c + 4*a^3*c^2)*d - (
a^2*b^3 + a^3*b*c)*e)*f^3 - 3*(a^2*b^2*c*e^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^2
- (a*b^3*c - a^2*b*c^2)*d*e)*f^2 + (3*a*b*c^3*d^2*e - 3*a*b^2*c^2*d*e^2 + 3
*a^2*b*c^2*e^3 + (b^2*c^3 - 4*a*c^4)*d^3)*f)*x - sqrt(1/2)*((b^2*c^4 - 4*a*
c^5)*d^3 - (a*b^2*c^3 - 4*a^2*c^4)*d*e^2 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c
^2)*f^3 - ((a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d + 2*(a^2*b^3*c - 4*a^3*
b*c^2)*e)*f^2 - (3*(a*b^2*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3
)*d*e - (a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*f - ((a*b^3*c^4 - 4*a^2*b*c^5)*d - 2
*(a^2*b^2*c^4 - 4*a^3*c^5)*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*f)*sqrt((c^6*d^4
 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 +
4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*
c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2
- 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2
*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 -
 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f + (a*b^2*c^3 -
4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3
*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c
^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*
c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^
2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) + sqr
t(1/2)*c*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)
*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f - (a*b^2*c^3 - 4*a^2*c^4)*
sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^
4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3
- 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c
^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3
)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c^5*d^4 -
b*c^4*d^3*e + a*b*c^3*d*e^3 - a^2*c^3*e^4 - (a^3*b^2 - a^4*c)*f^4 - ((a*b^4
 - 3*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 + a^3*b*c)*e)*f^3 - 3*(a^2*b^2*c*e
^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^2 - (a*b^3*c - a^2*b*c^2)*d*e)*f^2 + (3*a*b*
c^3*d^2*e - 3*a*b^2*c^2*d*e^2 + 3*a^2*b*c^2*e^3 + (b^2*c^3 - 4*a*c^4)*d^3)*
f)*x + sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^3 - (a*b^2*c^3 - 4*a^2*c^4)*d*e^2 +
 (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*f^3 - ((a*b^4*c - 7*a^2*b^2*c^2 + 12*a
^3*c^3)*d + 2*(a^2*b^3*c - 4*a^3*b*c^2)*e)*f^2 - (3*(a*b^2*c^3 - 4*a^2*c^4)
*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d*e - (a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*f +
 ((a*b^3*c^4 - 4*a^2*b*c^5)*d - 2*(a^2*b^2*c^4 - 4*a^3*c^5)*e + (a^2*b^3*c^
3 - 4*a^3*b*c^4)*f)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^
4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c
- a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3
*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d
*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^2 - 4*a
*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c
- 2*a^2*c^2)*e)*f - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2
 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 -
a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c
^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a
*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7))
)/(a*b^2*c^3 - 4*a^2*c^4))) - sqrt(1/2)*c*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e +
a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2
)*e)*f - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*
e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d
- (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*
c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*
```

```
e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^
3 - 4*a^2*c^4))*log(2*(c^5*d^4 - b*c^4*d^3*e + a*b*c^3*d*e^3 - a^2*c^3*e^4
- (a^3*b^2 - a^4*c)*f^4 - ((a*b^4 - 3*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 +
 a^3*b*c)*e)*f^3 - 3*(a^2*b^2*c*e^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^2 - (a*b^3*
c - a^2*b*c^2)*d*e)*f^2 + (3*a*b*c^3*d^2*e - 3*a*b^2*c^2*d*e^2 + 3*a^2*b*c^
2*e^3 + (b^2*c^3 - 4*a*c^4)*d^3)*f)*x - sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^3
- (a*b^2*c^3 - 4*a^2*c^4)*d*e^2 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*f^3 -
 ((a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d + 2*(a^2*b^3*c - 4*a^3*b*c^2)*e)
*f^2 - (3*(a*b^2*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d*e - (
a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*f + ((a*b^3*c^4 - 4*a^2*b*c^5)*d - 2*(a^2*b^2
*c^4 - 4*a^3*c^5)*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*f)*sqrt((c^6*d^4 - 2*a*c^
5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b
^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e +
 (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^
5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*
a^3*c^7)))*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*
c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f - (a*b^2*c^3 - 4*a^2*c^4
)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c +
a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^
3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3
*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e
^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) - 2*f*x)/c
```

Sympy [B] time = 90.1555, size = 1151, normalized size = 5.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**5 - 128*a**2*b**2*c**4 + 16*a*b**4*c**3) + _t**2
*(48*a**3*b*c**2*f**2 - 64*a**3*c**3*e*f - 28*a**2*b**3*c*f**2 + 48*a**2*b*
*2*c**2*e*f - 32*a**2*b*c**3*d*f - 16*a**2*b*c**3*e**2 + 64*a**2*c**4*d*e +
 4*a*b**5*f**2 - 8*a*b**4*c*e*f + 8*a*b**3*c**2*d*f + 4*a*b**3*c**2*e**2 -
16*a*b**2*c**3*d*e - 16*a*b*c**4*d**2 + 4*b**3*c**3*d**2) + a**4*f**4 - 2*a
**3*b*e*f**3 - 4*a**3*c*d*f**3 + 2*a**3*c*e**2*f**2 + 2*a**2*b**2*d*f**3 +
a**2*b**2*e**2*f**2 + 2*a**2*b*c*d*e*f**2 - 2*a**2*b*c*e**3*f + 6*a**2*c**2
*d**2*f**2 - 4*a**2*c**2*d*e**2*f + a**2*c**2*e**4 - 2*a*b**3*d*e*f**2 - 4*
a*b**2*c*d**2*f**2 + 4*a*b**2*c*d*e**2*f + 2*a*b*c**2*d**2*e*f - 2*a*b*c**2
*d*e**3 - 4*a*c**3*d**3*f + 2*a*c**3*d**2*e**2 + b**4*d**2*f**2 - 2*b**3*c*
d**2*e*f + 2*b**2*c**2*d**3*f + b**2*c**2*d**2*e**2 - 2*b*c**3*d**3*e + c**
4*d**4, Lambda(_t, _t*log(x + (32*_t**3*a**3*b*c**4*f - 64*_t**3*a**3*c**5*
e - 8*_t**3*a**2*b**3*c**3*f + 16*_t**3*a**2*b**2*c**4*e + 32*_t**3*a**2*b*
c**5*d - 8*_t**3*a*b**3*c**4*d - 4*_t*a**4*c**2*f**3 + 8*_t*a**3*b**2*c*f**
3 - 18*_t*a**3*b*c**2*e*f**2 + 12*_t*a**3*c**3*d*f**2 + 12*_t*a**3*c**3*e**
2*f - 2*_t*a**2*b**4*f**3 + 6*_t*a**2*b**3*c*e*f**2 - 6*_t*a**2*b**2*c**2*d
*f**2 - 6*_t*a**2*b**2*c**2*e**2*f + 12*_t*a**2*b*c**3*d*e*f + 2*_t*a**2*b*
c**3*e**3 - 12*_t*a**2*c**4*d**2*f - 12*_t*a**2*c**4*d*e**2 + 6*_t*a*b*c**4
*d**2*e + 4*_t*a*c**5*d**3 - 2*_t*b**2*c**4*d**3)/(a**4*c*f**4 - a**3*b**2*
f**4 + a**3*b*c*e*f**3 - 4*a**3*c**2*d*f**3 + a**2*b**3*e*f**3 + 3*a**2*b**
2*c*d*f**3 - 3*a**2*b**2*c*e**2*f**2 - 3*a**2*b*c**2*d*e*f**2 + 3*a**2*b*c*
*2*e**3*f + 6*a**2*c**3*d**2*f**2 - a**2*c**3*e**4 - a*b**4*d*f**3 + 3*a*b*
*3*c*d*e*f**2 - 3*a*b**2*c**2*d**2*f**2 - 3*a*b**2*c**2*d*e**2*f + 3*a*b*c*
*3*d**2*e*f + a*b*c**3*d*e**3 - 4*a*c**4*d**3*f + b**2*c**3*d**3*f - b*c**4
*d**3*e + c**5*d**4)))) + f*x/c
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.58 $\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=213

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}a\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{ax}$$

```
[Out] -(d/(a*x)) - ((c*d - a*f + (b*c*d - 2*a*c*e + a*b*f)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[c]*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) - ((c*d - a*f - (b*c*d - 2*a*c*e + a*b*f)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*a*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.839172, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1664, 1166, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}a\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(d/(a*x)) - ((c*d - a*f + (b*c*d - 2*a*c*e + a*b*f)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[c]*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) - ((c*d - a*f - (b*c*d - 2*a*c*e + a*b*f)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*a*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)}dx = \int \left(\frac{d}{ax^2}+\frac{-bd+ae-(cd-af)x^2}{a\left(a+bx^2+cx^4\right)}\right)dx$$
$$= -\frac{d}{ax}+\frac{\int \frac{-bd+ae+(-cd+af)x^2}{a+bx^2+cx^4}dx}{a}$$
$$= -\frac{d}{ax}-\frac{\left(cd-af-\frac{bcd-2ace+abf}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}dx}{2a}+\frac{\left(-cd+af+\frac{2ace-b(cd+af)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}}$$
$$= -\frac{d}{ax}-\frac{\left(cd-af-\frac{2ace-b(cd+af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(cd-af-\frac{bcd-2ace+abf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-}}}\right.}{\sqrt{2}a\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.327722, size = 253, normalized size = 1.19

$$\frac{-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(cd\sqrt{b^2-4ac}-af\sqrt{b^2-4ac}+abf-2ace+bcd\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+abf-2ace+bcd\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2d}{x}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*d)/x - (Sqrt[2]*(b*c*d + c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f - a*S
qrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b*c*d
 - c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f + a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b^2 - 4*a*c
]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a)
```

Maple [B] time = 0.025, size = 563, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -d/a/x-1/2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((
-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f+1/2/a*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)
^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d+1/2/(-4*a*c+
b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((
-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*f-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b
^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))
*e+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arct
anh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d+1/2*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*f-1/2/a*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
b*f-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^
```

```
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{(cd-af)x^2+bd-ae}{cx^4+bx^2+a}\,dx}{a}-\frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(-((c*d - a*f)*x^2 + b*d - a*e)/(c*x^4 + b*x^2 + a), x)/a - d/(a*x
)
```

Fricas [B] time = 5.20909, size = 11429, normalized size = 53.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a*x*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^
2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*a^3*c*e)*f + (a^3*b^2*c
- 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f
^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*
e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a
^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^
2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))/(a^3*b^2*c - 4*a^4
*c^2))*log(-2*(3*a*b^2*c^2*d^2*e^2 - 3*a^2*b*c^2*d*e^3 + a^3*c^2*e^4 - a^5*
f^4 + (b^2*c^3 - a*c^4)*d^4 - (b^3*c^2 + a*b*c^3)*d^3*e + (a^4*b*e - (a^3*b
^2 - 4*a^4*c)*d)*f^3 - 3*(a^3*b*c*d*e - (a^2*b^2*c - 2*a^3*c^2)*d^2)*f^2 +
(3*a^2*b^2*c*d*e^2 - a^3*b*c*e^3 + (b^4*c - 3*a*b^2*c^2 + 4*a^2*c^3)*d^3 -
3*(a*b^3*c - a^2*b*c^2)*d^2*e)*f)*x + sqrt(1/2)*((b^5*c - 5*a*b^3*c^2 + 4*a
^2*b*c^3)*d^3 - (3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e + 3*(a^2*b^3
*c - 4*a^3*b*c^2)*d*e^2 - (a^3*b^2*c - 4*a^4*c^2)*e^3 - ((a^3*b^3 - 4*a^4*b
*c)*d - (a^4*b^2 - 4*a^5*c)*e)*f^2 + 2*((a^2*b^3*c - 4*a^3*b*c^2)*d^2 - (a^
3*b^2*c - 4*a^4*c^2)*d*e)*f - ((a^3*b^4*c - 6*a^4*b^2*c^2 + 8*a^5*c^3)*d -
(a^4*b^3*c - 4*a^5*b*c^2)*e + 2*(a^5*b^2*c - 4*a^6*c^2)*f)*sqrt(-(4*a^3*b*c
^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 +
 a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^
3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^
2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/
(a^6*b^2*c^2 - 4*a^7*c^3)))*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2 + (b^3*c - 3*a*b
*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*a^3*c*e)*f + (a^
3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3
 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c
^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*
e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e
^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))/(a^3*b^2*c
 - 4*a^4*c^2))) - sqrt(1/2)*a*x*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2 + (b^3*c - 3
*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*a^3*c*e)*f +
 (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d
```

```
*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2
*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^
5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2
*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))/(a^3*b
^2*c - 4*a^4*c^2))*log(-2*(3*a*b^2*c^2*d^2*e^2 - 3*a^2*b*c^2*d*e^3 + a^3*c^
2*e^4 - a^5*f^4 + (b^2*c^3 - a*c^4)*d^4 - (b^3*c^2 + a*b*c^3)*d^3*e + (a^4*
b*e - (a^3*b^2 - 4*a^4*c)*d)*f^3 - 3*(a^3*b*c*d*e - (a^2*b^2*c - 2*a^3*c^2)
*d^2)*f^2 + (3*a^2*b^2*c*d*e^2 - a^3*b*c*e^3 + (b^4*c - 3*a*b^2*c^2 + 4*a^2
*c^3)*d^3 - 3*(a*b^3*c - a^2*b*c^2)*d^2*e)*f)*x - sqrt(1/2)*((b^5*c - 5*a*b
^3*c^2 + 4*a^2*b*c^3)*d^3 - (3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e
+ 3*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^2 - (a^3*b^2*c - 4*a^4*c^2)*e^3 - ((a^3*b
^3 - 4*a^4*b*c)*d - (a^4*b^2 - 4*a^5*c)*e)*f^2 + 2*((a^2*b^3*c - 4*a^3*b*c^
2)*d^2 - (a^3*b^2*c - 4*a^4*c^2)*d*e)*f - ((a^3*b^4*c - 6*a^4*b^2*c^2 + 8*a
^5*c^3)*d - (a^4*b^3*c - 4*a^5*b*c^2)*e + 2*(a^5*b^2*c - 4*a^6*c^2)*f)*sqrt
(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2
*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*
c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*
c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c
^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2 + (b
^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*a^3*
c*e)*f + (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4
*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c
^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*
d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e -
 a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3))
)/(a^3*b^2*c - 4*a^4*c^2))) + sqrt(1/2)*a*x*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2
+ (b^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*
a^3*c*e)*f - (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4
 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b
^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*
b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2
*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c
^3)))/(a^3*b^2*c - 4*a^4*c^2))*log(-2*(3*a*b^2*c^2*d^2*e^2 - 3*a^2*b*c^2*d*
e^3 + a^3*c^2*e^4 - a^5*f^4 + (b^2*c^3 - a*c^4)*d^4 - (b^3*c^2 + a*b*c^3)*d
^3*e + (a^4*b*e - (a^3*b^2 - 4*a^4*c)*d)*f^3 - 3*(a^3*b*c*d*e - (a^2*b^2*c
- 2*a^3*c^2)*d^2)*f^2 + (3*a^2*b^2*c*d*e^2 - a^3*b*c*e^3 + (b^4*c - 3*a*b^2
*c^2 + 4*a^2*c^3)*d^3 - 3*(a*b^3*c - a^2*b*c^2)*d^2*e)*f)*x + sqrt(1/2)*((b
^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^3 - (3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3
*c^3)*d^2*e + 3*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^2 - (a^3*b^2*c - 4*a^4*c^2)*e
^3 - ((a^3*b^3 - 4*a^4*b*c)*d - (a^4*b^2 - 4*a^5*c)*e)*f^2 + 2*((a^2*b^3*c
- 4*a^3*b*c^2)*d^2 - (a^3*b^2*c - 4*a^4*c^2)*d*e)*f + ((a^3*b^4*c - 6*a^4*b
^2*c^2 + 8*a^5*c^3)*d - (a^4*b^3*c - 4*a^5*b*c^2)*e + 2*(a^5*b^2*c - 4*a^6*
c^2)*f)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 -
(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2
*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^
2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2
*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))*sqrt(-(a^2*b*c*e^2 + a^
3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*
c*d - 2*a^3*c*e)*f - (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4
*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4
+ 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2
*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b
*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 -
 4*a^7*c^3)))/(a^3*b^2*c - 4*a^4*c^2))) - sqrt(1/2)*a*x*sqrt(-(a^2*b*c*e^2
+ a^3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^
2*b*c*d - 2*a^3*c*e)*f - (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 -
 a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*
d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2
 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a
```

```
^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c
^2 - 4*a^7*c^3)))/(a^3*b^2*c - 4*a^4*c^2))*log(-2*(3*a*b^2*c^2*d^2*e^2 - 3*
a^2*b*c^2*d*e^3 + a^3*c^2*e^4 - a^5*f^4 + (b^2*c^3 - a*c^4)*d^4 - (b^3*c^2
+ a*b*c^3)*d^3*e + (a^4*b*e - (a^3*b^2 - 4*a^4*c)*d)*f^3 - 3*(a^3*b*c*d*e -
 (a^2*b^2*c - 2*a^3*c^2)*d^2)*f^2 + (3*a^2*b^2*c*d*e^2 - a^3*b*c*e^3 + (b^4
*c - 3*a*b^2*c^2 + 4*a^2*c^3)*d^3 - 3*(a*b^3*c - a^2*b*c^2)*d^2*e)*f)*x - s
qrt(1/2)*((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^3 - (3*a*b^4*c - 13*a^2*b^2
*c^2 + 4*a^3*c^3)*d^2*e + 3*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^2 - (a^3*b^2*c -
4*a^4*c^2)*e^3 - ((a^3*b^3 - 4*a^4*b*c)*d - (a^4*b^2 - 4*a^5*c)*e)*f^2 + 2*
((a^2*b^3*c - 4*a^3*b*c^2)*d^2 - (a^3*b^2*c - 4*a^4*c^2)*d*e)*f + ((a^3*b^4
*c - 6*a^4*b^2*c^2 + 8*a^5*c^3)*d - (a^4*b^3*c - 4*a^5*b*c^2)*e + 2*(a^5*b^
2*c - 4*a^6*c^2)*f)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3
- a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^
3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e
^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^
2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))*sqrt(-(a^2*
b*c*e^2 + a^3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e
 + 2*(a^2*b*c*d - 2*a^3*c*e)*f - (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2
*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a
^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)
*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2
+ 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a
^6*b^2*c^2 - 4*a^7*c^3)))/(a^3*b^2*c - 4*a^4*c^2))) + 2*d)/(a*x)
```

Sympy [B] time = 96.5926, size = 1192, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**3 - 128*a**4*b**2*c**2 + 16*a**3*b**4*c) + _t**2
*(-16*a**4*b*c*f**2 + 64*a**4*c**2*e*f + 4*a**3*b**3*f**2 - 16*a**3*b**2*c*
e*f - 32*a**3*b*c**2*d*f - 16*a**3*b*c**2*e**2 - 64*a**3*c**3*d*e + 8*a**2*
b**3*c*d*f + 4*a**2*b**3*c*e**2 + 48*a**2*b**2*c**2*d*e + 48*a**2*b*c**3*d*
*2 - 8*a*b**4*c*d*e - 28*a*b**3*c**2*d**2 + 4*b**5*c*d**2) + a**4*f**4 - 2*
a**3*b*e*f**3 - 4*a**3*c*d*f**3 + 2*a**3*c*e**2*f**2 + 2*a**2*b**2*d*f**3 +
 a**2*b**2*e**2*f**2 + 2*a**2*b*c*d*e*f**2 - 2*a**2*b*c*e**3*f + 6*a**2*c**
2*d**2*f**2 - 4*a**2*c**2*d*e**2*f + a**2*c**2*e**4 - 2*a*b**3*d*e*f**2 - 4
*a*b**2*c*d**2*f**2 + 4*a*b**2*c*d*e**2*f + 2*a*b*c**2*d**2*e*f - 2*a*b*c**
2*d*e**3 - 4*a*c**3*d**3*f + 2*a*c**3*d**2*e**2 + b**4*d**2*f**2 - 2*b**3*c
*d**2*e*f + 2*b**2*c**2*d**3*f + b**2*c**2*d**2*e**2 - 2*b*c**3*d**3*e + c*
*4*d**4, Lambda(_t, _t*log(x + (64*_t**3*a**6*c**2*f - 16*_t**3*a**5*b**2*c
*f - 32*_t**3*a**5*b*c**2*e - 64*_t**3*a**5*c**3*d + 8*_t**3*a**4*b**3*c*e
+ 48*_t**3*a**4*b**2*c**2*d - 8*_t**3*a**3*b**4*c*d - 2*_t*a**5*b*f**3 + 12
*_t*a**5*c*e*f**2 - 6*_t*a**4*b*c*d*f**2 - 6*_t*a**4*b*c*e**2*f - 24*_t*a**
4*c**2*d*e*f - 4*_t*a**4*c**2*e**3 + 12*_t*a**3*b**2*c*d*e*f + 2*_t*a**3*b*
*2*c*e**3 + 18*_t*a**3*b*c**2*d**2*f + 18*_t*a**3*b*c**2*d*e**2 + 12*_t*a**
3*c**3*d**2*e - 6*_t*a**2*b**3*c*d**2*f - 6*_t*a**2*b**3*c*d*e**2 - 24*_t*a
**2*b**2*c**2*d**2*e - 10*_t*a**2*b*c**3*d**3 + 6*_t*a*b**4*c*d**2*e + 10*_
t*a*b**3*c**2*d**3 - 2*_t*b**5*c*d**3)/(a**5*f**4 - a**4*b*e*f**3 - 4*a**4*
c*d*f**3 + a**3*b**2*d*f**3 + 3*a**3*b*c*d*e*f**2 + a**3*b*c*e**3*f + 6*a**
3*c**2*d**2*f**2 - a**3*c**2*e**4 - 3*a**2*b**2*c*d**2*f**2 - 3*a**2*b**2*c
*d*e**2*f - 3*a**2*b*c**2*d**2*e*f + 3*a**2*b*c**2*d*e**3 - 4*a**2*c**3*d**
3*f + 3*a*b**3*c*d**2*e*f + 3*a*b**2*c**2*d**3*f - 3*a*b**2*c**2*d**2*e**2
+ a*b*c**3*d**3*e + a*c**4*d**4 - b**4*c*d**3*f + b**3*c**2*d**3*e - b**2*c
```

```
**3*d**4)))) - d/(a*x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.59 $\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=267

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-abe-2a(cd-af)+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-a\left(-e\sqrt{b^2-4ac}-2af+2cd\right)-b\left(d\sqrt{\right.\right.}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -d/(3*a*x^3) + (b*d - a*e)/(a^2*x) + (Sqrt[c]*(b*d - a*e + (b^2*d - a*b*e -
 2*a*(c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^
2*d - b*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*(2*c*d - Sqrt[b^2 - 4*a*c]*e - 2*a*
f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.0651, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1664, 1166, 205}

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-abe-2a(cd-af)+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-a\left(-e\sqrt{b^2-4ac}-2af+2cd\right)-b\left(d\sqrt{\right.\right.}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -d/(3*a*x^3) + (b*d - a*e)/(a^2*x) + (Sqrt[c]*(b*d - a*e + (b^2*d - a*b*e -
 2*a*(c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^
2*d - b*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*(2*c*d - Sqrt[b^2 - 4*a*c]*e - 2*a*
f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)}dx = \int \left(\frac{d}{ax^4}+\frac{-bd+ae}{a^2x^2}+\frac{b^2d-abe-a(cd-af)+c(bd-ae)x^2}{a^2\left(a+bx^2+cx^4\right)}\right)dx$$
$$= -\frac{d}{3ax^3}+\frac{bd-ae}{a^2x}+\frac{\int \frac{b^2d-abe-a(cd-af)+c(bd-ae)x^2}{a+bx^2+cx^4}dx}{a^2}$$
$$= -\frac{d}{3ax^3}+\frac{bd-ae}{a^2x}+\frac{\left(c\left(bd-ae-\frac{b^2d-abe-2a(cd-af)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}dx}{2a^2}+\frac{\left(c\left(bd-ae+\frac{b^2d-ab}{\sqrt{}}\right.\right.}{}$$
$$= -\frac{d}{3ax^3}+\frac{bd-ae}{a^2x}+\frac{\sqrt{c}\left(bd-ae+\frac{b^2d-abe-2a(cd-af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(bd-ae-\frac{b^2d-ab}{\sqrt{}}\right.}{\sqrt{2}a^2}$$

Mathematica [A] time = 0.372637, size = 284, normalized size = 1.06

$$\frac{\frac{3\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(a\left(-e\sqrt{b^2-4ac}+2af-2cd\right)+b\left(d\sqrt{b^2-4ac}-ae\right)+b^2d\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-a\left(e\sqrt{b^2-4ac}+2af-2cd\right)+b\left(d\sqrt{b^2-4ac}+ae\right)+b^2(-\right.}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*d)/x^3 + (6*b*d - 6*a*e)/x + (3*Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^2
 - 4*a*c]*d - a*e) + a*(-2*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f))*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-(b^2*d) + b*(Sqrt[b^2 - 4*a*c]*d +
 a*e) - a*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*
c]]))/(6*a^2)
```

Maple [B] time = 0.029, size = 727, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/3*d/a/x^3-1/a/x*e+1/a^2/x*b*d+1/2/a*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)
^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e-1/2/a^2*c*2^
(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2))*b*d-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*
c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f+1/2/a*c/(-
4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/
2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*e+1/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)
/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b)*c)^(1/2))*d-1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)
*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*d-1/2/a
*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+
```

```
b^2)^(1/2))*c)^(1/2))*e+1/2/a^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d-c/(-4*a*c+b^2)^(1/
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*f+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e+1
/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c
*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d-1/2/a^2*c/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*b^2*d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 27.6739, size = 19478, normalized size = 72.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(1/2)*a^2*x^3*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2
)*d^2 - 2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2
 + 2*((a^2*b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f + (a^5*b^2 - 4*a^6
*c)*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2
*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5
 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4
 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*
a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2
 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 -
9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^
2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)
)*log(2*(a^6*c*f^4 + (b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d^4 - (b^5*c^2 - a*b
^3*c^3 - 3*a^2*b*c^4)*d^3*e + 3*(a*b^4*c^2 - 2*a^2*b^2*c^3)*d^2*e^2 - (3*a^
2*b^3*c^2 - 5*a^3*b*c^3)*d*e^3 + (a^3*b^2*c^2 - a^4*c^3)*e^4 - (3*a^5*b*c*e
 - (3*a^4*b^2*c - 4*a^5*c^2)*d)*f^3 + 3*(a^4*b^2*c*e^2 + (a^2*b^4*c - 3*a^3
*b^2*c^2 + 2*a^4*c^3)*d^2 - (2*a^3*b^3*c - 3*a^4*b*c^2)*d*e)*f^2 + ((b^6*c
- 5*a*b^4*c^2 + 9*a^2*b^2*c^3 - 4*a^3*c^4)*d^3 - 3*(a*b^5*c - 3*a^2*b^3*c^2
 + 3*a^3*b*c^3)*d^2*e + 3*(a^2*b^4*c - a^3*b^2*c^2)*d*e^2 - (a^3*b^3*c + a^
4*b*c^2)*e^3)*f)*x + sqrt(1/2)*((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*
b^2*c^3 + 4*a^4*c^4)*d^3 - (3*a*b^7 - 21*a^2*b^5*c + 41*a^3*b^3*c^2 - 20*a^
4*b*c^3)*d^2*e + (3*a^2*b^6 - 18*a^3*b^4*c + 25*a^4*b^2*c^2 - 4*a^5*c^3)*d*
e^2 - (a^3*b^5 - 5*a^4*b^3*c + 4*a^5*b*c^2)*e^3 + (a^6*b^2 - 4*a^7*c)*f^3 +
 3*((a^4*b^4 - 5*a^5*b^2*c + 4*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*f^2 +
((3*a^2*b^6 - 19*a^3*b^4*c + 31*a^4*b^2*c^2 - 12*a^5*c^3)*d^2 - 2*(3*a^3*b^
5 - 16*a^4*b^3*c + 16*a^5*b*c^2)*d*e + (3*a^4*b^4 - 13*a^5*b^2*c + 4*a^6*c^
2)*e^2)*f - ((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*d - (a^6*b^4 - 6*a^7*b^
```

2*c + 8*a^8*c^2)*e + (a^7*b^3 - 4*a^8*b*c)*f)*sqrt((a^8*f^4 + (b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*
c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a
^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e
^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)
*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a
^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*
a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e
+ (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(
a^10*b^2 - 4*a^11*c)))*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d
^2 - 2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 +
2*((a^2*b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f + (a^5*b^2 - 4*a^6*c)
*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^
4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3
*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 -
3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 -
4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6
*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 +
4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a
^4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 -
(a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)))
- 3*sqrt(1/2)*a^2*x^3*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^
2 - 2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + 2
*((a^2*b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f + (a^5*b^2 - 4*a^6*c)*
sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4
)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*
a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3
*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4
*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*
c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4
*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^
4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 -
(a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*lo
g(2*(a^6*c*f^4 + (b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d^4 - (b^5*c^2 - a*b^3*c
^3 - 3*a^2*b*c^4)*d^3*e + 3*(a*b^4*c^2 - 2*a^2*b^2*c^3)*d^2*e^2 - (3*a^2*b^
3*c^2 - 5*a^3*b*c^3)*d*e^3 + (a^3*b^2*c^2 - a^4*c^3)*e^4 - (3*a^5*b*c*e - (
3*a^4*b^2*c - 4*a^5*c^2)*d)*f^3 + 3*(a^4*b^2*c*e^2 + (a^2*b^4*c - 3*a^3*b^2
*c^2 + 2*a^4*c^3)*d^2 - (2*a^3*b^3*c - 3*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 5*
a*b^4*c^2 + 9*a^2*b^2*c^3 - 4*a^3*c^4)*d^3 - 3*(a*b^5*c - 3*a^2*b^3*c^2 + 3
*a^3*b*c^3)*d^2*e + 3*(a^2*b^4*c - a^3*b^2*c^2)*d*e^2 - (a^3*b^3*c + a^4*b*
c^2)*e^3)*f)*x - sqrt(1/2)*((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*
c^3 + 4*a^4*c^4)*d^3 - (3*a*b^7 - 21*a^2*b^5*c + 41*a^3*b^3*c^2 - 20*a^4*b*
c^3)*d^2*e + (3*a^2*b^6 - 18*a^3*b^4*c + 25*a^4*b^2*c^2 - 4*a^5*c^3)*d*e^2
- (a^3*b^5 - 5*a^4*b^3*c + 4*a^5*b*c^2)*e^3 + (a^6*b^2 - 4*a^7*c)*f^3 + 3*(
(a^4*b^4 - 5*a^5*b^2*c + 4*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*f^2 + ((3*
a^2*b^6 - 19*a^3*b^4*c + 31*a^4*b^2*c^2 - 12*a^5*c^3)*d^2 - 2*(3*a^3*b^5 -
16*a^4*b^3*c + 16*a^5*b*c^2)*d*e + (3*a^4*b^4 - 13*a^5*b^2*c + 4*a^6*c^2)*e
^2)*f - ((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*d - (a^6*b^4 - 6*a^7*b^2*c
+ 8*a^8*c^2)*e + (a^7*b^3 - 4*a^8*b*c)*f)*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c +
7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b
^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 +
(a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*
f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b
*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*
b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3
*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10
*b^2 - 4*a^11*c)))*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2 -
2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + 2*((

a^2*b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f + (a^5*b^2 - 4*a^6*c)*sqr
t((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d
^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2
*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^
4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a
^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2
)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((
a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b
^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^
5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) + 3*
sqrt(1/2)*a^2*x^3*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2 -
2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + 2*((a
^2*b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f - (a^5*b^2 - 4*a^6*c)*sqrt
((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^
4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*
b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4
*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^
7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)
d^2 - 2(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a
^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^
3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5
*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*
(a^6*c*f^4 + (b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d^4 - (b^5*c^2 - a*b^3*c^3 -
3*a^2*b*c^4)*d^3*e + 3*(a*b^4*c^2 - 2*a^2*b^2*c^3)*d^2*e^2 - (3*a^2*b^3*c^
2 - 5*a^3*b*c^3)*d*e^3 + (a^3*b^2*c^2 - a^4*c^3)*e^4 - (3*a^5*b*c*e - (3*a^
4*b^2*c - 4*a^5*c^2)*d)*f^3 + 3*(a^4*b^2*c*e^2 + (a^2*b^4*c - 3*a^3*b^2*c^2
+ 2*a^4*c^3)*d^2 - (2*a^3*b^3*c - 3*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 5*a*b^
4*c^2 + 9*a^2*b^2*c^3 - 4*a^3*c^4)*d^3 - 3*(a*b^5*c - 3*a^2*b^3*c^2 + 3*a^3
*b*c^3)*d^2*e + 3*(a^2*b^4*c - a^3*b^2*c^2)*d*e^2 - (a^3*b^3*c + a^4*b*c^2)
*e^3)*f)*x + sqrt(1/2)*((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3
+ 4*a^4*c^4)*d^3 - (3*a*b^7 - 21*a^2*b^5*c + 41*a^3*b^3*c^2 - 20*a^4*b*c^3)
*d^2*e + (3*a^2*b^6 - 18*a^3*b^4*c + 25*a^4*b^2*c^2 - 4*a^5*c^3)*d*e^2 - (a
^3*b^5 - 5*a^4*b^3*c + 4*a^5*b*c^2)*e^3 + (a^6*b^2 - 4*a^7*c)*f^3 + 3*((a^4
*b^4 - 5*a^5*b^2*c + 4*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*f^2 + ((3*a^2*
b^6 - 19*a^3*b^4*c + 31*a^4*b^2*c^2 - 12*a^5*c^3)*d^2 - 2*(3*a^3*b^5 - 16*a
^4*b^3*c + 16*a^5*b*c^2)*d*e + (3*a^4*b^4 - 13*a^5*b^2*c + 4*a^6*c^2)*e^2)*
f + ((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*d - (a^6*b^4 - 6*a^7*b^2*c + 8*
a^8*c^2)*e + (a^7*b^3 - 4*a^8*b*c)*f)*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^
3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c
^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^
4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3
+ 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*
d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*
c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4
*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2
- 4*a^11*c)))*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2 - 2*(
a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + 2*((a^2*
b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f - (a^5*b^2 - 4*a^6*c)*sqrt((a
^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 -
4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6
- 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^
3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b
*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^
2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*
b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c
+ 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^
3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 3*sqrt
(1/2)*a^2*x^3*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2 - 2*(a

```
*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + 2*((a^2*b
^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f - (a^5*b^2 - 4*a^6*c)*sqrt((a^
8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 -
4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6
- 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3
*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*
e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2
 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b
^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c
+ 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3
 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(a^6
*c*f^4 + (b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d^4 - (b^5*c^2 - a*b^3*c^3 - 3*a
^2*b*c^4)*d^3*e + 3*(a*b^4*c^2 - 2*a^2*b^2*c^3)*d^2*e^2 - (3*a^2*b^3*c^2 -
5*a^3*b*c^3)*d*e^3 + (a^3*b^2*c^2 - a^4*c^3)*e^4 - (3*a^5*b*c*e - (3*a^4*b^
2*c - 4*a^5*c^2)*d)*f^3 + 3*(a^4*b^2*c*e^2 + (a^2*b^4*c - 3*a^3*b^2*c^2 + 2
*a^4*c^3)*d^2 - (2*a^3*b^3*c - 3*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 5*a*b^4*c^
2 + 9*a^2*b^2*c^3 - 4*a^3*c^4)*d^3 - 3*(a*b^5*c - 3*a^2*b^3*c^2 + 3*a^3*b*c
^3)*d^2*e + 3*(a^2*b^4*c - a^3*b^2*c^2)*d*e^2 - (a^3*b^3*c + a^4*b*c^2)*e^3
)*f)*x - sqrt(1/2)*((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*
a^4*c^4)*d^3 - (3*a*b^7 - 21*a^2*b^5*c + 41*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2
*e + (3*a^2*b^6 - 18*a^3*b^4*c + 25*a^4*b^2*c^2 - 4*a^5*c^3)*d*e^2 - (a^3*b
^5 - 5*a^4*b^3*c + 4*a^5*b*c^2)*e^3 + (a^6*b^2 - 4*a^7*c)*f^3 + 3*((a^4*b^4
 - 5*a^5*b^2*c + 4*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*f^2 + ((3*a^2*b^6
- 19*a^3*b^4*c + 31*a^4*b^2*c^2 - 12*a^5*c^3)*d^2 - 2*(3*a^3*b^5 - 16*a^4*b
^3*c + 16*a^5*b*c^2)*d*e + (3*a^4*b^4 - 13*a^5*b^2*c + 4*a^6*c^2)*e^2)*f +
((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*d - (a^6*b^4 - 6*a^7*b^2*c + 8*a^8*
c^2)*e + (a^7*b^3 - 4*a^8*b*c)*f)*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^
3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 -
 a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^
4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*
((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e
+ (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2
- a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4
 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4
*a^11*c)))*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2 - 2*(a*b^
4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + 2*((a^2*b^3
- 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f - (a^5*b^2 - 4*a^6*c)*sqrt((a^8*f
^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(
a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 1
2*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c
+ 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e -
 (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2 -
2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6
- 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c + 5
*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3 -
a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 6*(b*d - a
*e)*x^2 + 2*a*d)/(a^2*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.60 $\int \frac{d+ex^2+fx^4}{x^6\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=329

$$-\frac{-abe-a(cd-af)+b^2d}{a^3x}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}}-abe-a(cd-af)+b^2d\right)}{\sqrt{2}a^3\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\cdots}{\sqrt{\sqrt{b^2-\cdots}}}\right)\cdots}{\cdots}$$

```
[Out] -d/(5*a*x^5) + (b*d - a*e)/(3*a^2*x^3) - (b^2*d - a*b*e - a*(c*d - a*f))/(a
^3*x) - (Sqrt[c]*(b^2*d - a*b*e - a*(c*d - a*f) + (b^3*d - a*b^2*e + 2*a^2*
c*e - a*b*(3*c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt
[c]*(b^2*d - a*b*e - a*(c*d - a*f) - (b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*
c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.94197, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1664, 1166, 205}

$$-\frac{-abe-a(cd-af)+b^2d}{a^3x}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}}-abe-a(cd-af)+b^2d\right)}{\sqrt{2}a^3\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\cdots}{\sqrt{\sqrt{b^2-\cdots}}}\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -d/(5*a*x^5) + (b*d - a*e)/(3*a^2*x^3) - (b^2*d - a*b*e - a*(c*d - a*f))/(a
^3*x) - (Sqrt[c]*(b^2*d - a*b*e - a*(c*d - a*f) + (b^3*d - a*b^2*e + 2*a^2*
c*e - a*b*(3*c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt
[c]*(b^2*d - a*b*e - a*(c*d - a*f) - (b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*
c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^6\left(a+bx^2+cx^4\right)}dx = \int \left(\frac{d}{ax^6}+\frac{-bd+ae}{a^2x^4}+\frac{b^2d-abe-a(cd-af)}{a^3x^2}+\frac{-b^3d+ab^2e-a^2ce+ab(2cd-af)-c\left(b^2\right.}{a^3\left(a+bx^2+cx^4\right)}\right.$$

$$= -\frac{d}{5ax^5}+\frac{bd-ae}{3a^2x^3}-\frac{b^2d-abe-a(cd-af)}{a^3x}+\frac{\int \frac{-b^3d+ab^2e-a^2ce+ab(2cd-af)-c\left(b^2d-abe-a(cd-af)\right)x^2}{a+bx^2+cx^4}}{a^3}$$

$$= -\frac{d}{5ax^5}+\frac{bd-ae}{3a^2x^3}-\frac{b^2d-abe-a(cd-af)}{a^3x}-\frac{\left(c\left(b^2d-abe-a(cd-af)-\frac{b^3d-ab^2e+2a^2ce-ab(}{\sqrt{b^2-4ac}}\right.\right.}{2a^3}$$

$$= -\frac{d}{5ax^5}+\frac{bd-ae}{3a^2x^3}-\frac{b^2d-abe-a(cd-af)}{a^3x}-\frac{\sqrt{c}\left(b^2d-abe-a(cd-af)+\frac{b^3d-ab^2e+2a^2ce-ab}{\sqrt{b^2-4ac}}\right.}{\sqrt{2}a^3\sqrt{b-\sqrt{b^2-4}}}$$

Mathematica [A] time = 0.604115, size = 394, normalized size = 1.2

$$\frac{-\frac{6a^2d}{x^5}+\frac{30\left(abe+a(cd-af)+b^2(-d)\right)}{x}-\frac{15\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(ab\left(-e\sqrt{b^2-4ac}+af-3cd\right)+a\left(-cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+2ace\right)+b^2\left(d\sqrt{b^2-4ac}-ae\right)+b^3\right.}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}}{30a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-6*a^2*d)/x^5 + (10*a*(b*d - a*e))/x^3 + (30*(-(b^2*d) + a*b*e + a*(c*d -
 a*f)))/x - (15*Sqrt[2]*Sqrt[c]*(b^3*d + b^2*(Sqrt[b^2 - 4*a*c]*d - a*e) +
a*b*(-3*c*d - Sqrt[b^2 - 4*a*c]*e + a*f) + a*(-(c*Sqrt[b^2 - 4*a*c]*d) + 2*
a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (15*Sqrt[2]
*Sqrt[c]*(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*b*(-3*c*d + Sqrt[b^2
- 4*a*c]*e + a*f) + a*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e - a*Sqrt[b^2 - 4*a*c
]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(30*a^3)
```

Maple [B] time = 0.036, size = 1121, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/a^3*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d+1/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*e-1/2/a^2*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(
1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*e+1/2/a^3*c*2^(1/2)/(((-4*a*c+b^2)
```

```
^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^
2*d+1/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arc
tanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e+1/2/a^2*c*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*b*e-1/a^3/x*b^2*d+1/3/a^2/x^3*b*d+1/a^2/x*b*e+1/a^2/x*c*d-3/2/a^2*c^
2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2
^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d+1/2/a^3*c/(-4*a*c+b^2)^(1/2)*2
^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^
(1/2)-b)*c)^(1/2))*b^3*d+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*f-
1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e-3/2/a^2*c^2/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d+1/2/a^3*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))*b^3*d+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*f-1/3/a/x^3*e-
1/a/x*f+1/2/a*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2
)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f-1/2/a^2*c^2*2^(1/2)/(((-4*a*c+b^2)^(1
/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d-1/2
/a*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*f+1/2/a^2*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d-1/5*d/a/x^5-1/2/
a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c
*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*e
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 102.526, size = 31905, normalized size = 96.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/30*(15*sqrt(1/2)*a^3*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^
3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a
^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + (a^4*b^3 - 3*a^5*b*c)*f^2 + 2*((a
^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d - (a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)
*e)*f + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62
*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 -
 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c
^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3
+ 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5
*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^
6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^
```

2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5
- 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6
*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c +
21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*
c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*
a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48
*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^
6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b
^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^
2 - 4*a^8*c))*log(-2*((b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^3*c^7)*d^4
- (b^7*c^3 - 3*a*b^5*c^4 - 2*a^2*b^3*c^5 + 5*a^3*b*c^6)*d^3*e + 3*(a*b^6*c
^3 - 4*a^2*b^4*c^4 + 3*a^3*b^2*c^5)*d^2*e^2 - (3*a^2*b^5*c^3 - 11*a^3*b^3*c
^4 + 7*a^4*b*c^5)*d*e^3 + (a^3*b^4*c^3 - 3*a^4*b^2*c^4 + a^5*c^5)*e^4 + (a^
6*b^2*c^2 - a^7*c^3)*f^4 + ((3*a^4*b^4*c^2 - 9*a^5*b^2*c^3 + 4*a^6*c^4)*d -
(3*a^5*b^3*c^2 - 5*a^6*b*c^3)*e)*f^3 + 3*((a^2*b^6*c^2 - 5*a^3*b^4*c^3 + 7
*a^4*b^2*c^4 - 2*a^5*c^5)*d^2 - (2*a^3*b^5*c^2 - 7*a^4*b^3*c^3 + 5*a^5*b*c^
4)*d*e + (a^4*b^4*c^2 - 2*a^5*b^2*c^3)*e^2)*f^2 + ((b^8*c^2 - 7*a*b^6*c^3 +
18*a^2*b^4*c^4 - 19*a^3*b^2*c^5 + 4*a^4*c^6)*d^3 - 3*(a*b^7*c^2 - 5*a^2*b^
5*c^3 + 8*a^3*b^3*c^4 - 5*a^4*b*c^5)*d^2*e + 3*(a^2*b^6*c^2 - 3*a^3*b^4*c^3
+ a^4*b^2*c^4)*d*e^2 - (a^3*b^5*c^2 - a^4*b^3*c^3 - 3*a^5*b*c^4)*e^3)*f)*x
+ sqrt(1/2)*((b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 54*a^4
*b^3*c^4 - 8*a^5*b*c^5)*d^3 - (3*a*b^10 - 30*a^2*b^8*c + 105*a^3*b^6*c^2 -
151*a^4*b^4*c^3 + 77*a^5*b^2*c^4 - 4*a^6*c^5)*d^2*e + (3*a^2*b^9 - 27*a^3*b
^7*c + 81*a^4*b^5*c^2 - 92*a^5*b^3*c^3 + 32*a^6*b*c^4)*d*e^2 - (a^3*b^8 - 8
*a^4*b^6*c + 20*a^5*b^4*c^2 - 17*a^6*b^2*c^3 + 4*a^7*c^4)*e^3 + (a^6*b^5 -
5*a^7*b^3*c + 4*a^8*b*c^2)*f^3 + ((3*a^4*b^7 - 21*a^5*b^5*c + 40*a^6*b^3*c^
2 - 16*a^7*b*c^3)*d - (3*a^5*b^6 - 18*a^6*b^4*c + 25*a^7*b^2*c^2 - 4*a^8*c^
3)*e)*f^2 + ((3*a^2*b^9 - 27*a^3*b^7*c + 80*a^4*b^5*c^2 - 85*a^5*b^3*c^3 +
20*a^6*b*c^4)*d^2 - 2*(3*a^3*b^8 - 24*a^4*b^6*c + 59*a^5*b^4*c^2 - 45*a^6*b
^2*c^3 + 4*a^7*c^4)*d*e + (3*a^4*b^7 - 21*a^5*b^5*c + 41*a^6*b^3*c^2 - 20*a
^7*b*c^3)*e^2)*f - ((a^7*b^6 - 8*a^8*b^4*c + 18*a^9*b^2*c^2 - 8*a^10*c^3)*d
- (a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*e + (a^9*b^4 - 6*a^10*b^2*c + 8*
a^11*c^2)*f)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 +
46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c +
29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(
3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^
4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b
^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a
^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^
6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c +
2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a
^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2
- 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*
e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9
*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 -
39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^
4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^
3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)))*sqrt(-((b^7 - 7*a*b^5*
c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*
c^2 - 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + (a^4*b^3
- 3*a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d - (a^3*b^4 -
4*a^4*b^2*c + 2*a^5*c^2)*e)*f + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^1
0*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a
^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 2
2*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*
b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 -
7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*
b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^
4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2

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 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^
8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*
a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 1
2*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8
*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^
3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e
 + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d
*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*
b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))) - 15*sqrt(1/2)*a^3*x^5*sqrt(-((b^7
- 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c +
9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2
+ (a^4*b^3 - 3*a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d -
(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)*e)*f + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12
- 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b
^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b
^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c
 + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*
(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e
^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4
 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a
^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*
((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d
^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a
^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 -
 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d
^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*
c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 +
a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)
*f)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(-2*((b^6*c^4 - 5*a*b^4
*c^5 + 6*a^2*b^2*c^6 - a^3*c^7)*d^4 - (b^7*c^3 - 3*a*b^5*c^4 - 2*a^2*b^3*c^
5 + 5*a^3*b*c^6)*d^3*e + 3*(a*b^6*c^3 - 4*a^2*b^4*c^4 + 3*a^3*b^2*c^5)*d^2*
e^2 - (3*a^2*b^5*c^3 - 11*a^3*b^3*c^4 + 7*a^4*b*c^5)*d*e^3 + (a^3*b^4*c^3 -
 3*a^4*b^2*c^4 + a^5*c^5)*e^4 + (a^6*b^2*c^2 - a^7*c^3)*f^4 + ((3*a^4*b^4*c
^2 - 9*a^5*b^2*c^3 + 4*a^6*c^4)*d - (3*a^5*b^3*c^2 - 5*a^6*b*c^3)*e)*f^3 +
3*((a^2*b^6*c^2 - 5*a^3*b^4*c^3 + 7*a^4*b^2*c^4 - 2*a^5*c^5)*d^2 - (2*a^3*b
^5*c^2 - 7*a^4*b^3*c^3 + 5*a^5*b*c^4)*d*e + (a^4*b^4*c^2 - 2*a^5*b^2*c^3)*e
^2)*f^2 + ((b^8*c^2 - 7*a*b^6*c^3 + 18*a^2*b^4*c^4 - 19*a^3*b^2*c^5 + 4*a^4
*c^6)*d^3 - 3*(a*b^7*c^2 - 5*a^2*b^5*c^3 + 8*a^3*b^3*c^4 - 5*a^4*b*c^5)*d^2
*e + 3*(a^2*b^6*c^2 - 3*a^3*b^4*c^3 + a^4*b^2*c^4)*d*e^2 - (a^3*b^5*c^2 - a
^4*b^3*c^3 - 3*a^5*b*c^4)*e^3)*f)*x - sqrt(1/2)*((b^11 - 11*a*b^9*c + 44*a^
2*b^7*c^2 - 77*a^3*b^5*c^3 + 54*a^4*b^3*c^4 - 8*a^5*b*c^5)*d^3 - (3*a*b^10
- 30*a^2*b^8*c + 105*a^3*b^6*c^2 - 151*a^4*b^4*c^3 + 77*a^5*b^2*c^4 - 4*a^6
*c^5)*d^2*e + (3*a^2*b^9 - 27*a^3*b^7*c + 81*a^4*b^5*c^2 - 92*a^5*b^3*c^3 +
 32*a^6*b*c^4)*d*e^2 - (a^3*b^8 - 8*a^4*b^6*c + 20*a^5*b^4*c^2 - 17*a^6*b^2
*c^3 + 4*a^7*c^4)*e^3 + (a^6*b^5 - 5*a^7*b^3*c + 4*a^8*b*c^2)*f^3 + ((3*a^4
*b^7 - 21*a^5*b^5*c + 40*a^6*b^3*c^2 - 16*a^7*b*c^3)*d - (3*a^5*b^6 - 18*a^
6*b^4*c + 25*a^7*b^2*c^2 - 4*a^8*c^3)*e)*f^2 + ((3*a^2*b^9 - 27*a^3*b^7*c +
 80*a^4*b^5*c^2 - 85*a^5*b^3*c^3 + 20*a^6*b*c^4)*d^2 - 2*(3*a^3*b^8 - 24*a^
4*b^6*c + 59*a^5*b^4*c^2 - 45*a^6*b^2*c^3 + 4*a^7*c^4)*d*e + (3*a^4*b^7 - 2
1*a^5*b^5*c + 41*a^6*b^3*c^2 - 20*a^7*b*c^3)*e^2)*f - ((a^7*b^6 - 8*a^8*b^4
*c + 18*a^9*b^2*c^2 - 8*a^10*c^3)*d - (a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^
2)*e + (a^9*b^4 - 6*a^10*b^2*c + 8*a^11*c^2)*f)*sqrt(((b^12 - 10*a*b^10*c +
 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^
6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5
*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c
^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^
4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 -
 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2
*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^
```

9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 1
8*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b
^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7
*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c +
22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9
 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3
*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2
- (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 -
 4*a^15*c)))*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 -
2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*
b^3*c + 5*a^4*b*c^2)*e^2 + (a^4*b^3 - 3*a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*
b^3*c + 5*a^4*b*c^2)*d - (a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)*e)*f + (a^7*b^
2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 +
46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c +
29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(
3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^
4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b
^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a
^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^
6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c +
 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a
^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2
- 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*
e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9
*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 -
39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^
4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^
3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c)))
+ 15*sqrt(1/2)*a^3*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c
^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a^2*b^
5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + (a^4*b^3 - 3*a^5*b*c)*f^2 + 2*((a^2*b^
5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d - (a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)*e)*f
 - (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*
b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^
2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d
^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*
a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2
- 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4
*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^
4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a
^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*
c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^
7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 -
 a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b
^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*
b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c +
 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c
+ 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4
*a^8*c))*log(-2*((b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^3*c^7)*d^4 - (b
^7*c^3 - 3*a*b^5*c^4 - 2*a^2*b^3*c^5 + 5*a^3*b*c^6)*d^3*e + 3*(a*b^6*c^3 -
4*a^2*b^4*c^4 + 3*a^3*b^2*c^5)*d^2*e^2 - (3*a^2*b^5*c^3 - 11*a^3*b^3*c^4 +
7*a^4*b*c^5)*d*e^3 + (a^3*b^4*c^3 - 3*a^4*b^2*c^4 + a^5*c^5)*e^4 + (a^6*b^2
*c^2 - a^7*c^3)*f^4 + ((3*a^4*b^4*c^2 - 9*a^5*b^2*c^3 + 4*a^6*c^4)*d - (3*a
^5*b^3*c^2 - 5*a^6*b*c^3)*e)*f^3 + 3*((a^2*b^6*c^2 - 5*a^3*b^4*c^3 + 7*a^4*
b^2*c^4 - 2*a^5*c^5)*d^2 - (2*a^3*b^5*c^2 - 7*a^4*b^3*c^3 + 5*a^5*b*c^4)*d*
e + (a^4*b^4*c^2 - 2*a^5*b^2*c^3)*e^2)*f^2 + ((b^8*c^2 - 7*a*b^6*c^3 + 18*a
^2*b^4*c^4 - 19*a^3*b^2*c^5 + 4*a^4*c^6)*d^3 - 3*(a*b^7*c^2 - 5*a^2*b^5*c^3
 + 8*a^3*b^3*c^4 - 5*a^4*b*c^5)*d^2*e + 3*(a^2*b^6*c^2 - 3*a^3*b^4*c^3 + a^
4*b^2*c^4)*d*e^2 - (a^3*b^5*c^2 - a^4*b^3*c^3 - 3*a^5*b*c^4)*e^3)*f)*x + sq

rt(1/2)*((b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 54*a^4*b^3*
c^4 - 8*a^5*b*c^5)*d^3 - (3*a*b^10 - 30*a^2*b^8*c + 105*a^3*b^6*c^2 - 151*a
^4*b^4*c^3 + 77*a^5*b^2*c^4 - 4*a^6*c^5)*d^2*e + (3*a^2*b^9 - 27*a^3*b^7*c
+ 81*a^4*b^5*c^2 - 92*a^5*b^3*c^3 + 32*a^6*b*c^4)*d*e^2 - (a^3*b^8 - 8*a^4*
b^6*c + 20*a^5*b^4*c^2 - 17*a^6*b^2*c^3 + 4*a^7*c^4)*e^3 + (a^6*b^5 - 5*a^7
*b^3*c + 4*a^8*b*c^2)*f^3 + ((3*a^4*b^7 - 21*a^5*b^5*c + 40*a^6*b^3*c^2 - 1
6*a^7*b*c^3)*d - (3*a^5*b^6 - 18*a^6*b^4*c + 25*a^7*b^2*c^2 - 4*a^8*c^3)*e)
*f^2 + ((3*a^2*b^9 - 27*a^3*b^7*c + 80*a^4*b^5*c^2 - 85*a^5*b^3*c^3 + 20*a^
6*b*c^4)*d^2 - 2*(3*a^3*b^8 - 24*a^4*b^6*c + 59*a^5*b^4*c^2 - 45*a^6*b^2*c^
3 + 4*a^7*c^4)*d*e + (3*a^4*b^7 - 21*a^5*b^5*c + 41*a^6*b^3*c^2 - 20*a^7*b*
c^3)*e^2)*f + ((a^7*b^6 - 8*a^8*b^4*c + 18*a^9*b^2*c^2 - 8*a^10*c^3)*d - (a
^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*e + (a^9*b^4 - 6*a^10*b^2*c + 8*a^11*
c^2)*f)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^
4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^
3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2
*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a
^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^
3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^
2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6
 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^
9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^
2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a
^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*
f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*
b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^
6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2
 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2
 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)))*sqrt(-((b^7 - 7*a*b^5*c + 1
4*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 -
 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + (a^4*b^3 - 3*
a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d - (a^3*b^4 - 4*a^
4*b^2*c + 2*a^5*c^2)*e)*f - (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c +
 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^
6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5
*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c
^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^
4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 -
 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2
*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^
9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 1
8*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b
^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7
*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c +
22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9
 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3
*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2
- (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 -
 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))) - 15*sqrt(1/2)*a^3*x^5*sqrt(-((b^7 - 7*a
*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3
*b^2*c^2 - 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + (a^
4*b^3 - 3*a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d - (a^3*
b^4 - 4*a^4*b^2*c + 2*a^5*c^2)*e)*f - (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*
a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^
5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^
3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66
*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*
b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 +
(a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a
^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^

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2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a
^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 -
2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^
6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^
3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 -
(3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*
d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c
^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(
a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(-2*((b^6*c^4 - 5*a*b^4*c^5
+ 6*a^2*b^2*c^6 - a^3*c^7)*d^4 - (b^7*c^3 - 3*a*b^5*c^4 - 2*a^2*b^3*c^5 + 5
*a^3*b*c^6)*d^3*e + 3*(a*b^6*c^3 - 4*a^2*b^4*c^4 + 3*a^3*b^2*c^5)*d^2*e^2 -
 (3*a^2*b^5*c^3 - 11*a^3*b^3*c^4 + 7*a^4*b*c^5)*d*e^3 + (a^3*b^4*c^3 - 3*a^
4*b^2*c^4 + a^5*c^5)*e^4 + (a^6*b^2*c^2 - a^7*c^3)*f^4 + ((3*a^4*b^4*c^2 -
9*a^5*b^2*c^3 + 4*a^6*c^4)*d - (3*a^5*b^3*c^2 - 5*a^6*b*c^3)*e)*f^3 + 3*((a
^2*b^6*c^2 - 5*a^3*b^4*c^3 + 7*a^4*b^2*c^4 - 2*a^5*c^5)*d^2 - (2*a^3*b^5*c^
2 - 7*a^4*b^3*c^3 + 5*a^5*b*c^4)*d*e + (a^4*b^4*c^2 - 2*a^5*b^2*c^3)*e^2)*f
^2 + ((b^8*c^2 - 7*a*b^6*c^3 + 18*a^2*b^4*c^4 - 19*a^3*b^2*c^5 + 4*a^4*c^6)
*d^3 - 3*(a*b^7*c^2 - 5*a^2*b^5*c^3 + 8*a^3*b^3*c^4 - 5*a^4*b*c^5)*d^2*e +
3*(a^2*b^6*c^2 - 3*a^3*b^4*c^3 + a^4*b^2*c^4)*d*e^2 - (a^3*b^5*c^2 - a^4*b^
3*c^3 - 3*a^5*b*c^4)*e^3)*f)*x - sqrt(1/2)*((b^11 - 11*a*b^9*c + 44*a^2*b^7
*c^2 - 77*a^3*b^5*c^3 + 54*a^4*b^3*c^4 - 8*a^5*b*c^5)*d^3 - (3*a*b^10 - 30*
a^2*b^8*c + 105*a^3*b^6*c^2 - 151*a^4*b^4*c^3 + 77*a^5*b^2*c^4 - 4*a^6*c^5)
*d^2*e + (3*a^2*b^9 - 27*a^3*b^7*c + 81*a^4*b^5*c^2 - 92*a^5*b^3*c^3 + 32*a
^6*b*c^4)*d*e^2 - (a^3*b^8 - 8*a^4*b^6*c + 20*a^5*b^4*c^2 - 17*a^6*b^2*c^3
+ 4*a^7*c^4)*e^3 + (a^6*b^5 - 5*a^7*b^3*c + 4*a^8*b*c^2)*f^3 + ((3*a^4*b^7
- 21*a^5*b^5*c + 40*a^6*b^3*c^2 - 16*a^7*b*c^3)*d - (3*a^5*b^6 - 18*a^6*b^4
*c + 25*a^7*b^2*c^2 - 4*a^8*c^3)*e)*f^2 + ((3*a^2*b^9 - 27*a^3*b^7*c + 80*a
^4*b^5*c^2 - 85*a^5*b^3*c^3 + 20*a^6*b*c^4)*d^2 - 2*(3*a^3*b^8 - 24*a^4*b^6
*c + 59*a^5*b^4*c^2 - 45*a^6*b^2*c^3 + 4*a^7*c^4)*d*e + (3*a^4*b^7 - 21*a^5
*b^5*c + 41*a^6*b^3*c^2 - 20*a^7*b*c^3)*e^2)*f + ((a^7*b^6 - 8*a^8*b^4*c +
18*a^9*b^2*c^2 - 8*a^10*c^3)*d - (a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*e
+ (a^9*b^4 - 6*a^10*b^2*c + 8*a^11*c^2)*f)*sqrt(((b^12 - 10*a*b^10*c + 37*a
^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^
4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*
c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 -
72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7
*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^
5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*
b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3
)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5
*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 -
15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*
c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^
4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21
*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*
b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^
5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^
15*c)))*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 - 2*(a*
b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*b^3*c
 + 5*a^4*b*c^2)*e^2 + (a^4*b^3 - 3*a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*b^3*c
 + 5*a^4*b*c^2)*d - (a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)*e)*f - (a^7*b^2 - 4
*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^
4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^
3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2
*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a
^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^
3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^
2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6
 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^
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9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^
2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a
^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*
f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*
b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^
6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2
 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2
 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))) - 30*
(a*b*e - a^2*f - (b^2 - a*c)*d)*x^4 + 6*a^2*d - 10*(a*b*d - a^2*e)*x^2)/(a^
3*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**6/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.61 $\int \frac{x^7\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=320

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2c^3e-12ab^2c^2e-b^3c(cd-20af)+6abc^2(cd-5af)+2b^4ce-3b^5f\right)}{2c^4\left(b^2-4ac\right)^{3/2}}+\frac{x^6\left(x^2\left(-\left(-2acf+b^2f\right.\right.\right.}{2c\left(b^2-\right.}$$

```
[Out] ((2*b^2*c*e - 6*a*c^2*e - 3*b^3*f - b*c*(c*d - 11*a*f))*x^2)/(2*c^3*(b^2 -
4*a*c)) + ((4*c^2*d + 3*b^2*f - 2*c*(b*e + 4*a*f))*x^4)/(4*c^2*(b^2 - 4*a*c
)) + (x^6*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^
2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*b^4*c*e - 12*a*b^2*c^2*e
+ 12*a^2*c^3*e - 3*b^5*f - b^3*c*(c*d - 20*a*f) + 6*a*b*c^2*(c*d - 5*a*f))*
ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*(b^2 - 4*a*c)^(3/2)) + ((c
^2*d + 3*b^2*f - 2*c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Rubi [A] time = 1.23253, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1663, 1644, 800, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2c^3e-12ab^2c^2e-b^3c(cd-20af)+6abc^2(cd-5af)+2b^4ce-3b^5f\right)}{2c^4\left(b^2-4ac\right)^{3/2}}+\frac{x^6\left(x^2\left(-\left(-2acf+b^2f\right.\right.\right.}{2c\left(b^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] ((2*b^2*c*e - 6*a*c^2*e - 3*b^3*f - b*c*(c*d - 11*a*f))*x^2)/(2*c^3*(b^2 -
4*a*c)) + ((4*c^2*d + 3*b^2*f - 2*c*(b*e + 4*a*f))*x^4)/(4*c^2*(b^2 - 4*a*c
)) + (x^6*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^
2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*b^4*c*e - 12*a*b^2*c^2*e
+ 12*a^2*c^3*e - 3*b^5*f - b^3*c*(c*d - 20*a*f) + 6*a*b*c^2*(c*d - 5*a*f))*
ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*(b^2 - 4*a*c)^(3/2)) + ((c
^2*d + 3*b^2*f - 2*c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^4)
```


Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
```

```
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^7\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{x^6\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(3\left(2ae-\frac{b(cd+af)}{c}\right)-\frac{\left(4c^2d-2bc\right.}{}\right.}{a+bx+cx^2}\right.}{2\left(b^2-4ac\right)}$$

$$= \frac{x^6\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int\left(-\frac{2b^2ce-6ac^2e-3b^3f-bc(cd-}{c^3}\right.\right.}{}$$

$$= \frac{\left(2b^2ce-6ac^2e-3b^3f-bc(cd-11af)\right)x^2}{2c^3\left(b^2-4ac\right)} + \frac{\left(4c^2d+3b^2f-2c(be+4af)\right)x^4}{4c^2\left(b^2-4ac\right)} + \frac{x^6\left(2ace\right.}{}$$

$$= \frac{\left(2b^2ce-6ac^2e-3b^3f-bc(cd-11af)\right)x^2}{2c^3\left(b^2-4ac\right)} + \frac{\left(4c^2d+3b^2f-2c(be+4af)\right)x^4}{4c^2\left(b^2-4ac\right)} + \frac{x^6\left(2ace\right.}{}$$

$$= \frac{\left(2b^2ce-6ac^2e-3b^3f-bc(cd-11af)\right)x^2}{2c^3\left(b^2-4ac\right)} + \frac{\left(4c^2d+3b^2f-2c(be+4af)\right)x^4}{4c^2\left(b^2-4ac\right)} + \frac{x^6\left(2ace\right.}{}$$

$$= \frac{\left(2b^2ce-6ac^2e-3b^3f-bc(cd-11af)\right)x^2}{2c^3\left(b^2-4ac\right)} + \frac{\left(4c^2d+3b^2f-2c(be+4af)\right)x^4}{4c^2\left(b^2-4ac\right)} + \frac{x^6\left(2ace\right.}{}$$

Mathematica [A] time = 0.551008, size = 309, normalized size = 0.97

$$\frac{\frac{2\left(a^2c\left(-4b^2f+bc\left(3e+5fx^2\right)-2c^2\left(d+ex^2\right)\right)+2a^3c^2f+ab\left(-b^2c\left(e+5fx^2\right)+b^3f+bc^2\left(d+4ex^2\right)-3c^3dx^2\right)+b^3x^2\left(b^2f-bce+c^2d\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(-12a^2c^3e+12a\right.}{}}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*c*(c*e - 2*b*f)*x^2 + c^2*f*x^4 + (2*(2*a^3*c^2*f + b^3*(c^2*d - b*c*e +
 b^2*f)*x^2 + a*b*(b^3*f - 3*c^3*d*x^2 + b*c^2*(d + 4*e*x^2) - b^2*c*(e + 5
*f*x^2)) + a^2*c*(-4*b^2*f - 2*c^2*(d + e*x^2) + b*c*(3*e + 5*f*x^2))))/((b
^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(-2*b^4*c*e + 12*a*b^2*c^2*e - 12*a^2
*c^3*e + 3*b^5*f + b^3*c*(c*d - 20*a*f) + 6*a*b*c^2*(-(c*d) + 5*a*f))*ArcTa
n[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (c^2*d + 3*b^2*
f - 2*c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Maple [B] time = 0.018, size = 1167, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/c^3*b*f*x^2+5/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b^3*f-5/2/c^2/(c*x
^4+b*x^2+a)/(4*a*c-b^2)*x^2*a^2*b*f-2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a
```

```
*b^2*e+3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b*d-1/c^2/(c*x^4+b*x^2+a)*a^
3/(4*a*c-b^2)*f+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*d+1/2/c^3/(4*a*c-b^2)*l
n(c*x^4+b*x^2+a)*b^3*e-1/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^
2)^(1/2))*b^4*e-2/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a^2*f+1/c/(4*a*c-b^2)*l
n(c*x^4+b*x^2+a)*a*d-3/4/c^4/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^4*f-1/4/c^2/(4
*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2*d-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/
(4*a*c-b^2)^(1/2))*a^2*e+3/2/c^4/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2))*b^5*f+1/2/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2
)^(1/2))*b^3*d+1/4/c^2*f*x^4+1/2/c^2*e*x^2+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*
x^2*a^2*e-1/2/c^4/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b^5*f+1/2/c^3/(c*x^4+b*x^
2+a)/(4*a*c-b^2)*x^2*b^4*e+2/c^3/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*b^2*f+1/2/
c^3/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b^3*e-1/2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)
*x^2*b^3*d-3/2/c^2/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*b*e-1/2/c^4/(c*x^4+b*x^2
+a)*a/(4*a*c-b^2)*b^4*f-1/2/c^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b^2*d+15/c^2/
(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*b*f+6/c^2/(4*a*
c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^2*e-3/c/(4*a*c-b^2)^
(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*d-2/c^2/(4*a*c-b^2)*ln(c*x^
4+b*x^2+a)*a*b*e+7/2/c^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a*b^2*f-10/c^3/(4*a*
c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^3*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 3.46226, size = 4393, normalized size = 13.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] [1/4*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*f*x^8 + (2*(b^4*c^3 - 8*a*b^2*c^
4 + 16*a^2*c^5)*e - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*f)*x^6 + (2*(b
^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b
^2*c^3 - 16*a^3*c^4)*f)*x^4 + 2*((b^5*c^2 - 7*a*b^3*c^3 + 12*a^2*b*c^4)*d -
 (b^6*c - 9*a*b^4*c^2 + 26*a^2*b^2*c^3 - 24*a^3*c^4)*e + (b^7 - 11*a*b^5*c
+ 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*f)*x^2 - (((b^3*c^3 - 6*a*b*c^4)*d - 2*(b^
4*c^2 - 6*a*b^2*c^3 + 6*a^2*c^4)*e + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3
)*f)*x^4 + ((b^4*c^2 - 6*a*b^2*c^3)*d - 2*(b^5*c - 6*a*b^3*c^2 + 6*a^2*b*c^
3)*e + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*f)*x^2 + (a*b^3*c^2 - 6*a^2*b*
c^3)*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (3*a*b^5 - 20*a^2*b^3*
c + 30*a^3*b*c^2)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + 2*(a*b^4*c^2
 - 6*a^2*b^2*c^3 + 8*a^3*c^4)*d - 2*(a*b^5*c - 7*a^2*b^3*c^2 + 12*a^3*b*c^3
)*e + 2*(a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*f + (((b^4*c^3 -
 8*a*b^2*c^4 + 16*a^2*c^5)*d - 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e +
 (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*f)*x^4 + ((b^5*c^2
- 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*
```

```
e + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*f)*x^2 + (a*b^4*c^
2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c
^3)*e + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*f)*log(c*x^4
 + b*x^2 + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*c^5 - 8*a*b^2
*c^6 + 16*a^2*c^7)*x^4 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^2), 1/4*(
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*f*x^8 + (2*(b^4*c^3 - 8*a*b^2*c^4 + 16
*a^2*c^5)*e - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*f)*x^6 + (2*(b^5*c^2
 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3
 - 16*a^3*c^4)*f)*x^4 + 2*((b^5*c^2 - 7*a*b^3*c^3 + 12*a^2*b*c^4)*d - (b^6*
c - 9*a*b^4*c^2 + 26*a^2*b^2*c^3 - 24*a^3*c^4)*e + (b^7 - 11*a*b^5*c + 41*a
^2*b^3*c^2 - 52*a^3*b*c^3)*f)*x^2 + 2*(((b^3*c^3 - 6*a*b*c^4)*d - 2*(b^4*c^
2 - 6*a*b^2*c^3 + 6*a^2*c^4)*e + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*f)
*x^4 + ((b^4*c^2 - 6*a*b^2*c^3)*d - 2*(b^5*c - 6*a*b^3*c^2 + 6*a^2*b*c^3)*e
 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*f)*x^2 + (a*b^3*c^2 - 6*a^2*b*c^3)
*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (3*a*b^5 - 20*a^2*b^3*c +
30*a^3*b*c^2)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c
)/(b^2 - 4*a*c)) + 2*(a*b^4*c^2 - 6*a^2*b^2*c^3 + 8*a^3*c^4)*d - 2*(a*b^5*c
 - 7*a^2*b^3*c^2 + 12*a^3*b*c^3)*e + 2*(a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^
2 - 8*a^4*c^3)*f + (((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 2*(b^5*c^2 -
8*a*b^3*c^3 + 16*a^2*b*c^4)*e + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 -
32*a^3*c^4)*f)*x^4 + ((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(b^6*c -
 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*e + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 3
2*a^3*b*c^3)*f)*x^2 + (a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d - 2*(a*b^5
*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2
*c^2 - 32*a^4*c^3)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 +
16*a^3*c^6 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + (b^5*c^4 - 8*a*b^3*
c^5 + 16*a^2*b*c^6)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

Giac [A] time = 19.8252, size = 572, normalized size = 1.79

$$-\frac{\left(b^3c^2d - 6\,abc^3d + 3\,b^5f - 20\,ab^3cf + 30\,a^2bc^2f - 2\,b^4ce + 12\,ab^2c^2e - 12\,a^2c^3e\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(b^2c^4 - 4\,ac^5\right)\sqrt{-b^2+4\,ac}} - \frac{b^2c^3dx^4 - 4\,ac}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] -1/2*(b^3*c^2*d - 6*a*b*c^3*d + 3*b^5*f - 20*a*b^3*c*f + 30*a^2*b*c^2*f - 2
*b^4*c*e + 12*a*b^2*c^2*e - 12*a^2*c^3*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 +
4*a*c))/((b^2*c^4 - 4*a*c^5)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c^3*d*x^4 - 4*a
*c^4*d*x^4 + 3*b^4*c*f*x^4 - 14*a*b^2*c^2*f*x^4 + 8*a^2*c^3*f*x^4 - 2*b^3*c
^2*x^4*e + 8*a*b*c^3*x^4*e - b^3*c^2*d*x^2 + 2*a*b*c^3*d*x^2 + b^5*f*x^2 -
4*a*b^3*c*f*x^2 - 2*a^2*b*c^2*f*x^2 + 4*a^2*c^3*x^2*e - a*b^2*c^2*d + a*b^4
```

```
*f - 6*a^2*b^2*c*f + 4*a^3*c^2*f + 2*a^2*b*c^2*e)/((b^2*c^4 - 4*a*c^5)*(c*x
^4 + b*x^2 + a)) + 1/4*(c^2*d + 3*b^2*f - 2*a*c*f - 2*b*c*e)*log(c*x^4 + b*
x^2 + a)/c^4 + 1/4*(c^2*f*x^4 - 4*b*c*f*x^2 + 2*c^2*x^2*e)/c^4
```

3.62 $\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=236

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2c^2f-2ac\left(6b^2f-3bce+2c^2d\right)+b^3(-(ce-2bf))\right)}{2c^3\left(b^2-4ac\right)^{3/2}}+\frac{x^4\left(x^2\left(-\left(-2acf+b^2f-bce+2c^2d\right)\right)\right.}{2c\left(b^2-4ac\right)\left(a+bx^2+\right.}$$

```
[Out] ((2*c^2*d + 2*b^2*f - c*(b*e + 6*a*f))*x^2)/(2*c^2*(b^2 - 4*a*c)) + (x^4*(2
*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2))/(2*c*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((12*a^2*c^2*f - b^3*(c*e - 2*b*f) - 2*a*
c*(2*c^2*d - 3*b*c*e + 6*b^2*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*c^3*(b^2 - 4*a*c)^(3/2)) + ((c*e - 2*b*f)*Log[a + b*x^2 + c*x^4])/(4*c^3
)
```

Rubi [A] time = 0.440226, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1663, 1644, 773, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2c^2f-2ac\left(6b^2f-3bce+2c^2d\right)+b^3(-(ce-2bf))\right)}{2c^3\left(b^2-4ac\right)^{3/2}}+\frac{x^4\left(x^2\left(-\left(-2acf+b^2f-bce+2c^2d\right)\right)\right.}{2c\left(b^2-4ac\right)\left(a+bx^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] ((2*c^2*d + 2*b^2*f - c*(b*e + 6*a*f))*x^2)/(2*c^2*(b^2 - 4*a*c)) + (x^4*(2
*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2))/(2*c*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((12*a^2*c^2*f - b^3*(c*e - 2*b*f) - 2*a*
c*(2*c^2*d - 3*b*c*e + 6*b^2*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*c^3*(b^2 - 4*a*c)^(3/2)) + ((c*e - 2*b*f)*Log[a + b*x^2 + c*x^4])/(4*c^3
)
```


Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
```

```
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^5\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^2}dx, x, x^2\right)$$

$$= \frac{x^4\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\mathrm{Subst}\left(\int \frac{x\left(2\left(2ae-\frac{b(cd+af)}{c}\right)-\frac{\left(2c^2d-bce+\cdots\right.}{}\right.}{a+bx+cx^2}\right.}{2\left(b^2-4ac\right)}$$

$$= \frac{\left(2c^2d+2b^2f-c(be+6af)\right)x^2}{2c^2\left(b^2-4ac\right)} + \frac{x^4\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \cdots$$

$$= \frac{\left(2c^2d+2b^2f-c(be+6af)\right)x^2}{2c^2\left(b^2-4ac\right)} + \frac{x^4\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \cdots$$

$$= \frac{\left(2c^2d+2b^2f-c(be+6af)\right)x^2}{2c^2\left(b^2-4ac\right)} + \frac{x^4\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \cdots$$

$$= \frac{\left(2c^2d+2b^2f-c(be+6af)\right)x^2}{2c^2\left(b^2-4ac\right)} + \frac{x^4\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \cdots$$

Mathematica [A] time = 0.385163, size = 236, normalized size = 1.

$$\frac{-\frac{2\left(a^2c\left(2c\left(e+fx^2\right)-3bf\right)+a\left(-b^2c\left(e+4fx^2\right)+b^3f+bc^2\left(d+3ex^2\right)-2c^3dx^2\right)+b^2x^2\left(b^2f-bce+c^2d\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(12a^2c^2f-2ac\left(6b^2f-3bce+2c^2d\right)+b^3(2bf\cdots\right.}{\left(4ac-b^2\right)^{3/2}}\cdots}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*c*f*x^2 - (2*(b^2*(c^2*d - b*c*e + b^2*f)*x^2 + a^2*c*(-3*b*f + 2*c*(e +
 f*x^2)) + a*(b^3*f - 2*c^3*d*x^2 + b*c^2*(d + 3*e*x^2) - b^2*c*(e + 4*f*x^
2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*(12*a^2*c^2*f + b^3*(-(c*e)
+ 2*b*f) - 2*a*c*(2*c^2*d - 3*b*c*e + 6*b^2*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-
b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (c*e - 2*b*f)*Log[a + b*x^2 + c*x^4])
/(4*c^3)
```

Maple [B] time = 0.017, size = 832, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*f*x^2/c^2+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a^2*f-2/c^2/(c*x^4+b*x^2+
a)/(4*a*c-b^2)*x^2*a*b^2*f+3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b*e-1/(c
*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*d+1/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b
^4*f-1/2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b^3*e+1/2/c/(c*x^4+b*x^2+a)/(4
```

```
*a*c-b^2)*x^2*b^2*d-3/2/c^2/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*b*f+1/c/(c*x^4+
b*x^2+a)*a^2/(4*a*c-b^2)*e+1/2/c^3/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b^3*f-1/2/
c^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b^2*e+1/2/c/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)
*b*d-2/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a*b*f+1/c/(4*a*c-b^2)*ln(c*x^4+b*x
^2+a)*a*e+1/2/c^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^3*f-1/4/c^2/(4*a*c-b^2)*l
n(c*x^4+b*x^2+a)*b^2*e-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)
^(1/2))*a^2*f+6/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
*a*b^2*f-3/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*e+
2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*d-1/c^3/(4*a*c-
b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*f+1/2/c^2/(4*a*c-b^2)^
(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.30547, size = 3043, normalized size = 12.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f*x^6 + 2*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*f*x^4 - 2*((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d - (b^5*c
- 7*a*b^3*c^2 + 12*a^2*b*c^3)*e + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^
3*c^3)*f)*x^2 + (4*a^2*c^3*d + (4*a*c^4*d + (b^3*c^2 - 6*a*b*c^3)*e - 2*(b^
4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*f)*x^4 + (4*a*b*c^3*d + (b^4*c - 6*a*b^2*c^2
)*e - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*f)*x^2 + (a*b^3*c - 6*a^2*b*c^2)*e
- 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 +
 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
+ a)) - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d + 2*(a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*
c^3)*e - 2*(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*f + (((b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*e - 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f)*x^4 + ((b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e - 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*f)*
x^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e - 2*(a*b^5 - 8*a^2*b^3*c + 1
6*a^3*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3
*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 +
16*a^2*b*c^5)*x^2), 1/4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f*x^6 + 2*(
b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f*x^4 - 2*((b^4*c^2 - 6*a*b^2*c^3 + 8*a
^2*c^4)*d - (b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*e + (b^6 - 9*a*b^4*c + 26*
a^2*b^2*c^2 - 24*a^3*c^3)*f)*x^2 + 2*(4*a^2*c^3*d + (4*a*c^4*d + (b^3*c^2 -
 6*a*b*c^3)*e - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*f)*x^4 + (4*a*b*c^3*d +
 (b^4*c - 6*a*b^2*c^2)*e - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*f)*x^2 + (a*b^
3*c - 6*a^2*b*c^2)*e - 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*f)*sqrt(-b^2 + 4
*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(a*b^3*c^
2 - 4*a^2*b*c^3)*d + 2*(a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*e - 2*(a*b^5 -
 7*a^2*b^3*c + 12*a^3*b*c^2)*f + (((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e -
```

```
 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f)*x^4 + ((b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3)*e - 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*f)*x^2 + (a*b^4*c - 8*
a^2*b^2*c^2 + 16*a^3*c^3)*e - 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f)*log
(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8
*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 19.2062, size = 377, normalized size = 1.6

$$\frac{fx^2}{2\,c^2} - \frac{\left(4\,ac^3d - 2\,b^4f + 12\,ab^2cf - 12\,a^2c^2f + b^3ce - 6\,abc^2e\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(b^2c^3 - 4\,ac^4\right)\sqrt{-b^2 + 4\,ac}} + \frac{2\,b^3fx^4 - 8\,abcfx^4 - b^2cx^4e + 4\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/c^2 - 1/2*(4*a*c^3*d - 2*b^4*f + 12*a*b^2*c*f - 12*a^2*c^2*f + b^
3*c*e - 6*a*b*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4
*a*c^4)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*b^3*f*x^4 - 8*a*b*c*f*x^4 - b^2*c*x^4*
e + 4*a*c^2*x^4*e - 2*b^2*c*d*x^2 + 4*a*c^2*d*x^2 - 4*a^2*c*f*x^2 + b^3*x^2
*e - 2*a*b*c*x^2*e - 2*a*b*c*d - 2*a^2*b*f + a*b^2*e)/((c*x^4 + b*x^2 + a)*
(b^2*c^2 - 4*a*c^3)) - 1/4*(2*b*f - c*e)*log(c*x^4 + b*x^2 + a)/c^3
```

3.63 $\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=165

$$\frac{x^2\left(x^2\left(-\left(-2acf+b^2f-bce+2c^2d\right)\right)-b(af+cd)+2ace\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2bc(3af+cd)+4ac^2e+b^3f\right)}{2c^2\left(b^2-4ac\right)^{3/2}}+\frac{f\log}{}$$

```
[Out] (x^2*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2))/(
2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((4*a*c^2*e + b^3*f - 2*b*c*(c*d +
 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/
2)) + (f*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] time = 0.286901, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1663, 1644, 634, 618, 206, 628}

$$\frac{x^2\left(x^2\left(-\left(-2acf+b^2f-bce+2c^2d\right)\right)-b(af+cd)+2ace\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2bc(3af+cd)+4ac^2e+b^3f\right)}{2c^2\left(b^2-4ac\right)^{3/2}}+\frac{f\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^2*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2))/(
2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((4*a*c^2*e + b^3*f - 2*b*c*(c*d +
 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/
2)) + (f*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{x^2\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{2ae-\frac{b(cd+af)}{c}-\frac{\left(b^2-4ac\right)fx}{c}}{a+bx+cx^2}\,dx,\right.}{2\left(b^2-4ac\right)}$$

$$= \frac{x^2\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{f\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2} -$$

$$= \frac{x^2\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{f\log\left(a+bx^2+cx^4\right)}{4c^2} + \frac{\left(4ac^2e+\right.}{}$$

$$= \frac{x^2\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(4ac^2e+b^3f-2bc(cd+3af)\right)\text{ta}}{2c^2\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.287673, size = 175, normalized size = 1.06

$$\frac{\frac{2\left(-2a^2cf+a\left(b^2f-bc\left(e+3fx^2\right)+2c^2\left(d+ex^2\right)\right)+bx^2\left(b^2f-bce+c^2d\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(-2bc(3af+cd)+4ac^2e+b^3f\right)}{\left(4ac-b^2\right)^{3/2}}+f\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(-2*a^2*c*f + b*(c^2*d - b*c*e + b^2*f)*x^2 + a*(b^2*f + 2*c^2*(d + e*x
^2) - b*c*(e + 3*f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(4*a*c^
2*e + b^3*f - 2*b*c*(c*d + 3*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]
```

```
)/(-b^2 + 4*a*c)^(3/2) + f*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [B] time = 0.013, size = 336, normalized size = 2.

$$\frac{1}{2\,c x^4+2\,b x^2+2\,a}\left(\frac{\left(3\,abcf-2\,a c^2 e-b^3 f+b^2 ce-b c^2 d\right) x^2}{\left(4\,ac-b^2\right) c^2}+\frac{a\left(2\,acf-b^2 f+bce-2\,c^2 d\right)}{\left(4\,ac-b^2\right) c^2}\right)+\frac{\ln\left(c x^4+b x^2+a\right) af}{\left(4\,ac-b^2\right) c}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*((3*a*b*c*f-2*a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)/(4*a*c-b^2)/c^2*x^2+a*(2*a
*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)/c^2)/(c*x^4+b*x^2+a)+1/c/(4*a*c-b^2)*
ln(c*x^4+b*x^2+a)*a*f-1/4/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2*f-3/c/(4*a*
c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*f+2/(4*a*c-b^2)^(3/2
)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*e*a-1/(4*a*c-b^2)^(3/2)*arctan((2*c
*x^2+b)/(4*a*c-b^2)^(1/2))*b*d+1/2/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2))*b^3*f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.69974, size = 2033, normalized size = 12.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*e + (b
^5 - 7*a*b^3*c + 12*a^2*b*c^2)*f)*x^2 - (2*a*b*c^2*d - 4*a^2*c^2*e + (2*b*c
^3*d - 4*a*c^3*e - (b^3*c - 6*a*b*c^2)*f)*x^4 + (2*b^2*c^2*d - 4*a*b*c^2*e
- (b^4 - 6*a*b^2*c)*f)*x^2 - (a*b^3 - 6*a^2*b*c)*f)*sqrt(b^2 - 4*a*c)*log((
2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x
^4 + b*x^2 + a)) + 4*(a*b^2*c^2 - 4*a^2*c^3)*d - 2*(a*b^3*c - 4*a^2*b*c^2)*
e + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*f + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*
c^3)*f*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*f*x^2 + (a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^
3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 +
 16*a^2*b*c^4)*x^2), 1/4*(2*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*b^2*c^2
 + 8*a^2*c^3)*e + (b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*f)*x^2 - 2*(2*a*b*c^2*d
- 4*a^2*c^2*e + (2*b*c^3*d - 4*a*c^3*e - (b^3*c - 6*a*b*c^2)*f)*x^4 + (2*b^
2*c^2*d - 4*a*b*c^2*e - (b^4 - 6*a*b^2*c)*f)*x^2 - (a*b^3 - 6*a^2*b*c)*f)*s
```

```
qrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) +
 4*(a*b^2*c^2 - 4*a^2*c^3)*d - 2*(a*b^3*c - 4*a^2*b*c^2)*e + 2*(a*b^4 - 6*a
^2*b^2*c + 8*a^3*c^2)*f + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*f*x^4 + (b^5
- 8*a*b^3*c + 16*a^2*b*c^2)*f*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*f)*l
og(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 -
 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2
)]
```

Sympy [B] time = 145.522, size = 1030, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (f/(4*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*f - 4*a*c**2*e - b**3*f + 2
*b*c**2*d)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))
)*log(x**2 + (-32*a**2*c**3*(f/(4*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c
*f - 4*a*c**2*e - b**3*f + 2*b*c**2*d)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2
*c**2 + 12*a*b**4*c - b**6))) + 8*a**2*c*f + 16*a*b**2*c**2*(f/(4*c**2) - s
qrt(-(4*a*c - b**2)**3)*(6*a*b*c*f - 4*a*c**2*e - b**3*f + 2*b*c**2*d)/(4*c
**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - a*b**2*f -
2*a*b*c*e - 2*b**4*c*(f/(4*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*f - 4*
a*c**2*e - b**3*f + 2*b*c**2*d)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
 12*a*b**4*c - b**6))) + b**2*c*d)/(6*a*b*c*f - 4*a*c**2*e - b**3*f + 2*b*c
**2*d)) + (f/(4*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*f - 4*a*c**2*e -
b**3*f + 2*b*c**2*d)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*
c - b**6)))*log(x**2 + (-32*a**2*c**3*(f/(4*c**2) + sqrt(-(4*a*c - b**2)**3
)*(6*a*b*c*f - 4*a*c**2*e - b**3*f + 2*b*c**2*d)/(4*c**2*(64*a**3*c**3 - 48
*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 8*a**2*c*f + 16*a*b**2*c**2*(f/(4
*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*f - 4*a*c**2*e - b**3*f + 2*b*c*
*2*d)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - a
*b**2*f - 2*a*b*c*e - 2*b**4*c*(f/(4*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*a*
b*c*f - 4*a*c**2*e - b**3*f + 2*b*c**2*d)/(4*c**2*(64*a**3*c**3 - 48*a**2*b
**2*c**2 + 12*a*b**4*c - b**6))) + b**2*c*d)/(6*a*b*c*f - 4*a*c**2*e - b**3
*f + 2*b*c**2*d)) + (2*a**2*c*f - a*b**2*f + a*b*c*e - 2*a*c**2*d + x**2*(3
*a*b*c*f - 2*a*c**2*e - b**3*f + b**2*c*e - b*c**2*d))/(8*a**2*c**3 - 2*a*b
**2*c**2 + x**4*(8*a*c**4 - 2*b**2*c**3) + x**2*(8*a*b*c**3 - 2*b**3*c**2))
```

Giac [A] time = 19.6017, size = 263, normalized size = 1.59

$$\frac{\left(2\,bc^2d - b^3f + 6\,abcf - 4\,ac^2e\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(b^2c^2 - 4\,ac^3\right)\sqrt{-b^2+4\,ac}} + \frac{f\log\left(cx^4+bx^2+a\right)}{4\,c^2} + \frac{2\,ac^2d + ab^2f - 2\,a^2cf - abce + \left(bc^2d \cdots\right.}{2\left(cx^4+bx^2+a\right)\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b*c^2*d - b^3*f + 6*a*b*c*f - 4*a*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-
b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/4*f*log(c*x^4 +
b*x^2 + a)/c^2 + 1/2*(2*a*c^2*d + a*b^2*f - 2*a^2*c*f - a*b*c*e + (b*c^2*d
+ b^3*f - 3*a*b*c*f - b^2*c*e + 2*a*c^2*e)*x^2)/((c*x^4 + b*x^2 + a)*(b^2 -
 4*a*c)*c^2)
```

3.64 $\int \frac{x\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=123

$$\frac{x^2\left(-\left(-2acf+b^2f-bce+2c^2d\right)\right)-b(af+cd)+2ace}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2af-be+2cd)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2)/(2*c*(b
^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((2*c*d - b*e + 2*a*f)*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.183795, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1663, 1660, 12, 618, 206}

$$\frac{x^2\left(-\left(-2acf+b^2f-bce+2c^2d\right)\right)-b(af+cd)+2ace}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2af-be+2cd)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2)/(2*c*(b
^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((2*c*d - b*e + 2*a*f)*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)\\
&= \frac{2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\operatorname{Subst}\left(\int \frac{2cd-be+2af}{a+bx+cx^2}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)}\\
&= \frac{2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(2cd-be+2af)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,\right.}{2\left(b^2-4ac\right)}\\
&= \frac{2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{(2cd-be+2af)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx\right.}{b^2-4ac}\\
&= \frac{2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{(2cd-be+2af)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.122431, size = 130, normalized size = 1.06

$$\frac{abf-2ac\left(e+fx^2\right)+b^2fx^2+bc\left(d-ex^2\right)+2c^2dx^2}{2c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}-\frac{\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)(-2af+be-2cd)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a*b*f + 2*c^2*d*x^2 + b^2*f*x^2 + b*c*(d - e*x^2) - 2*a*c*(e + f*x^2))/(2*
c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) - ((-2*c*d + b*e - 2*a*f)*ArcTan[(b +
 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] time = 0.011, size = 205, normalized size = 1.7

$$\frac{1}{2\,cx^4+2\,bx^2+2\,a}\left(-\frac{\left(2\,acf-b^2f+bce-2\,c^2d\right)x^2}{\left(4\,ac-b^2\right)c}+\frac{abf-2\,cea+bcd}{\left(4\,ac-b^2\right)c}\right)+2\,\frac{af}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)-be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)/c*x^2+1/c*(a*b*f-2*a*c*e+b*
c*d)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4
```

```
*a*c-b^2)^(1/2))*a*f-1/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/
2))*b*e+2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.44558, size = 1374, normalized size = 11.17

$$\left[-\frac{\left(2\left(b^2c^2-4\,ac^3\right)d-\left(b^3c-4\,abc^2\right)e+\left(b^4-6\,ab^2c+8\,a^2c^2\right)f\right)x^2+\left(\left(2\,c^3d-bc^2e+2\,ac^2f\right)x^4+2\,ac^2d-abce+2\,a^2cf\right.}{2\left(ab^4c-8\,a^2b^2c^2+16\,a^3c^3+\left(b^4c^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] [-1/2*((2*(b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 6*a*b^2*c
+ 8*a^2*c^2)*f)*x^2 + ((2*c^3*d - b*c^2*e + 2*a*c^2*f)*x^4 + 2*a*c^2*d - a*
b*c*e + 2*a^2*c*f + (2*b*c^2*d - b^2*c*e + 2*a*b*c*f)*x^2)*sqrt(b^2 - 4*a*c
)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c
))/(c*x^4 + b*x^2 + a)) + (b^3*c - 4*a*b*c^2)*d - 2*(a*b^2*c - 4*a^2*c^2)*e
 + (a*b^3 - 4*a^2*b*c)*f)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)
, -1/2*((2*(b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 6*a*b^2*c
 + 8*a^2*c^2)*f)*x^2 - 2*((2*c^3*d - b*c^2*e + 2*a*c^2*f)*x^4 + 2*a*c^2*d -
 a*b*c*e + 2*a^2*c*f + (2*b*c^2*d - b^2*c*e + 2*a*b*c*f)*x^2)*sqrt(-b^2 + 4
*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3*c - 4*
a*b*c^2)*d - 2*(a*b^2*c - 4*a^2*c^2)*e + (a*b^3 - 4*a^2*b*c)*f)/(a*b^4*c -
8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]
```

Sympy [B] time = 40.1526, size = 474, normalized size = 3.85

$$-\frac{\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(2af-be+2cd\right)\log\left(x^2+\frac{-16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(2af-be+2cd\right)+8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(2af-be+2cd\right)+2abf-b^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(2af-be+2c\right.}{4acf-2bce+4c^2d}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d)*log(x**2 + (-16*a**2*c**2
*sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) + 8*a*b**2*c*sqrt(-1/(4*a
*c - b**2)**3)*(2*a*f - b*e + 2*c*d) + 2*a*b*f - b**4*sqrt(-1/(4*a*c - b**2
)**3)*(2*a*f - b*e + 2*c*d) - b**2*e + 2*b*c*d)/(4*a*c*f - 2*b*c*e + 4*c**2
*d))/2 + sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d)*log(x**2 + (16*a*
*2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) - 8*a*b**2*c*sqrt(
-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) + 2*a*b*f + b**4*sqrt(-1/(4*a*c
 - b**2)**3)*(2*a*f - b*e + 2*c*d) - b**2*e + 2*b*c*d)/(4*a*c*f - 2*b*c*e +
 4*c**2*d))/2 - (-a*b*f + 2*a*c*e - b*c*d + x**2*(2*a*c*f - b**2*f + b*c*e
- 2*c**2*d))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x*
*2*(8*a*b*c**2 - 2*b**3*c))
```

Giac [A] time = 19.2514, size = 189, normalized size = 1.54

$$-\frac{\left(2\,cd+2\,af-be\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{2\,c^2dx^2+b^2fx^2-2\,acfx^2-bcx^2e+bcd+abf-2\,ace}{2\left(cx^4+bx^2+a\right)\left(b^2c-4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -(2*c*d + 2*a*f - b*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a
*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c^2*d*x^2 + b^2*f*x^2 - 2*a*c*f*x^2 - b*c*
x^2*e + b*c*d + a*b*f - 2*a*c*e)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
```

3.65 $\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=166

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(4a^2ce-2ab(af+3cd)+b^3d\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{d\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{d\log(x)}{a^2}+\frac{x^2(abf-2ace+bcd)-abe-2a(c}{2a\left(b^2-4ac\right)\left(a+bx^2+\right.}$$

```
[Out] (b^2*d - a*b*e - 2*a*(c*d - a*f) + (b*c*d - 2*a*c*e + a*b*f)*x^2)/(2*a*(b^2
 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^3*d + 4*a^2*c*e - 2*a*b*(3*c*d + a*f))
*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (d
*Log[x])/a^2 - (d*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rubi [A] time = 0.393699, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1663, 1646, 800, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(4a^2ce-2ab(af+3cd)+b^3d\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{d\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{d\log(x)}{a^2}+\frac{x^2(abf-2ace+bcd)-abe-2a(c}{2a\left(b^2-4ac\right)\left(a+bx^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (b^2*d - a*b*e - 2*a*(c*d - a*f) + (b*c*d - 2*a*c*e + a*b*f)*x^2)/(2*a*(b^2
 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^3*d + 4*a^2*c*e - 2*a*b*(3*c*d + a*f))
*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (d
*Log[x])/a^2 - (d*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{d+ex+fx^2}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\mathrm{Subst}\left(\int \frac{-\left(\frac{b^2}{a}-4c\right)d-\frac{(bcd-2ace+abf)x}{a}}{x\left(a+bx+cx^2\right)}\,dx, x, x^2 \ldots}{2\left(b^2-4ac\right) \ldots}$$

$$= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\mathrm{Subst}\left(\int\left(\frac{\left(-b^2+4ac\right)d}{a^2x}+\frac{b^3d+2a^2ce-ab(5cd+a \ldots}{a^2\left(a+bx+ \ldots\right.}\right.\right.}{2\left(b^2-4ac\right) \ldots}$$

$$= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{d\log(x)}{a^2} - \frac{\mathrm{Subst}\left(\int \frac{b^3d+2a^2ce-ab(5cd+af) \ldots}{a+bx+cx^2 \ldots}\right.}{2a^2\left(b^2-4 \ldots\right.}$$

$$= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{d\log(x)}{a^2} - \frac{d\,\mathrm{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2 \ldots\right.}{4a^2 \ldots}$$

$$= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{d\log(x)}{a^2} - \frac{d\log\left(a+bx^2+cx^4\right)}{4a^2} + \frac{\left(b^3d \ldots\right.}{\ldots}$$

$$= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^3d+4a^2ce-2ab(3cd+af)\right)\tanh^{-1}\left(\ldots\right.}{2a^2\left(b^2-4ac\right)^{3/2} \ldots}$$

Mathematica [A] time = 0.469313, size = 268, normalized size = 1.61

$$-\frac{-\frac{2a\left(b\left(-ae+afx^2+cdx^2\right)+2a\left(af-c\left(d+ex^2\right)\right)+b^2d\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(4ac\left(ae-d\sqrt{b^2-4ac}\right)+b^2d\sqrt{b^2-4ac}-2ab(af+3cd)+b^3d\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\log\left(\sqrt{b^2-4ac}+b+2cx\right.}{}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -((-2*a*(b^2*d + b*(-(a*e) + c*d*x^2 + a*f*x^2) + 2*a*(a*f - c*(d + e*x^2))
))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - 4*d*Log[x] + ((b^3*d + b^2*Sqrt[b^
2 - 4*a*c]*d + 4*a*c*(-(Sqrt[b^2 - 4*a*c]*d) + a*e) - 2*a*b*(3*c*d + a*f))*
Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-(b^3*d) + b^
2*Sqrt[b^2 - 4*a*c]*d - 4*a*c*(Sqrt[b^2 - 4*a*c]*d + a*e) + 2*a*b*(3*c*d +
a*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

Maple [B] time = 0.018, size = 462, normalized size = 2.8

$$\frac{d\ln(x)}{a^2}-\frac{bx^2f}{\left(2cx^4+2bx^2+2a\right)\left(4ac-b^2\right)}+\frac{cx^2e}{\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)}-\frac{bx^2cd}{2a\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)}-\frac{a}{\left(cx^4+bx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] d*ln(x)/a^2-1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b*f+c/(c*x^4+b*x^2+a)/(4*a*
c-b^2)*x^2*e-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b*c*d-a/(c*x^4+b*x^2+a)/
(4*a*c-b^2)*f+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*e+1/(c*x^4+b*x^2+a)/(4*a*c-
b^2)*c*d-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2*d-1/a/(4*a*c-b^2)*c*ln(c*x^4
+b*x^2+a)*d+1/4/a^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2*d-1/(4*a*c-b^2)^(3/2)
*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*f+2/(4*a*c-b^2)^(3/2)*arctan((2*c*
x^2+b)/(4*a*c-b^2)^(1/2))*c*e-3/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a
*c-b^2)^(1/2))*b*c*d+1/2/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^
2)^(1/2))*b^3*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.60329, size = 2333, normalized size = 14.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e + (a^2*b^3
 - 4*a^3*b*c)*f)*x^2 + (4*a^3*c*e - 2*a^3*b*f + (4*a^2*c^2*e - 2*a^2*b*c*f
+ (b^3*c - 6*a*b*c^2)*d)*x^4 + (4*a^2*b*c*e - 2*a^2*b^2*f + (b^4 - 6*a*b^2*
c)*d)*x^2 + (a*b^3 - 6*a^2*b*c)*d)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c
*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))
+ 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*d - 2*(a^2*b^3 - 4*a^3*b*c)*e + 4*(a^
3*b^2 - 4*a^4*c)*f - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^4 + (b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*log(c*
x^4 + b*x^2 + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^4 + (b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*log(x)
)/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4
*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2*((a*b^3*c -
 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^
2 + 2*(4*a^3*c*e - 2*a^3*b*f + (4*a^2*c^2*e - 2*a^2*b*c*f + (b^3*c - 6*a*b*
c^2)*d)*x^4 + (4*a^2*b*c*e - 2*a^2*b^2*f + (b^4 - 6*a*b^2*c)*d)*x^2 + (a*b^
3 - 6*a^2*b*c)*d)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*
c)/(b^2 - 4*a*c)) + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*d - 2*(a^2*b^3 - 4*
a^3*b*c)*e + 4*(a^3*b^2 - 4*a^4*c)*f - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*
d*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2)*d)*log(c*x^4 + b*x^2 + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*
d*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2)*d)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^
3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 19.5116, size = 306, normalized size = 1.84

$$-\frac{\left(b^3d-6\,abcd-2\,a^2bf+4\,a^2ce\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^2b^2-4\,a^3c\right)\sqrt{-b^2+4\,ac}}-\frac{d\log\left(cx^4+bx^2+a\right)}{4\,a^2}+\frac{d\log\left(x^2\right)}{2\,a^2}+\frac{b^2cdx^4-4\,ac^2dx^4+b^3dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3*d - 6*a*b*c*d - 2*a^2*b*f + 4*a^2*c*e)*arctan((2*c*x^2 + b)/sqrt(
-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*c)*sqrt(-b^2 + 4*a*c)) - 1/4*d*log(c*x^4 +
 b*x^2 + a)/a^2 + 1/2*d*log(x^2)/a^2 + 1/4*(b^2*c*d*x^4 - 4*a*c^2*d*x^4 + b
^3*d*x^2 - 2*a*b*c*d*x^2 + 2*a^2*b*f*x^2 - 4*a^2*c*x^2*e + 3*a*b^2*d - 8*a^
2*c*d + 4*a^3*f - 2*a^2*b*e)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c))
```

3.66 $\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=234

$$-\frac{2a^2ce+cx^2\left(-abe-2a(cd-af)+b^2d\right)-ab^2e-ab(3cd-af)+b^3d}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(6a^2bce+4a^2c(3cd-af)-12a\right.}{2a^3\left(b^2-4ac\right)^{3/2}}$$

```
[Out] -d/(2*a^2*x^2) - (b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f) + c*(b^2*
d - a*b*e - 2*a*(c*d - a*f))*x^2)/(2*a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
 - ((2*b^4*d - 12*a*b^2*c*d - a*b^3*e + 6*a^2*b*c*e + 4*a^2*c*(3*c*d - a*f)
)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(3/2)) - (
(2*b*d - a*e)*Log[x])/a^3 + ((2*b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rubi [A] time = 0.725096, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1663, 1646, 1628, 634, 618, 206, 628}

$$-\frac{2a^2ce+cx^2\left(-abe-2a(cd-af)+b^2d\right)-ab^2e-ab(3cd-af)+b^3d}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(6a^2bce+4a^2c(3cd-af)-12a\right.}{2a^3\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -d/(2*a^2*x^2) - (b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f) + c*(b^2*
d - a*b*e - 2*a*(c*d - a*f))*x^2)/(2*a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
 - ((2*b^4*d - 12*a*b^2*c*d - a*b^3*e + 6*a^2*b*c*e + 4*a^2*c*(3*c*d - a*f)
)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(3/2)) - (
(2*b*d - a*e)*Log[x])/a^3 + ((2*b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{d+ex+fx^2}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c\left(b^2d-abe-2a(cd-af)\right)x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-\left(\frac{b^2}{a}-4c\right)\cdots}{\cdots}\right)}{\cdots}$$

$$= -\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c\left(b^2d-abe-2a(cd-af)\right)x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int\left(\frac{\left(-b^2+4a\cdots}{a^2x^2}\cdots\right.\right.}{\cdots}$$

$$= -\frac{d}{2a^2x^2} - \frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c\left(b^2d-abe-2a(cd-af)\right)x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{(2bd-ae\cdots}{a^3\cdots}$$

$$= -\frac{d}{2a^2x^2} - \frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c\left(b^2d-abe-2a(cd-af)\right)x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{(2bd-ae\cdots}{a^3\cdots}$$

$$= -\frac{d}{2a^2x^2} - \frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c\left(b^2d-abe-2a(cd-af)\right)x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{(2bd-ae\cdots}{a^3\cdots}$$

$$= -\frac{d}{2a^2x^2} - \frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c\left(b^2d-abe-2a(cd-af)\right)x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2b^4d-1\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.691677, size = 403, normalized size = 1.72

$$\frac{\frac{\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(4a^2c\left(e\sqrt{b^2-4ac}-af+3cd\right)+b^3\left(2d\sqrt{b^2-4ac}-ae\right)-ab^2\left(e\sqrt{b^2-4ac}+12cd\right)+2abc\left(3ae-4d\sqrt{b^2-4ac}\right)+2b^4d\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(4a^2c\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*d)/x^2 - (2*a*(b^3*d + b^2*(-(a*e) + c*d*x^2) + a*b*(a*f - c*(3*d +
e*x^2)) + 2*a*c*(-(c*d*x^2) + a*(e + f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) + 4*(-2*b*d + a*e)*Log[x] + ((2*b^4*d + b^3*(2*Sqrt[b^2 - 4*a*c]*d
- a*e) + 2*a*b*c*(-4*Sqrt[b^2 - 4*a*c]*d + 3*a*e) - a*b^2*(12*c*d + Sqrt[b^
2 - 4*a*c]*e) + 4*a^2*c*(3*c*d + Sqrt[b^2 - 4*a*c]*e - a*f))*Log[b - Sqrt[b
^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-2*b^4*d + b^3*(2*Sqrt[b^2
- 4*a*c]*d + a*e) - 2*a*b*c*(4*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*b^2*(12*c*d
 - Sqrt[b^2 - 4*a*c]*e) + 4*a^2*c*(-3*c*d + Sqrt[b^2 - 4*a*c]*e + a*f))*Log
[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^3)
```

Maple [B] time = 0.023, size = 722, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/2*d/a^2/x^2+1/a^2*ln(x)*e-2/a^3*ln(x)*b*d+1/(c*x^4+b*x^2+a)*c/(4*a*c-b^2
)*x^2*f-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*b*e-1/a/(c*x^4+b*x^2+a)*c^2
/(4*a*c-b^2)*x^2*d+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*b^2*d+1/2/(c*x
^4+b*x^2+a)/(4*a*c-b^2)*b*f+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c*e-1/2/a/(c*x^4+
b*x^2+a)/(4*a*c-b^2)*b^2*e-3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*c*d+1/2/a^2/
(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^3*d-1/a/(4*a*c-b^2)*c*ln(c*x^4+b*x^2+a)*e+1/4
/a^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2*e+2/a^2/(4*a*c-b^2)*c*ln(c*x^4+b*x^2
+a)*b*d-1/2/a^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^3*d+2/(4*a*c-b^2)^(3/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*f-3/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))*b*c*e-6/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a
*c-b^2)^(1/2))*c^2*d+1/2/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^
2)^(1/2))*b^3*e+6/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2
))*b^2*c*d-1/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^
4*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 16.6412, size = 3637, normalized size = 15.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d - (a^2*b^3*c - 4*a^3*b
*c^2)*e + 2*(a^3*b^2*c - 4*a^4*c^2)*f)*x^4 + 2*((2*a*b^5 - 15*a^2*b^3*c + 2
8*a^3*b*c^2)*d - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*e + (a^3*b^3 - 4*a^4*b
*c)*f)*x^2 + ((4*a^3*c^2*f - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d + (a*b^3
*c - 6*a^2*b*c^2)*e)*x^6 + (4*a^3*b*c*f - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)
*d + (a*b^4 - 6*a^2*b^2*c)*e)*x^4 + (4*a^4*c*f - 2*(a*b^4 - 6*a^2*b^2*c + 6
*a^3*c^2)*d + (a^2*b^3 - 6*a^3*b*c)*e)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^
4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x
^2 + a)) + 2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*d - ((2*(b^5*c - 8*a*b^3*
c^2 + 16*a^2*b*c^3)*d - (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*
(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)
*e)*x^4 + (2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*
c + 16*a^4*c^2)*e)*x^2)*log(c*x^4 + b*x^2 + a) + 4*((2*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d - (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*(b^
6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)
*x^4 + (2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c +
 16*a^4*c^2)*e)*x^2)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6
+ (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*
a^6*c^2)*x^2), -1/4*(2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d - (a^2*b
^3*c - 4*a^3*b*c^2)*e + 2*(a^3*b^2*c - 4*a^4*c^2)*f)*x^4 + 2*((2*a*b^5 - 15
*a^2*b^3*c + 28*a^3*b*c^2)*d - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*e + (a^3
*b^3 - 4*a^4*b*c)*f)*x^2 - 2*((4*a^3*c^2*f - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2
*c^3)*d + (a*b^3*c - 6*a^2*b*c^2)*e)*x^6 + (4*a^3*b*c*f - 2*(b^5 - 6*a*b^3*
c + 6*a^2*b*c^2)*d + (a*b^4 - 6*a^2*b^2*c)*e)*x^4 + (4*a^4*c*f - 2*(a*b^4 -
 6*a^2*b^2*c + 6*a^3*c^2)*d + (a^2*b^3 - 6*a^3*b*c)*e)*x^2)*sqrt(-b^2 + 4*a
*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 2*(a^2*b^4 -
8*a^3*b^2*c + 16*a^4*c^2)*d - ((2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d -
(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*(b^6 - 8*a*b^4*c + 16*a^
2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)*x^4 + (2*(a*b^5 - 8*
a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*e)*x^2)*
log(c*x^4 + b*x^2 + a) + 4*((2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d - (a*
b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*(b^6 - 8*a*b^4*c + 16*a^2*b
^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)*x^4 + (2*(a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*e)*x^2)*log
(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c
 + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 21.9743, size = 387, normalized size = 1.65

$$\frac{\left(2\,b^4d-12\,ab^2cd+12\,a^2c^2d-4\,a^3cf-ab^3e+6\,a^2bce\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^3b^2-4\,a^4c\right)\sqrt{-b^2+4\,ac}}-\frac{2\,b^2cdx^4-6\,ac^2dx^4+2\,a^2cfx^4-abcx^4e+2}{2\left(cx^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^4*d - 12*a*b^2*c*d + 12*a^2*c^2*d - 4*a^3*c*f - a*b^3*e + 6*a^2*b*
c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-b^
2 + 4*a*c)) - 1/2*(2*b^2*c*d*x^4 - 6*a*c^2*d*x^4 + 2*a^2*c*f*x^4 - a*b*c*x^
4*e + 2*b^3*d*x^2 - 7*a*b*c*d*x^2 + a^2*b*f*x^2 - a*b^2*x^2*e + 2*a^2*c*x^2
*e + a*b^2*d - 4*a^2*c*d)/((c*x^6 + b*x^4 + a*x^2)*(a^2*b^2 - 4*a^3*c)) + 1
/4*(2*b*d - a*e)*log(c*x^4 + b*x^2 + a)/a^3 - 1/2*(2*b*d - a*e)*log(x^2)/a^
3
```

3.67 $\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=329

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2b^2ce+6a^2bc(5cd-af)-12a^3c^2e-ab^3(20cd-af)-2ab^4e+3b^5d\right)}{2a^4\left(b^2-4ac\right)^{3/2}}+\frac{cx^2\left(2a^2ce-ab^2e-ab(3\right.}{}$$

```
[Out] -d/(4*a^2*x^4) + (2*b*d - a*e)/(2*a^3*x^2) + (b^4*d - a*b^3*e + 3*a^2*b*c*e
 + 2*a^2*c*(c*d - a*f) - a*b^2*(4*c*d - a*f) + c*(b^3*d - a*b^2*e + 2*a^2*c
*e - a*b*(3*c*d - a*f))*x^2)/(2*a^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((
3*b^5*d - 2*a*b^4*e + 12*a^2*b^2*c*e - 12*a^3*c^2*e + 6*a^2*b*c*(5*c*d - a*
f) - a*b^3*(20*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4
*(b^2 - 4*a*c)^(3/2)) + ((3*b^2*d - 2*a*b*e - a*(2*c*d - a*f))*Log[x])/a^4
- ((3*b^2*d - 2*a*b*e - a*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rubi [A] time = 1.15731, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1663, 1646, 1628, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2b^2ce+6a^2bc(5cd-af)-12a^3c^2e-ab^3(20cd-af)-2ab^4e+3b^5d\right)}{2a^4\left(b^2-4ac\right)^{3/2}}+\frac{cx^2\left(2a^2ce-ab^2e-ab(3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -d/(4*a^2*x^4) + (2*b*d - a*e)/(2*a^3*x^2) + (b^4*d - a*b^3*e + 3*a^2*b*c*e
 + 2*a^2*c*(c*d - a*f) - a*b^2*(4*c*d - a*f) + c*(b^3*d - a*b^2*e + 2*a^2*c
*e - a*b*(3*c*d - a*f))*x^2)/(2*a^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((
3*b^5*d - 2*a*b^4*e + 12*a^2*b^2*c*e - 12*a^3*c^2*e + 6*a^2*b*c*(5*c*d - a*
f) - a*b^3*(20*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4
*(b^2 - 4*a*c)^(3/2)) + ((3*b^2*d - 2*a*b*e - a*(2*c*d - a*f))*Log[x])/a^4
- ((3*b^2*d - 2*a*b*e - a*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
```

```
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x^3\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)$$

$$=\frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c\left(b^3d-ab^2e+2a^2ce-ab(3cd-a\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$=\frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c\left(b^3d-ab^2e+2a^2ce-ab(3cd-a\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$=-\frac{d}{4a^2x^4}+\frac{2bd-ae}{2a^3x^2}+\frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c\left(b^3d-ab^2\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$=-\frac{d}{4a^2x^4}+\frac{2bd-ae}{2a^3x^2}+\frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c\left(b^3d-ab^2\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$=-\frac{d}{4a^2x^4}+\frac{2bd-ae}{2a^3x^2}+\frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c\left(b^3d-ab^2\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$=-\frac{d}{4a^2x^4}+\frac{2bd-ae}{2a^3x^2}+\frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c\left(b^3d-ab^2\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 1.21921, size = 592, normalized size = 1.8

$$-\frac{\frac{2a\left(2a^2c\left(af-c\left(d+ex^2\right)\right)+ab^2\left(-af+4cd+cex^2\right)+b^3\left(ae-cdx^2\right)-abc\left(3ae+afx^2-3cdx^2\right)+b^4(-d)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(2a^2bc\left(4e\sqrt{b^2-4ac}-3af+15cd\right)-4\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -((a^2*d)/x^4 + (2*a*(-2*b*d + a*e))/x^2 + (2*a*(-(b^4*d) + b^3*(a*e - c*d*
x^2) + a*b^2*(4*c*d - a*f + c*e*x^2) - a*b*c*(3*a*e - 3*c*d*x^2 + a*f*x^2)
+ 2*a^2*c*(a*f - c*(d + e*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - 4*(
3*b^2*d - 2*a*b*e + a*(-2*c*d + a*f))*Log[x] + ((3*b^5*d + b^4*(3*Sqrt[b^2
- 4*a*c]*d - 2*a*e) + 2*a^2*b*c*(15*c*d + 4*Sqrt[b^2 - 4*a*c]*e - 3*a*f) +
a*b^3*(-20*c*d - 2*Sqrt[b^2 - 4*a*c]*e + a*f) - 4*a^2*c*(-2*c*Sqrt[b^2 - 4*
a*c]*d + 3*a*c*e + a*Sqrt[b^2 - 4*a*c]*f) + a*b^2*(-14*c*Sqrt[b^2 - 4*a*c]*
d + 12*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2]
)/(b^2 - 4*a*c)^(3/2) + ((-3*b^5*d + b^4*(3*Sqrt[b^2 - 4*a*c]*d + 2*a*e) -
a*b^3*(-20*c*d + 2*Sqrt[b^2 - 4*a*c]*e + a*f) + 2*a^2*b*c*(-15*c*d + 4*Sqrt
[b^2 - 4*a*c]*e + 3*a*f) + 4*a^2*c*(2*c*Sqrt[b^2 - 4*a*c]*d + 3*a*c*e - a*S
qrt[b^2 - 4*a*c]*f) + a*b^2*(-2*c*(7*Sqrt[b^2 - 4*a*c]*d + 6*a*e) + a*Sqrt[
b^2 - 4*a*c]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))
/(4*a^4)
```

Maple [B] time = 0.028, size = 1078, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2*f-1/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c^
2*d+1/a^3/x^2*b*d-2/a^3*ln(x)*b*e-2/a^3*ln(x)*c*d+3/a^4*ln(x)*b^2*d+1/(c*x^
4+b*x^2+a)/(4*a*c-b^2)*c*f-1/2/a^3/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*b^3*d+
1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*b^2*e+3/2/a^2/(c*x^4+b*x^2+a)*c^2
/(4*a*c-b^2)*x^2*b*d+1/2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^3*e-1/2/a^3/(c*x
^4+b*x^2+a)/(4*a*c-b^2)*b^4*d+3/4/a^4/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^4*d+3
/2/a^4/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^5*d-1/a/(4
*a*c-b^2)*c*ln(c*x^4+b*x^2+a)*f+1/4/a^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2*f
+2/a^2/(4*a*c-b^2)*c^2*ln(c*x^4+b*x^2+a)*d-1/2/a^3/(4*a*c-b^2)*ln(c*x^4+b*x
^2+a)*b^3*e-6/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c^2
*e+1/2/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*f-1/
a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*e-1/2/a/(c*
x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*b*f-1/2/a^2/x^2*e+1/a^2*ln(x)*f-1/4*d/a^2/x^
4+2/a^2/(4*a*c-b^2)*c*ln(c*x^4+b*x^2+a)*e*b-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-
b^2)*x^2*e-3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*c*e+2/a^2/(c*x^4+b*x^2+a)/(4
*a*c-b^2)*b^2*c*d-3/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2
))*b*c*f+6/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*
c*e+15/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c^2*d-
10/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*c*d-7/2/
a^3/(4*a*c-b^2)*c*ln(c*x^4+b*x^2+a)*b^2*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 39.111, size = 5341, normalized size = 16.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*((3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*d - 2*(a^2*b^4*c - 7*a
^3*b^2*c^2 + 12*a^4*c^3)*e + (a^3*b^3*c - 4*a^4*b*c^2)*f)*x^6 + ((6*a*b^6 -
 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(2*a^2*b^5 - 15*a^3*b^3
*c + 28*a^4*b*c^2)*e + 2*(a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*f)*x^4 + (3*(a
^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d - 2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*
c^2)*e)*x^2 + (((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*d - 2*(a*b^4*c - 6*
a^2*b^2*c^2 + 6*a^3*c^3)*e + (a^2*b^3*c - 6*a^3*b*c^2)*f)*x^8 + ((3*b^6 - 2
0*a*b^4*c + 30*a^2*b^2*c^2)*d - 2*(a*b^5 - 6*a^2*b^3*c + 6*a^3*b*c^2)*e + (
a^2*b^4 - 6*a^3*b^2*c)*f)*x^6 + ((3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*d
- 2*(a^2*b^4 - 6*a^3*b^2*c + 6*a^4*c^2)*e + (a^3*b^3 - 6*a^4*b*c)*f)*x^4)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
```

```
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)*d - (((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^
5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4
*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*d - 2*
(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*
b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*d -
 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*e + (a^3*b^4 - 8*a^4*b^2*c + 16*a
^5*c^2)*f)*x^4)*log(c*x^4 + b*x^2 + a) + 4*(((3*b^6*c - 26*a*b^4*c^2 + 64*a
^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e +
 (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^5*c + 6
4*a^2*b^3*c^2 - 32*a^3*b*c^3)*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*
e + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a^2*b^4*
c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^
2)*e + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*f)*x^4)*log(x))/((a^4*b^4*c - 8
*a^5*b^2*c^2 + 16*a^6*c^3)*x^8 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^6
 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^4), 1/4*(2*((3*a*b^5*c - 23*a^2*b
^3*c^2 + 44*a^3*b*c^3)*d - 2*(a^2*b^4*c - 7*a^3*b^2*c^2 + 12*a^4*c^3)*e + (
a^3*b^3*c - 4*a^4*b*c^2)*f)*x^6 + ((6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^
2 - 32*a^4*c^3)*d - 2*(2*a^2*b^5 - 15*a^3*b^3*c + 28*a^4*b*c^2)*e + 2*(a^3*
b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*f)*x^4 + (3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*
b*c^2)*d - 2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*e)*x^2 + 2*(((3*b^5*c - 2
0*a*b^3*c^2 + 30*a^2*b*c^3)*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e +
 (a^2*b^3*c - 6*a^3*b*c^2)*f)*x^8 + ((3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*
d - 2*(a*b^5 - 6*a^2*b^3*c + 6*a^3*b*c^2)*e + (a^2*b^4 - 6*a^3*b^2*c)*f)*x^
6 + ((3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*d - 2*(a^2*b^4 - 6*a^3*b^2*c +
 6*a^4*c^2)*e + (a^3*b^3 - 6*a^4*b*c)*f)*x^4)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (a^3*b^4 - 8*a^4*b^2*c + 16
*a^5*c^2)*d - (((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d -
2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (a^2*b^4*c - 8*a^3*b^2*c^2 +
 16*a^4*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)
*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e + (a^2*b^5 - 8*a^3*b^3*c +
16*a^4*b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c
^3)*d - 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*e + (a^3*b^4 - 8*a^4*b^2*c
 + 16*a^5*c^2)*f)*x^4)*log(c*x^4 + b*x^2 + a) + 4*(((3*b^6*c - 26*a*b^4*c^2
 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c
^3)*e + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^
5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^
2*c^2)*e + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a
^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a
^4*b*c^2)*e + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*f)*x^4)*log(x))/((a^4*b^
4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^8 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c
^2)*x^6 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**5/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 19.6479, size = 722, normalized size = 2.19

$$-\frac{\left(3\,b^{5}d-20\,ab^{3}cd+30\,a^{2}bc^{2}d+a^{2}b^{3}f-6\,a^{3}bcf-2\,ab^{4}e+12\,a^{2}b^{2}ce-12\,a^{3}c^{2}e\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\left(a^{4}b^{2}-4\,a^{5}c\right)\sqrt{-b^{2}+4\,ac}}+\frac{3\,b^{4}cdx^{4}-14\,ab^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^5*d - 20*a*b^3*c*d + 30*a^2*b*c^2*d + a^2*b^3*f - 6*a^3*b*c*f - 2
*a*b^4*e + 12*a^2*b^2*c*e - 12*a^3*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 +
4*a*c))/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(3*b^4*c*d*x^4 - 14*
a*b^2*c^2*d*x^4 + 8*a^2*c^3*d*x^4 + a^2*b^2*c*f*x^4 - 4*a^3*c^2*f*x^4 - 2*a
*b^3*c*x^4*e + 8*a^2*b*c^2*x^4*e + 3*b^5*d*x^2 - 12*a*b^3*c*d*x^2 + 2*a^2*b
*c^2*d*x^2 + a^2*b^3*f*x^2 - 2*a^3*b*c*f*x^2 - 2*a*b^4*x^2*e + 6*a^2*b^2*c*
x^2*e + 4*a^3*c^2*x^2*e + 5*a*b^4*d - 22*a^2*b^2*c*d + 12*a^3*c^2*d + 3*a^3
*b^2*f - 8*a^4*c*f - 4*a^2*b^3*e + 14*a^3*b*c*e)/((a^4*b^2 - 4*a^5*c)*(c*x^
4 + b*x^2 + a)) - 1/4*(3*b^2*d - 2*a*c*d + a^2*f - 2*a*b*e)*log(c*x^4 + b*x
^2 + a)/a^4 + 1/2*(3*b^2*d - 2*a*c*d + a^2*f - 2*a*b*e)*log(x^2)/a^4 - 1/4*
(9*b^2*d*x^4 - 6*a*c*d*x^4 + 3*a^2*f*x^4 - 6*a*b*x^4*e - 4*a*b*d*x^2 + 2*a^
2*x^2*e + a^2*d)/(a^4*x^4)
```

3.68 $\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=550

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{20a^2c^3e-19ab^2c^2e-b^3c(cd-34af)+4abc^2(2cd-13af)+3b^4ce-5b^5f}{\sqrt{b^2-4ac}}-b^2c(cd-24af)-13abc^2e+2ac^2(3cd-7a\right.}{2\sqrt{2}c^{7/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] ((c*e - 2*b*f)*x)/c^3 + (f*x^3)/(3*c^2) + (x*(a*(b^2*c*e - 2*a*c^2*e - b^3*
f - b*c*(c*d - 3*a*f)) + (b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*
f) + 2*a*c^2*(c*d - a*f))*x^2))/(2*c^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
 ((3*b^3*c*e - 13*a*b*c^2*e - 5*b^4*f - b^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c
*d - 7*a*f) - (3*b^4*c*e - 19*a*b^2*c^2*e + 20*a^2*c^3*e - 5*b^5*f - b^3*c*
(c*d - 34*a*f) + 4*a*b*c^2*(2*c*d - 13*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a
*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3*c*e - 13*a*b*c^2*e - 5*b^4*f - b
^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c*d - 7*a*f) + (3*b^4*c*e - 19*a*b^2*c^2*e
 + 20*a^2*c^3*e - 5*b^5*f - b^3*c*(c*d - 34*a*f) + 4*a*b*c^2*(2*c*d - 13*a*
f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 13.2272, antiderivative size = 550, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1668, 1676, 1166, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{20a^2c^3e-19ab^2c^2e-b^3c(cd-34af)+4abc^2(2cd-13af)+3b^4ce-5b^5f}{\sqrt{b^2-4ac}}-b^2c(cd-24af)-13abc^2e+2ac^2(3cd-7a\right.}{2\sqrt{2}c^{7/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((c*e - 2*b*f)*x)/c^3 + (f*x^3)/(3*c^2) + (x*(a*(b^2*c*e - 2*a*c^2*e - b^3*
f - b*c*(c*d - 3*a*f)) + (b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*
f) + 2*a*c^2*(c*d - a*f))*x^2))/(2*c^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
 ((3*b^3*c*e - 13*a*b*c^2*e - 5*b^4*f - b^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c
*d - 7*a*f) - (3*b^4*c*e - 19*a*b^2*c^2*e + 20*a^2*c^3*e - 5*b^5*f - b^3*c*
(c*d - 34*a*f) + 4*a*b*c^2*(2*c*d - 13*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a
*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3*c*e - 13*a*b*c^2*e - 5*b^4*f - b
^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c*d - 7*a*f) + (3*b^4*c*e - 19*a*b^2*c^2*e
 + 20*a^2*c^3*e - 5*b^5*f - b^3*c*(c*d - 34*a*f) + 4*a*b*c^2*(2*c*d - 13*a*
f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
```

```
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx=\frac{x\left(a\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)+\left(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-a\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$=\frac{x\left(a\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)+\left(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-a\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$=\frac{(ce-2bf)x}{c^3}+\frac{fx^3}{3c^2}+\frac{x\left(a\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)+\left(b^3ce-3abc^2e-b^4f-b^2c(c\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$=\frac{(ce-2bf)x}{c^3}+\frac{fx^3}{3c^2}+\frac{x\left(a\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)+\left(b^3ce-3abc^2e-b^4f-b^2c(c\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$=\frac{(ce-2bf)x}{c^3}+\frac{fx^3}{3c^2}+\frac{x\left(a\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)+\left(b^3ce-3abc^2e-b^4f-b^2c(c\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 2.3131, size = 648, normalized size = 1.18

$$-\frac{6\sqrt{c}x\left(a^2c\left(2c\left(e+fx^2\right)-3bf\right)+a\left(-b^2c\left(e+4fx^2\right)+b^3f+bc^2\left(d+3ex^2\right)-2c^3dx^2\right)+b^2x^2\left(b^2f-bce+c^2d\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(abc^2\left(13e\sqrt{b^2-4ac}-52af+8cd\right)+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (12*Sqrt[c]*(c*e - 2*b*f)*x + 4*c^(3/2)*f*x^3 - (6*Sqrt[c]*x*(b^2*(c^2*d -
b*c*e + b^2*f)*x^2 + a^2*c*(-3*b*f + 2*c*(e + f*x^2)) + a*(b^3*f - 2*c^3*d*
x^2 + b*c^2*(d + 3*e*x^2) - b^2*c*(e + 4*f*x^2))))/((b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)) + (3*Sqrt[2]*(-5*b^5*f + a*b*c^2*(8*c*d + 13*Sqrt[b^2 - 4*a*c]*
e - 52*a*f) - b^3*c*(c*d + 3*Sqrt[b^2 - 4*a*c]*e - 34*a*f) + b^4*(3*c*e + 5
*Sqrt[b^2 - 4*a*c]*f) + b^2*c*(c*Sqrt[b^2 - 4*a*c]*d - 19*a*c*e - 24*a*Sqrt
[b^2 - 4*a*c]*f) + 2*a*c^2*(-3*c*Sqrt[b^2 - 4*a*c]*d + 10*a*c*e + 7*a*Sqrt[
b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(5*b^5*f + b^
3*c*(c*d - 3*Sqrt[b^2 - 4*a*c]*e - 34*a*f) + a*b*c^2*(-8*c*d + 13*Sqrt[b^2
- 4*a*c]*e + 52*a*f) + b^4*(-3*c*e + 5*Sqrt[b^2 - 4*a*c]*f) + b^2*c*(c*Sqrt
[b^2 - 4*a*c]*d + 19*a*c*e - 24*a*Sqrt[b^2 - 4*a*c]*f) - 2*a*c^2*(3*c*Sqrt[
b^2 - 4*a*c]*d + 10*a*c*e - 7*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
- 4*a*c]]))/(12*c^(7/2))
```

Maple [B] time = 0.053, size = 2558, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -13/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/
2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a^2*b*f+17/2/c^2/(
4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arct
anh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*b^3*f-19/4/c/(4*a*c-b^2
)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2
^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*b^2*e-13/c/(4*a*c-b^2)/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*b*f+17/2/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*a*b^3*f-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*a*b^2*e+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*x*e+1/c/(c*x^4+b*x^2
+a)/(4*a*c-b^2)*x^3*a^2*f-1/2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b^3*e+1/2
/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b^2*d+1/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^2
)*x^3*b^4*f+3/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*d-3/2/(4*a*c-b^2)*2^(1/2)/
(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b
)*c)^(1/2))*a*d+1/c^2*e*x-2/c^3*b*f*x-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*d
+1/3*f*x^3/c^2+7/2/c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*a
rctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a^2*f+5/4/c^3/(4*a*c-b
^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+
b^2)^(1/2)-b)*c)^(1/2))*b^4*f-3/4/c^2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1
/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*e
+1/4/c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(
1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*d-7/2/c/(4*a*c-b^2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*a^2*f+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*d+2/(4
*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arcta
nh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*b*d-5/4/c^3/(4*a*c-b^2)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5*f+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
```

```
*a*c+b^2)^(1/2))*c)^(1/2))*b^4*e-1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))*b^3*d-5/4/c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*
c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/
2))*b^5*f+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/
2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^4*e-
1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/
2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*d-6/c^2/(4*a*c
-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*
c+b^2)^(1/2)-b)*c)^(1/2))*a*b^2*f+13/4/c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)
^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*
b*e+6/c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2*f-13/4/c/(4*a*c-b^2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))*a*b*e+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*e
+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*
arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a^2*e+1/2/c^3/(c*x^4+
b*x^2+a)*a/(4*a*c-b^2)*x*b^3*f-2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*b^2*
f+3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*b*e-3/2/c^2/(c*x^4+b*x^2+a)*a^2/(
4*a*c-b^2)*x*b*f-1/2/c^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*b^2*e+1/2/c/(c*x^4
+b*x^2+a)*a/(4*a*c-b^2)*x*b*d-5/4/c^3/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*f+
3/4/c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*e-1/4/c/(4*a*c-b^2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*b^2*d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.69 $\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=436

$$\frac{x\left(a\left(-2acf+b^2f-bce+2c^2d\right)-x^2\left(-bc(cd-3af)-2ac^2e+b^2ce+b^3(-f)\right)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{b^2c(19af+cd)-8abc^2e}{\sqrt{}}\right.}{2\sqrt{2}c^5}$$

```
[Out] (f*x)/c^2 + (x*(a*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f) - (b^2*c*e - 2*a*c^2*
e - b^3*f - b*c*(c*d - 3*a*f))*x^2))/(2*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + ((b^2*c*e - 6*a*c^2*e - 3*b^3*f + b*c*(c*d + 13*a*f) - (b^3*c*e - 8*a
*b*c^2*e - 3*b^4*f + 4*a*c^2*(c*d - 5*a*f) + b^2*c*(c*d + 19*a*f))/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt
[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*c*e - 6*a*c^
2*e - 3*b^3*f + b*c*(c*d + 13*a*f) + (b^3*c*e - 8*a*b*c^2*e - 3*b^4*f + 4*a
*c^2*(c*d - 5*a*f) + b^2*c*(c*d + 19*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c
)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 5.54118, antiderivative size = 436, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1668, 1676, 1166, 205}

$$\frac{x\left(a\left(-2acf+b^2f-bce+2c^2d\right)-x^2\left(-bc(cd-3af)-2ac^2e+b^2ce+b^3(-f)\right)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{b^2c(19af+cd)-8abc^2e}{\sqrt{}}\right.}{2\sqrt{2}c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (f*x)/c^2 + (x*(a*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f) - (b^2*c*e - 2*a*c^2*
e - b^3*f - b*c*(c*d - 3*a*f))*x^2))/(2*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + ((b^2*c*e - 6*a*c^2*e - 3*b^3*f + b*c*(c*d + 13*a*f) - (b^3*c*e - 8*a
*b*c^2*e - 3*b^4*f + 4*a*c^2*(c*d - 5*a*f) + b^2*c*(c*d + 19*a*f))/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt
[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*c*e - 6*a*c^
2*e - 3*b^3*f + b*c*(c*d + 13*a*f) + (b^3*c*e - 8*a*b*c^2*e - 3*b^4*f + 4*a
*c^2*(c*d - 5*a*f) + b^2*c*(c*d + 19*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c
)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
```

```
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}dx = \frac{x\left(a\left(2c^2d-bce+b^2f-2acf\right)-\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)x^2\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{a^2\left(2c^2d+b^2f-c(be\cdots\right.}{c^2}}{\cdots}$$

$$= \frac{x\left(a\left(2c^2d-bce+b^2f-2acf\right)-\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)x^2\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int\left(-\frac{2a\left(b^2-4ac\right)f\cdots}{c^2}\cdots\right.}{\cdots}$$

$$= \frac{fx}{c^2} + \frac{x\left(a\left(2c^2d-bce+b^2f-2acf\right)-\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)x^2\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{a^2\left(2c^2d\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{fx}{c^2} + \frac{x\left(a\left(2c^2d-bce+b^2f-2acf\right)-\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)x^2\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2ce-\cdots\right.}{\cdots}$$

$$= \frac{fx}{c^2} + \frac{x\left(a\left(2c^2d-bce+b^2f-2acf\right)-\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)x^2\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2ce-\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.73961, size = 511, normalized size = 1.17

$$\frac{\frac{2\sqrt{c}x\left(-2a^2cf+a\left(b^2f-bc\left(e+3fx^2\right)+2c^2\left(d+ex^2\right)\right)+bx^2\left(b^2f-bce+c^2d\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(2ac^2\left(3e\sqrt{b^2-4ac}-10af+2cd\right)+b^2c\left(-e\sqrt{b^2-4ac}+19af+cd\right)\cdots\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-\cdots}}}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*Sqrt[c]*f*x + (2*Sqrt[c]*x*(-2*a^2*c*f + b*(c^2*d - b*c*e + b^2*f)*x^2 +
 a*(b^2*f + 2*c^2*(d + e*x^2) - b*c*(e + 3*f*x^2))))/((b^2 - 4*a*c)*(a + b*
```

```
x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4*f + 2*a*c^2*(2*c*d + 3*Sqrt[b^2 - 4*a*c]*e
 - 10*a*f) + b^2*c*(c*d - Sqrt[b^2 - 4*a*c]*e + 19*a*f) + b^3*(c*e + 3*Sqrt
[b^2 - 4*a*c]*f) - b*c*(c*Sqrt[b^2 - 4*a*c]*d + 8*a*c*e + 13*a*Sqrt[b^2 - 4
*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 -
4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4*f + 2*a*c^2*(-2
*c*d + 3*Sqrt[b^2 - 4*a*c]*e + 10*a*f) - b^2*c*(c*d + Sqrt[b^2 - 4*a*c]*e +
 19*a*f) + b^3*(-(c*e) + 3*Sqrt[b^2 - 4*a*c]*f) - b*c*(c*Sqrt[b^2 - 4*a*c]*
d - 8*a*c*e + 13*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(
4*c^(5/2))
```

Maple [B] time = 0.042, size = 1977, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -3/4/c^2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2
^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*f+1/4/c/(4*a*c-b^2)*2^(1/2)/((
(-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*
c)^(1/2))*b^2*e+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/
2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a^2*f-
1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*d+5/(4*a*c-b^2)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*f-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*b^2*d+3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*b*f+1/
2/c/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*b*e-1/2/c^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^
2)*x*b^2*f+3/4/c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*f-1/4/c/(4*a*c-b^2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*b^2*e-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4
*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^
(1/2))*a*b^2*f-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b
^2*f-1/2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b^3*f+1/2/c/(c*x^4+b*x^2+a)/(4
*a*c-b^2)*x^3*b^2*e+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*x*f+1/4/(4*a*c-b^2)
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2))*b*d+3/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*e-1/4/(4*a*c-
b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*b*d-3/2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)
*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*e-1/(c*x^
4+b*x^2+a)*a/(4*a*c-b^2)*x*d-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*e-1/2/(c*x
^4+b*x^2+a)/(4*a*c-b^2)*x^3*b*d+f*x/c^2+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1
/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+
b^2)^(1/2)-b)*c)^(1/2))*b^4*f-1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/
(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b
)*c)^(1/2))*b^3*e-13/4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*f-c/(4*a*c-b^2)
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*d+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*b^4*f-1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2
```

```
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*b^3*e+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^
(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*b
*e+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*e+13/4/c/(4*a*c-
b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c
+b^2)^(1/2)-b)*c)^(1/2))*a*b*f-c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((
-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c
)^(1/2))*a*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(bc^2d-\left(b^2c-2\,ac^2\right)e+\left(b^3-3\,abc\right)f\right)x^3+\left(2\,ac^2d-abce+\left(ab^2-2\,a^2c\right)f\right)x}{2\left(ab^2c^2-4\,a^2c^3+\left(b^2c^3-4\,ac^4\right)x^4+\left(b^3c^2-4\,abc^3\right)x^2\right)}+\frac{fx}{c^2}+\frac{-\int\frac{2\,ac^2d-abce-\left(bc^2d+\left(b^2c-6\,ac^2\right)e-(3}{cx^4+bx}}{2\left(b^2c^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)*f)*x^3 + (2*a*c^2*d -
 a*b*c*e + (a*b^2 - 2*a^2*c)*f)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*
c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + f*x/c^2 + 1/2*integrate(-(2*a*c^2*d
 - a*b*c*e - (b*c^2*d + (b^2*c - 6*a*c^2)*e - (3*b^3 - 13*a*b*c)*f)*x^2 + (
3*a*b^2 - 10*a^2*c)*f)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4*a*c^3)
```

Fricas [B] time = 75.8106, size = 26999, normalized size = 61.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^2*c - 4*a*c^2)*f*x^5 + 2*(b*c^2*d - (b^2*c - 2*a*c^2)*e + (3*b^3
- 11*a*b*c)*f)*x^3 + sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)
*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^
4*c^3 - 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*
c^4)*e^2 + (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*
((3*b^5*c^2 - 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 15
0*a^2*b^2*c^3 - 120*a^3*c^4)*e)*f + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^
7 - 64*a^3*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e
^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*
e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4
*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)
*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 +
 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^
4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a
^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)
*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*
a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c
^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)
)*log(((3*b^2*c^6 + 4*a*c^7)*d^4 + (9*b^3*c^5 - 20*a*b*c^6)*d^3*e + 3*(3*b^
4*c^4 - 28*a*b^2*c^5)*d^2*e^2 + (3*b^5*c^3 - 65*a*b^3*c^4 + 324*a^2*b*c^5)*
d*e^3 - (5*a*b^4*c^3 - 81*a^2*b^2*c^4 + 324*a^3*c^5)*e^4 - (189*a^2*b^6 - 1
```

```
971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*f^4 - ((81*b^8 - 945*a*b^6
*c + 3213*a^2*b^4*c^2 - 3000*a^3*b^2*c^3 + 2000*a^4*c^4)*d - (135*a*b^7 - 1
323*a^2*b^5*c + 2727*a^3*b^3*c^2 + 2500*a^4*b*c^3)*e)*f^3 + 3*((27*b^6*c^2
- 117*a*b^4*c^3 - 150*a^2*b^2*c^4 + 200*a^3*c^5)*d^2 + (27*b^7*c - 405*a*b^
5*c^2 + 1461*a^2*b^3*c^3 - 500*a^3*b*c^4)*d*e - (45*a*b^6*c - 558*a^2*b^4*c
^2 + 1672*a^3*b^2*c^3)*e^2)*f^2 - ((27*b^4*c^4 + 80*a^2*c^6)*d^3 + 3*(18*b^
5*c^3 - 123*a*b^3*c^4 - 100*a^2*b*c^5)*d^2*e + 3*(9*b^6*c^2 - 165*a*b^4*c^3
 + 692*a^2*b^2*c^4)*d*e^2 - (45*a*b^5*c^2 - 647*a^2*b^3*c^3 + 2268*a^3*b*c^
4)*e^3)*f)*x + 1/2*sqrt(1/2)*(2*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^3 +
3*(b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*d^2*e - 18*(a*b^4*c^5 - 8*a^2*b^2*
c^6 + 16*a^3*c^7)*d*e^2 - (b^7*c^3 - 17*a*b^5*c^4 + 88*a^2*b^3*c^5 - 144*a^
3*b*c^6)*e^3 + (27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3
 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5)*f^3 - 3*(2*(12*a*b^6*c^3 - 121*a^2*b^4
*c^4 + 392*a^3*b^2*c^5 - 400*a^4*c^6)*d + (9*b^9*c - 153*a*b^7*c^2 + 947*a^
2*b^5*c^3 - 2536*a^3*b^3*c^4 + 2480*a^4*b*c^5)*e)*f^2 - 3*((3*b^6*c^4 - 14*
a*b^4*c^5 - 32*a^2*b^2*c^6 + 160*a^3*c^7)*d^2 - 26*(a*b^5*c^4 - 8*a^2*b^3*c
^5 + 16*a^3*b*c^6)*d*e - 3*(b^8*c^2 - 17*a*b^6*c^3 + 98*a^2*b^4*c^4 - 224*a
^3*b^2*c^5 + 160*a^4*c^6)*e^2)*f + (4*(b^7*c^7 - 12*a*b^5*c^8 + 48*a^2*b^3*
c^9 - 64*a^3*b*c^10)*d + (b^8*c^6 - 24*a*b^6*c^7 + 192*a^2*b^4*c^8 - 640*a^
3*b^2*c^9 + 768*a^4*c^10)*e - (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 -
 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*f)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^
2*c^6 - 3*a*c^7)*d^2*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*
b^2*c^5 + 81*a^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550
*a^3*b^2*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*
b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 5
50*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9
*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 +
484*a^2*b^2*c^4 - 75*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(
3*b^3*c^5 - 4*a*b*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*
e^2 + (3*b^5*c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^
2 + 2*(b^4*c^3 - 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 +
60*a^2*b*c^4)*e^2 + (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)
*f^2 - 2*((3*b^5*c^2 - 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4
*c^2 + 150*a^2*b^2*c^3 - 120*a^3*c^4)*e)*f + (b^6*c^5 - 12*a*b^4*c^6 + 48*a
^2*b^2*c^7 - 64*a^3*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c
^7)*d^2*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*
a^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3
+ 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125
*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)
*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51
*a*b^3*c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c
^4 - 75*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4
*a*b*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*
c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64
*a^3*c^8))) - sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 +
(b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^4*c^3 -
 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*c^4)*e^
2 + (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^5
*c^2 - 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*a^2*b
^2*c^3 - 120*a^3*c^4)*e)*f + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*
a^3*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 + 4*
(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*e^4 + (
81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*f
^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)*d + (2
7*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*
b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*
a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^5)
```

```
*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)*d^2*e
+ 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*a*b^3*c
^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 6
4*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log((
(3*b^2*c^6 + 4*a*c^7)*d^4 + (9*b^3*c^5 - 20*a*b*c^6)*d^3*e + 3*(3*b^4*c^4 -
 28*a*b^2*c^5)*d^2*e^2 + (3*b^5*c^3 - 65*a*b^3*c^4 + 324*a^2*b*c^5)*d*e^3 -
 (5*a*b^4*c^3 - 81*a^2*b^2*c^4 + 324*a^3*c^5)*e^4 - (189*a^2*b^6 - 1971*a^3
*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*f^4 - ((81*b^8 - 945*a*b^6*c + 32
13*a^2*b^4*c^2 - 3000*a^3*b^2*c^3 + 2000*a^4*c^4)*d - (135*a*b^7 - 1323*a^2
*b^5*c + 2727*a^3*b^3*c^2 + 2500*a^4*b*c^3)*e)*f^3 + 3*((27*b^6*c^2 - 117*a
*b^4*c^3 - 150*a^2*b^2*c^4 + 200*a^3*c^5)*d^2 + (27*b^7*c - 405*a*b^5*c^2 +
 1461*a^2*b^3*c^3 - 500*a^3*b*c^4)*d*e - (45*a*b^6*c - 558*a^2*b^4*c^2 + 16
72*a^3*b^2*c^3)*e^2)*f^2 - ((27*b^4*c^4 + 80*a^2*c^6)*d^3 + 3*(18*b^5*c^3 -
 123*a*b^3*c^4 - 100*a^2*b*c^5)*d^2*e + 3*(9*b^6*c^2 - 165*a*b^4*c^3 + 692*
a^2*b^2*c^4)*d*e^2 - (45*a*b^5*c^2 - 647*a^2*b^3*c^3 + 2268*a^3*b*c^4)*e^3)
*f)*x - 1/2*sqrt(1/2)*(2*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^3 + 3*(b^5*
c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*d^2*e - 18*(a*b^4*c^5 - 8*a^2*b^2*c^6 + 1
6*a^3*c^7)*d*e^2 - (b^7*c^3 - 17*a*b^5*c^4 + 88*a^2*b^3*c^5 - 144*a^3*b*c^6
)*e^3 + (27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3 + 1136
0*a^4*b^2*c^4 - 4000*a^5*c^5)*f^3 - 3*(2*(12*a*b^6*c^3 - 121*a^2*b^4*c^4 +
392*a^3*b^2*c^5 - 400*a^4*c^6)*d + (9*b^9*c - 153*a*b^7*c^2 + 947*a^2*b^5*c
^3 - 2536*a^3*b^3*c^4 + 2480*a^4*b*c^5)*e)*f^2 - 3*((3*b^6*c^4 - 14*a*b^4*c
^5 - 32*a^2*b^2*c^6 + 160*a^3*c^7)*d^2 - 26*(a*b^5*c^4 - 8*a^2*b^3*c^5 + 16
*a^3*b*c^6)*d*e - 3*(b^8*c^2 - 17*a*b^6*c^3 + 98*a^2*b^4*c^4 - 224*a^3*b^2*
c^5 + 160*a^4*c^6)*e^2)*f + (4*(b^7*c^7 - 12*a*b^5*c^8 + 48*a^2*b^3*c^9 - 6
4*a^3*b*c^10)*d + (b^8*c^6 - 24*a*b^6*c^7 + 192*a^2*b^4*c^8 - 640*a^3*b^2*c
^9 + 768*a^4*c^10)*e - (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 - 960*a^
3*b^3*c^8 + 1024*a^4*b*c^9)*f)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 -
 3*a*c^7)*d^2*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5
 + 81*a^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^
2*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4
 + 125*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*
b*c^4)*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^
3 - 51*a*b^3*c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2
*b^2*c^4 - 75*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c
^5 - 4*a*b*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (
3*b^5*c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(
b^4*c^3 - 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*
b*c^4)*e^2 + (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 -
2*((3*b^5*c^2 - 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 +
150*a^2*b^2*c^3 - 120*a^3*c^4)*e)*f + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*
c^7 - 64*a^3*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2
*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6
)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a
^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^
5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3
 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*
c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75
*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^
6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 4
9*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2
*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^
8))) + sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^
2 - 4*a*b*c^3)*x^2)*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^4*c^3 - 6*a*b^
2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*c^4)*e^2 + (9*
b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^5*c^2 -
13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*a^2*b^2*c^3
 - 120*a^3*c^4)*e)*f - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8
```

)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 + 4*(b^3*c^
5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*e^4 + (81*b^8
- 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*f^4 - 4*
((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c
 - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c^4
 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b*c
^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^5)*e^2)*f
^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)*d^2*e + 3*(3*
b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*a*b^3*c^4 + 19
8*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c
^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log(((3*b^2*
c^6 + 4*a*c^7)*d^4 + (9*b^3*c^5 - 20*a*b*c^6)*d^3*e + 3*(3*b^4*c^4 - 28*a*b
^2*c^5)*d^2*e^2 + (3*b^5*c^3 - 65*a*b^3*c^4 + 324*a^2*b*c^5)*d*e^3 - (5*a*b
^4*c^3 - 81*a^2*b^2*c^4 + 324*a^3*c^5)*e^4 - (189*a^2*b^6 - 1971*a^3*b^4*c
+ 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*f^4 - ((81*b^8 - 945*a*b^6*c + 3213*a^2*
b^4*c^2 - 3000*a^3*b^2*c^3 + 2000*a^4*c^4)*d - (135*a*b^7 - 1323*a^2*b^5*c
+ 2727*a^3*b^3*c^2 + 2500*a^4*b*c^3)*e)*f^3 + 3*((27*b^6*c^2 - 117*a*b^4*c^
3 - 150*a^2*b^2*c^4 + 200*a^3*c^5)*d^2 + (27*b^7*c - 405*a*b^5*c^2 + 1461*a
^2*b^3*c^3 - 500*a^3*b*c^4)*d*e - (45*a*b^6*c - 558*a^2*b^4*c^2 + 1672*a^3*
b^2*c^3)*e^2)*f^2 - ((27*b^4*c^4 + 80*a^2*c^6)*d^3 + 3*(18*b^5*c^3 - 123*a*
b^3*c^4 - 100*a^2*b*c^5)*d^2*e + 3*(9*b^6*c^2 - 165*a*b^4*c^3 + 692*a^2*b^2
*c^4)*d*e^2 - (45*a*b^5*c^2 - 647*a^2*b^3*c^3 + 2268*a^3*b*c^4)*e^3)*f)*x +
 1/2*sqrt(1/2)*(2*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^3 + 3*(b^5*c^5 - 8
*a*b^3*c^6 + 16*a^2*b*c^7)*d^2*e - 18*(a*b^4*c^5 - 8*a^2*b^2*c^6 + 16*a^3*c
^7)*d*e^2 - (b^7*c^3 - 17*a*b^5*c^4 + 88*a^2*b^3*c^5 - 144*a^3*b*c^6)*e^3 +
 (27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3 + 11360*a^4*b
^2*c^4 - 4000*a^5*c^5)*f^3 - 3*(2*(12*a*b^6*c^3 - 121*a^2*b^4*c^4 + 392*a^3
*b^2*c^5 - 400*a^4*c^6)*d + (9*b^9*c - 153*a*b^7*c^2 + 947*a^2*b^5*c^3 - 25
36*a^3*b^3*c^4 + 2480*a^4*b*c^5)*e)*f^2 - 3*((3*b^6*c^4 - 14*a*b^4*c^5 - 32
*a^2*b^2*c^6 + 160*a^3*c^7)*d^2 - 26*(a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*
c^6)*d*e - 3*(b^8*c^2 - 17*a*b^6*c^3 + 98*a^2*b^4*c^4 - 224*a^3*b^2*c^5 + 1
60*a^4*c^6)*e^2)*f - (4*(b^7*c^7 - 12*a*b^5*c^8 + 48*a^2*b^3*c^9 - 64*a^3*b
*c^10)*d + (b^8*c^6 - 24*a*b^6*c^7 + 192*a^2*b^4*c^8 - 640*a^3*b^2*c^9 + 76
8*a^4*c^10)*e - (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c
^8 + 1024*a^4*b*c^9)*f)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^
7)*d^2*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a
^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*
a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*
e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*
a*b^3*c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^
4 - 75*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*
a*b*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c
^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a
^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^4*c^3
 - 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*c^4)*
e^2 + (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b
^5*c^2 - 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*a^2
*b^2*c^3 - 120*a^3*c^4)*e)*f - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 6
4*a^3*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 +
4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*e^4 +
 (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)
f^4 - 4((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)*d +
(27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((
9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 6
5*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^
5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)*d^2*
e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*a*b^3
*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 -

```
 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) -
sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a
*b*c^3)*x^2)*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^4*c^3 - 6*a*b^2*c^4 -
 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*c^4)*e^2 + (9*b^7 - 1
05*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^5*c^2 - 13*a*b^
3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*a^2*b^2*c^3 - 120*a
^3*c^4)*e)*f - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt(
(c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 + 4*(b^3*c^5 - 9*a
*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*e^4 + (81*b^8 - 918*a
*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^
6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c - 351*
a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c^4 + 3*a*
b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b*c^5)*d*e
 + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^5)*e^2)*f^2 - 4*
((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)*d^2*e + 3*(3*b^4*c^4
 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*a*b^3*c^4 + 198*a^2*b
*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/
(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log(((3*b^2*c^6 + 4
*a*c^7)*d^4 + (9*b^3*c^5 - 20*a*b*c^6)*d^3*e + 3*(3*b^4*c^4 - 28*a*b^2*c^5)
*d^2*e^2 + (3*b^5*c^3 - 65*a*b^3*c^4 + 324*a^2*b*c^5)*d*e^3 - (5*a*b^4*c^3
 - 81*a^2*b^2*c^4 + 324*a^3*c^5)*e^4 - (189*a^2*b^6 - 1971*a^3*b^4*c + 5625*
a^4*b^2*c^2 - 2500*a^5*c^3)*f^4 - ((81*b^8 - 945*a*b^6*c + 3213*a^2*b^4*c^2
 - 3000*a^3*b^2*c^3 + 2000*a^4*c^4)*d - (135*a*b^7 - 1323*a^2*b^5*c + 2727*
a^3*b^3*c^2 + 2500*a^4*b*c^3)*e)*f^3 + 3*((27*b^6*c^2 - 117*a*b^4*c^3 - 150
*a^2*b^2*c^4 + 200*a^3*c^5)*d^2 + (27*b^7*c - 405*a*b^5*c^2 + 1461*a^2*b^3*
c^3 - 500*a^3*b*c^4)*d*e - (45*a*b^6*c - 558*a^2*b^4*c^2 + 1672*a^3*b^2*c^3
)*e^2)*f^2 - ((27*b^4*c^4 + 80*a^2*c^6)*d^3 + 3*(18*b^5*c^3 - 123*a*b^3*c^4
 - 100*a^2*b*c^5)*d^2*e + 3*(9*b^6*c^2 - 165*a*b^4*c^3 + 692*a^2*b^2*c^4)*d
*e^2 - (45*a*b^5*c^2 - 647*a^2*b^3*c^3 + 2268*a^3*b*c^4)*e^3)*f)*x - 1/2*sq
rt(1/2)*(2*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^3 + 3*(b^5*c^5 - 8*a*b^3*
c^6 + 16*a^2*b*c^7)*d^2*e - 18*(a*b^4*c^5 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*d*e
^2 - (b^7*c^3 - 17*a*b^5*c^4 + 88*a^2*b^3*c^5 - 144*a^3*b*c^6)*e^3 + (27*b^
10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4
 - 4000*a^5*c^5)*f^3 - 3*(2*(12*a*b^6*c^3 - 121*a^2*b^4*c^4 + 392*a^3*b^2*c^
5 - 400*a^4*c^6)*d + (9*b^9*c - 153*a*b^7*c^2 + 947*a^2*b^5*c^3 - 2536*a^3*
b^3*c^4 + 2480*a^4*b*c^5)*e)*f^2 - 3*((3*b^6*c^4 - 14*a*b^4*c^5 - 32*a^2*b^
2*c^6 + 160*a^3*c^7)*d^2 - 26*(a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*d*
e - 3*(b^8*c^2 - 17*a*b^6*c^3 + 98*a^2*b^4*c^4 - 224*a^3*b^2*c^5 + 160*a^4*
c^6)*e^2)*f - (4*(b^7*c^7 - 12*a*b^5*c^8 + 48*a^2*b^3*c^9 - 64*a^3*b*c^10)*
d + (b^8*c^6 - 24*a*b^6*c^7 + 192*a^2*b^4*c^8 - 640*a^3*b^2*c^9 + 768*a^4*c
^10)*e - (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 10
24*a^4*b*c^9)*f)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*
e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)
*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^
4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5
)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3
+ 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c
^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*
a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6
)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49
*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*
c^12 - 64*a^3*c^13)))*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^4*c^3 - 6*a*
b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*c^4)*e^2 + (
9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^5*c^2
- 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*a^2*b^2*c^
3 - 120*a^3*c^4)*e)*f - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c
^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 + 4*(b^3*
c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*e^4 + (81*b^
8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*f^4 -
```

```
4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)*d + (27*b^7
*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c
^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b
*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^5)*e^2)
*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)*d^2*e + 3*(
3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*a*b^3*c^4 +
198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3
*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + 2*(2*a*
c^2*d - a*b*c*e + (3*a*b^2 - 10*a^2*c)*f)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*
c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.70 $\int \frac{x^2\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=362

$$-\frac{x\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)+abf-2ace+bcd\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-4bc(2af+cd)+4ac^2e+b^2ce+b^3f}{c\sqrt{b^2-4ac}}+6af-\frac{b^2f}{c}-\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] -(x*(b*c*d - 2*a*c*e + a*b*f + (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2))/(2
*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*c*d - b*e + 6*a*f - (b^2*f)/c +
 (b^2*c*e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d + 2*a*f))/(c*Sqrt[b^2 - 4*a*c]))
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c
]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*c*d - b*e + 6*a*f - (b^2
*f)/c - (b^2*c*e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d + 2*a*f))/(c*Sqrt[b^2 - 4
*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 2.49751, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1668, 1166, 205}

$$-\frac{x\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)+abf-2ace+bcd\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-4bc(2af+cd)+4ac^2e+b^2ce+b^3f}{c\sqrt{b^2-4ac}}+6af-\frac{b^2f}{c}-\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] -(x*(b*c*d - 2*a*c*e + a*b*f + (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2))/(2
*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*c*d - b*e + 6*a*f - (b^2*f)/c +
 (b^2*c*e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d + 2*a*f))/(c*Sqrt[b^2 - 4*a*c]))
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c
]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*c*d - b*e + 6*a*f - (b^2
*f)/c - (b^2*c*e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d + 2*a*f))/(c*Sqrt[b^2 - 4
*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```


Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{x\left(bcd-2ace+abf+\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-\frac{a(bcd-2ace+abf)}{c}+a\left(2cd-be+6af-\frac{b^2f}{c}\right)x^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{x\left(bcd-2ace+abf+\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2cd-be+6af-\frac{b^2f}{c}-\frac{b^2ce+4ac^2e+b^3}{c\sqrt{b^2}}\right.}{4\left(b^2-\right.}$$

$$= -\frac{x\left(bcd-2ace+abf+\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2cd-be+6af-\frac{b^2f}{c}+\frac{b^2ce+4ac^2e+b^3}{c\sqrt{b^2}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)}$$

Mathematica [A] time = 1.23456, size = 414, normalized size = 1.14

$$\frac{-\frac{2\sqrt{c}x\left(abf-2ac\left(e+fx^2\right)+b^2fx^2+bc\left(d-ex^2\right)+2c^2dx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(bc\left(e\sqrt{b^2-4ac}+8af+4cd\right)-2c\left(cd\sqrt{b^2-4ac}+3af\sqrt{b^2-4ac}+2ace\right)+b^2\left(f\sqrt{b^2-4ac}-\right.\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*Sqrt[c]*x*(a*b*f + 2*c^2*d*x^2 + b^2*f*x^2 + b*c*(d - e*x^2) - 2*a*c*(
e + f*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(b^3*f) + b*c
*(4*c*d + Sqrt[b^2 - 4*a*c]*e + 8*a*f) + b^2*(-(c*e) + Sqrt[b^2 - 4*a*c]*f)
 - 2*c*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e + 3*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3*f + b*c*(-4*c*d + Sqrt[b^2 - 4*a*c
]*e - 8*a*f) + b^2*(c*e + Sqrt[b^2 - 4*a*c]*f) - 2*c*(c*Sqrt[b^2 - 4*a*c]*d
 - 2*a*c*e + 3*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*
c^(3/2))
```

Maple [B] time = 0.035, size = 1300, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2*(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)/c*x^3+1/2/c*(a*b*f-2*a*c*e+
b*c*d)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)-3/2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a
*f+1/4/(4*a*c-b^2)/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2
^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*f+1/4/(4*a*c-b^2)*2^(1/2)/(((-
4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)
^(1/2))*b*e-1/2/(4*a*c-b^2)*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arct
anh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d+2/(4*a*c-b^2)/(-4*a*c+b
^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-
4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*b*f-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/
2)-b)*c)^(1/2))*a*e-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+
b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
)*b^3*f-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*
c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*e+1/(4*a
*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arcta
nh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d+3/2/(4*a*c-b^2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*a*f-1/4/(4*a*c-b^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*f-1/4/(4*a*c-b^2)
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*b*e+1/2/(4*a*c-b^2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d+2/(4*a*c-b^2
)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*f-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*a*e-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*b^3*f-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e
+1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(2\,c^{2}d-bce+\left(b^{2}-2\,ac\right)f\right)x^{3}+\left(bcd-2\,ace+abf\right)x}{2\,\left(\left(b^{2}c^{2}-4\,ac^{3}\right)x^{4}+ab^{2}c-4\,a^{2}c^{2}+\left(b^{3}c-4\,abc^{2}\right)x^{2}\right)}-\frac{-\int\frac{bcd-2\,ace+abf-\left(2\,c^{2}d-bce-\left(b^{2}-6\,ac\right)f\right)x^{2}}{cx^{4}+bx^{2}+a}\,dx}{2\,\left(b^{2}c-4\,ac^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*x^3 + (b*c*d - 2*a*c*e + a*b*f)*x
)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
 - 1/2*integrate(-(b*c*d - 2*a*c*e + a*b*f - (2*c^2*d - b*c*e - (b^2 - 6*a*
c)*f)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Fricas [B] time = 37.949, size = 18090, normalized size = 49.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] -1/4*(2*(2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*x^3 + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f + (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))*log(((3*b^2*c^5 + 4*a*c^6)*d^4 - (b^3*c^4 + 12*a*b*c^5)*d^3*e + (a*b^3*c^3 + 12*a^2*b*c^4)*d*e^3 - (3*a^2*b^2*c^3 + 4*a^3*c^4)*e^4 + (5*a^3*b^4 - 81*a^4*b^2*c + 324*a^5*c^2)*f^4 + ((a*b^6 - 15*a^2*b^4*c + 432*a^4*c^3)*d - (3*a^2*b^5 - 65*a^3*b^3*c + 324*a^4*b*c^2)*e)*f^3 - 3*(3*(a*b^4*c^2 - 6*a^2*b^2*c^3 - 24*a^3*c^4)*d^2 - (a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d*e + (3*a^2*b^4*c - 28*a^3*b^2*c^2)*e^2)*f^2 - ((b^4*c^3 - 24*a*b^2*c^4 - 48*a^2*c^5)*d^3 + 9*(a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e - 3*(a*b^4*c^2 + 12*a^2*b^2*c^3)*d*e^2 + (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*e^3)*f)*x + 1/2*sqrt(1/2)*((b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*d^3 - 2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^2*e - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d*e^2 + 2*(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5)*e^3 - (a^2*b^7 - 17*a^3*b^5*c + 88*a^4*b^3*c^2 - 144*a^5*b*c^3)*f^3 - ((a*b^7*c - 23*a^2*b^5*c^2 + 136*a^3*b^3*c^3 - 240*a^4*b*c^4)*d + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e)*f^2 + (7*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^2 - 2*(a*b^6*c^2 - 2*a^2*b^4*c^3 - 32*a^3*b^2*c^4 + 96*a^4*c^5)*d*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*e^2)*f - ((a*b^8*c^4 - 8*a^2*b^6*c^5 + 128*a^4*b^2*c^7 - 256*a^5*c^8)*d - 4*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*e - (a^2*b^8*c^3 - 24*a^3*b^6*c^4 + 192*a^4*b^4*c^5 - 640*a^5*b^2*c^6 + 768*a^6*c^7)*f)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f + (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f + (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))*log(((3*b^2*c^5 + 4*a*c^6)*d^4 - (b^3*c^4 + 12*a*b*c^5)*d^3*e + (a*b^3*c^3 + 12*a^2*b*c^4)*d*e^3

- (3*a^2*b^2*c^3 + 4*a^3*c^4)*e^4 + (5*a^3*b^4 - 81*a^4*b^2*c + 324*a^5*c^
2)*f^4 + ((a*b^6 - 15*a^2*b^4*c + 432*a^4*c^3)*d - (3*a^2*b^5 - 65*a^3*b^3*
c + 324*a^4*b*c^2)*e)*f^3 - 3*(3*(a*b^4*c^2 - 6*a^2*b^2*c^3 - 24*a^3*c^4)*d
^2 - (a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d*e + (3*a^2*b^4*c - 28*a^3*
b^2*c^2)*e^2)*f^2 - ((b^4*c^3 - 24*a*b^2*c^4 - 48*a^2*c^5)*d^3 + 9*(a*b^3*c
^3 + 12*a^2*b*c^4)*d^2*e - 3*(a*b^4*c^2 + 12*a^2*b^2*c^3)*d*e^2 + (9*a^2*b^
3*c^2 - 20*a^3*b*c^3)*e^3)*f)*x - 1/2*sqrt(1/2)*((b^5*c^4 - 8*a*b^3*c^5 + 1
6*a^2*b*c^6)*d^3 - 2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^2*e - (a*b^
5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d*e^2 + 2*(a^2*b^4*c^3 - 8*a^3*b^2*c^
4 + 16*a^4*c^5)*e^3 - (a^2*b^7 - 17*a^3*b^5*c + 88*a^4*b^3*c^2 - 144*a^5*b*
c^3)*f^3 - ((a*b^7*c - 23*a^2*b^5*c^2 + 136*a^3*b^3*c^3 - 240*a^4*b*c^4)*d
+ 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e)*f^2 + (7*(a*b^5*c^3 - 8*
a^2*b^3*c^4 + 16*a^3*b*c^5)*d^2 - 2*(a*b^6*c^2 - 2*a^2*b^4*c^3 - 32*a^3*b^2
*c^4 + 96*a^4*c^5)*d*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*e^2
)*f - ((a*b^8*c^4 - 8*a^2*b^6*c^5 + 128*a^4*b^2*c^7 - 256*a^5*c^8)*d - 4*(a
^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*e - (a^2*b^8*c
^3 - 24*a^3*b^6*c^4 + 192*a^4*b^4*c^5 - 640*a^5*b^2*c^6 + 768*a^6*c^7)*f)*s
qrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81
*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2
)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*
c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^
2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a
^5*c^9)))*sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^4)*d
*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3*b*c^2)
f^2 - 2((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2 - 24*a^
3*c^3)*e)*f + (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*sq
rt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*
a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)
*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c
^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2
+ a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^
5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))) + sqr
t(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)
*x^2)*sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^4)*d*e +
(a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3*b*c^2)*f^2
- 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^
3)*e)*f - (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*sqrt((
c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*
c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*
f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 -
3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a
^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^
9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))*log(((3*b^
2*c^5 + 4*a*c^6)*d^4 - (b^3*c^4 + 12*a*b*c^5)*d^3*e + (a*b^3*c^3 + 12*a^2*b
*c^4)*d*e^3 - (3*a^2*b^2*c^3 + 4*a^3*c^4)*e^4 + (5*a^3*b^4 - 81*a^4*b^2*c +
324*a^5*c^2)*f^4 + ((a*b^6 - 15*a^2*b^4*c + 432*a^4*c^3)*d - (3*a^2*b^5 -
65*a^3*b^3*c + 324*a^4*b*c^2)*e)*f^3 - 3*(3*(a*b^4*c^2 - 6*a^2*b^2*c^3 - 24
*a^3*c^4)*d^2 - (a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d*e + (3*a^2*b^4*
c - 28*a^3*b^2*c^2)*e^2)*f^2 - ((b^4*c^3 - 24*a*b^2*c^4 - 48*a^2*c^5)*d^3 +
9*(a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e - 3*(a*b^4*c^2 + 12*a^2*b^2*c^3)*d*e^2
+ (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*e^3)*f)*x + 1/2*sqrt(1/2)*((b^5*c^4 - 8*a*
b^3*c^5 + 16*a^2*b*c^6)*d^3 - 2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^
2*e - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d*e^2 + 2*(a^2*b^4*c^3 - 8
*a^3*b^2*c^4 + 16*a^4*c^5)*e^3 - (a^2*b^7 - 17*a^3*b^5*c + 88*a^4*b^3*c^2 -
144*a^5*b*c^3)*f^3 - ((a*b^7*c - 23*a^2*b^5*c^2 + 136*a^3*b^3*c^3 - 240*a^
4*b*c^4)*d + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e)*f^2 + (7*(a*b
^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^2 - 2*(a*b^6*c^2 - 2*a^2*b^4*c^3 -
32*a^3*b^2*c^4 + 96*a^4*c^5)*d*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4
*b*c^4)*e^2)*f + ((a*b^8*c^4 - 8*a^2*b^6*c^5 + 128*a^4*b^2*c^7 - 256*a^5*c^

```
8)*d - 4*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*e -
 (a^2*b^8*c^3 - 24*a^3*b^6*c^4 + 192*a^4*b^4*c^5 - 640*a^5*b^2*c^6 + 768*a^
6*c^7)*f)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3
*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c -
9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 -
3*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a
^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2
*c^8 - 64*a^5*c^9)))*sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4
*a^2*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60
*a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*
c^2 - 24*a^3*c^3)*e)*f - (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*
a^4*c^6)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*
b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9
*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3
*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^
2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*
c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c
^6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c -
 4*a*b*c^2)*x^2)*sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2
*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3
*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2
 - 24*a^3*c^3)*e)*f - (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*
c^6)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*
c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3
*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^
2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^
4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8
 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))
*log(((3*b^2*c^5 + 4*a*c^6)*d^4 - (b^3*c^4 + 12*a*b*c^5)*d^3*e + (a*b^3*c^3
 + 12*a^2*b*c^4)*d*e^3 - (3*a^2*b^2*c^3 + 4*a^3*c^4)*e^4 + (5*a^3*b^4 - 81*
a^4*b^2*c + 324*a^5*c^2)*f^4 + ((a*b^6 - 15*a^2*b^4*c + 432*a^4*c^3)*d - (3
*a^2*b^5 - 65*a^3*b^3*c + 324*a^4*b*c^2)*e)*f^3 - 3*(3*(a*b^4*c^2 - 6*a^2*b
^2*c^3 - 24*a^3*c^4)*d^2 - (a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d*e +
(3*a^2*b^4*c - 28*a^3*b^2*c^2)*e^2)*f^2 - ((b^4*c^3 - 24*a*b^2*c^4 - 48*a^2
*c^5)*d^3 + 9*(a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e - 3*(a*b^4*c^2 + 12*a^2*b^2*
c^3)*d*e^2 + (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*e^3)*f)*x - 1/2*sqrt(1/2)*((b^5
*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*d^3 - 2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*
a^3*c^6)*d^2*e - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d*e^2 + 2*(a^2*
b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5)*e^3 - (a^2*b^7 - 17*a^3*b^5*c + 88*a^
4*b^3*c^2 - 144*a^5*b*c^3)*f^3 - ((a*b^7*c - 23*a^2*b^5*c^2 + 136*a^3*b^3*c
^3 - 240*a^4*b*c^4)*d + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e)*f^
2 + (7*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^2 - 2*(a*b^6*c^2 - 2*a^
2*b^4*c^3 - 32*a^3*b^2*c^4 + 96*a^4*c^5)*d*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c
^3 + 16*a^4*b*c^4)*e^2)*f + ((a*b^8*c^4 - 8*a^2*b^6*c^5 + 128*a^4*b^2*c^7 -
 256*a^5*c^8)*d - 4*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5
*b*c^7)*e - (a^2*b^8*c^3 - 24*a^3*b^6*c^4 + 192*a^4*b^4*c^5 - 640*a^5*b^2*c
^6 + 768*a^6*c^7)*f)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b
^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a
^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*
c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*
d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 +
 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*
b^2*c^3 + 4*a^2*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2
*b^3*c + 60*a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c -
 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f - (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^
2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^
4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^
2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c
^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d
```

```
^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 +
48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5
 - 64*a^4*c^6))) + 2*(b*c*d - 2*a*c*e + a*b*f)*x)/((b^2*c^2 - 4*a*c^3)*x^4
+ a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.71 $\int \frac{d+ex^2+fx^4}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=346

$$\frac{x\left(x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf-2ace+bcd\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] (x*(b^2*d - a*b*e - 2*a*(c*d - a*f) + (b*c*d - 2*a*c*e + a*b*f)*x^2))/(2*a*
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d - 2*a*c*e + a*b*f + (4*a*b*c*e
 + b^2*(c*d - a*f) - 4*a*c*(3*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*
c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d - 2*a*c*e + a*b*f - (4*a*b*c*e +
b^2*(c*d - a*f) - 4*a*c*(3*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.89649, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1678, 1166, 205}

$$\frac{x\left(x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf-2ace+bcd\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d - a*b*e - 2*a*(c*d - a*f) + (b*c*d - 2*a*c*e + a*b*f)*x^2))/(2*a*
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d - 2*a*c*e + a*b*f + (4*a*b*c*e
 + b^2*(c*d - a*f) - 4*a*c*(3*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*
c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d - 2*a*c*e + a*b*f - (4*a*b*c*e +
b^2*(c*d - a*f) - 4*a*c*(3*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-b^2d-abe+2a(3cd+af)+(-bcd+2ace-abf)x^2}{a+bx^2+cx^4}\,d}{2a\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(bcd-2ace+abf-\frac{4abce+b^2(cd-af)-4ac}{\sqrt{b^2-4ac}}\right.}{4a\left(b^2-4a\right.}$$

$$= \frac{x\left(b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(bcd-2ace+abf+\frac{4abce+b^2(cd-af)-4ac}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b}}$$

Mathematica [A] time = 1.19601, size = 382, normalized size = 1.1

$$\frac{\frac{2x\left(b\left(-ae+afx^2+cdx^2\right)+2a\left(af-c\left(d+ex^2\right)\right)+b^2d\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(b\left(cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+4ace\right)-2ac\left(e\sqrt{b^2-4ac}+2af+6cd\right)+b^2(cd-af)\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \cdots}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(b^2*d + b*(-(a*e) + c*d*x^2 + a*f*x^2) + 2*a*(a*f - c*(d + e*x^2))))
/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(b^2*(c*d - a*f) - 2*a*c*(6
*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c*e +
a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*
(b^2*(-(c*d) + a*f) + 2*a*c*(6*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b*(c*Sq
rt[b^2 - 4*a*c]*d - 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [B] time = 0.033, size = 1182, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2/a*(a*b*f-2*a*c*e+b*c*d)/(4*a*c-b^2)*x^3-1/2*(2*a^2*f-a*b*e-2*a*c*d+b^
2*d)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2
```

```
)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b
*f-1/2/(4*a*c-b^2)*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2
^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e+1/4/a/(4*a*c-b^2)*c*2^(1/2)/(((-
4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)
^(1/2))*b*d-a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)
-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f-1/4/(4
*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arcta
nh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*f+1/(4*a*c-b^2)*c/(-4*
a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)
/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*e-3/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2)
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2))*d+1/4/a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4
*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^
(1/2))*b^2*d-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*f+1/2/(4*a*c-b^2)*c*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))*e-1/4/a/(4*a*c-b^2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d-a/(4*a*c-b^2)*c/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*b^2*f+1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*b*e-3/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d+1/4/a/(4*
a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 35.917, size = 18375, normalized size = 53.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*c*d - 2*a*c*e + a*b*f)*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4
 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^
2 + 60*a^2*b*c^3)*d^2 + 2*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2
*b^3*c + 12*a^3*b*c^2)*e^2 + (a^3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c -
 28*a^3*b*c^2)*d + 2*(3*a^3*b^2*c + 4*a^4*c^2)*e)*f + (a^3*b^6*c - 12*a^4*b
^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4
 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4
*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*
(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*
```

```
b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^
2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4
*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))*log(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a
^2*c^5)*d^4 - (3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*d^3*e - 3*(3*a*b^4
*c^2 - 28*a^2*b^2*c^3)*d^2*e^2 - (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*d*e^3 - (3*
a^3*b^2*c^2 + 4*a^4*c^3)*e^4 + (3*a^5*b^2 + 4*a^6*c)*f^4 - ((a^3*b^4 - 24*a
^4*b^2*c - 48*a^5*c^2)*d + (a^4*b^3 + 12*a^5*b*c)*e)*f^3 - 9*((a^2*b^4*c -
6*a^3*b^2*c^2 - 24*a^4*c^3)*d^2 + (a^3*b^3*c + 12*a^4*b*c^2)*d*e)*f^2 + ((b
^6*c - 15*a*b^4*c^2 + 432*a^3*c^4)*d^3 + 3*(a*b^5*c + 3*a^2*b^3*c^2 - 108*a
^3*b*c^3)*d^2*e + 3*(a^2*b^4*c + 12*a^3*b^2*c^2)*d*e^2 + (a^3*b^3*c + 12*a^
4*b*c^2)*e^3)*f)*x + 1/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3
 - 672*a^3*b^2*c^4 + 864*a^4*c^5)*d^3 + 3*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^
3*b^3*c^3 - 112*a^4*b*c^4)*d^2*e + 3*(a^2*b^6*c - 10*a^3*b^4*c^2 + 32*a^4*b
^2*c^3 - 32*a^5*c^4)*d*e^2 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*e^3
 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*f^3 - ((a^3*b^6 - 26*a^4*b^4*c +
160*a^5*b^2*c^2 - 288*a^6*c^3)*d + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*e
)*f^2 - 2*((4*a^2*b^6*c - 59*a^3*b^4*c^2 + 280*a^4*b^2*c^3 - 432*a^5*c^4)*d
^2 + 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e + (a^4*b^4*c - 8*a^5*
b^2*c^2 + 16*a^6*c^3)*e^2)*f - ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c
^3 - 448*a^6*b^3*c^4 + 512*a^7*b*c^5)*d + (a^4*b^8*c - 8*a^5*b^6*c^2 + 128*
a^7*b^2*c^4 - 256*a^8*c^5)*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c
^3 - 64*a^8*b*c^4)*f)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^
3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*
a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e +
 a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^
4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c
^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b
*c^3)*d^2 + 2*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12*
a^3*b*c^2)*e^2 + (a^3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c^
2)*d + 2*(3*a^3*b^2*c + 4*a^4*c^2)*e)*f + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*
a^5*b^2*c^3 - 64*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*
d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2
- 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d
*e + a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e
+ a^4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b
^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^
5*b^2*c^3 - 64*a^6*c^4))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2
- 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*
b*c^3)*d^2 + 2*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12
*a^3*b*c^2)*e^2 + (a^3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c
^2)*d + 2*(3*a^3*b^2*c + 4*a^4*c^2)*e)*f + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48
*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c
*d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2
 - 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*
d*e + a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e
 + a^4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*
b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a
^5*b^2*c^3 - 64*a^6*c^4))*log(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^4
 - (3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*d^3*e - 3*(3*a*b^4*c^2 - 28*a
^2*b^2*c^3)*d^2*e^2 - (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*d*e^3 - (3*a^3*b^2*c^2
 + 4*a^4*c^3)*e^4 + (3*a^5*b^2 + 4*a^6*c)*f^4 - ((a^3*b^4 - 24*a^4*b^2*c -
48*a^5*c^2)*d + (a^4*b^3 + 12*a^5*b*c)*e)*f^3 - 9*((a^2*b^4*c - 6*a^3*b^2*c
^2 - 24*a^4*c^3)*d^2 + (a^3*b^3*c + 12*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 15*a
*b^4*c^2 + 432*a^3*c^4)*d^3 + 3*(a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d
^2*e + 3*(a^2*b^4*c + 12*a^3*b^2*c^2)*d*e^2 + (a^3*b^3*c + 12*a^4*b*c^2)*e^
3)*f)*x - 1/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*
b^2*c^4 + 864*a^4*c^5)*d^3 + 3*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 -
 112*a^4*b*c^4)*d^2*e + 3*(a^2*b^6*c - 10*a^3*b^4*c^2 + 32*a^4*b^2*c^3 - 32
*a^5*c^4)*d*e^2 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*e^3 + 2*(a^5*b
```

```
^4 - 8*a^6*b^2*c + 16*a^7*c^2)*f^3 - ((a^3*b^6 - 26*a^4*b^4*c + 160*a^5*b^2
*c^2 - 288*a^6*c^3)*d + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*e)*f^2 - 2*(
(4*a^2*b^6*c - 59*a^3*b^4*c^2 + 280*a^4*b^2*c^3 - 432*a^5*c^4)*d^2 + 5*(a^3
*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e + (a^4*b^4*c - 8*a^5*b^2*c^2 + 1
6*a^6*c^3)*e^2)*f - ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^
6*b^3*c^4 + 512*a^7*b*c^5)*d + (a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4
 - 256*a^8*c^5)*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8
*b*c^4)*f)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4
 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*
d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2
+ (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d*e^2
 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8
*b^2*c^4 - 64*a^9*c^5)))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^2 +
 2*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12*a^3*b*c^2)*
e^2 + (a^3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c^2)*d + 2*(3
*a^3*b^2*c + 4*a^4*c^2)*e)*f + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3
 - 64*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6
*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c
^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*
e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d
*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48
*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 -
 64*a^6*c^4))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c +
 (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^2
+ 2*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12*a^3*b*c^2)
*e^2 + (a^3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c^2)*d + 2*(
3*a^3*b^2*c + 4*a^4*c^2)*e)*f - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^
3 - 64*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^
6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*
c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c
*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*
d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 4
8*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3
- 64*a^6*c^4))*log(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^4 - (3*b^5*c
^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*d^3*e - 3*(3*a*b^4*c^2 - 28*a^2*b^2*c^3)
*d^2*e^2 - (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*d*e^3 - (3*a^3*b^2*c^2 + 4*a^4*c^
3)*e^4 + (3*a^5*b^2 + 4*a^6*c)*f^4 - ((a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2)
*d + (a^4*b^3 + 12*a^5*b*c)*e)*f^3 - 9*((a^2*b^4*c - 6*a^3*b^2*c^2 - 24*a^4
*c^3)*d^2 + (a^3*b^3*c + 12*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 15*a*b^4*c^2 +
432*a^3*c^4)*d^3 + 3*(a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d^2*e + 3*(a
^2*b^4*c + 12*a^3*b^2*c^2)*d*e^2 + (a^3*b^3*c + 12*a^4*b*c^2)*e^3)*f)*x + 1
/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 + 8
64*a^4*c^5)*d^3 + 3*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*
c^4)*d^2*e + 3*(a^2*b^6*c - 10*a^3*b^4*c^2 + 32*a^4*b^2*c^3 - 32*a^5*c^4)*d
*e^2 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*e^3 + 2*(a^5*b^4 - 8*a^6*
b^2*c + 16*a^7*c^2)*f^3 - ((a^3*b^6 - 26*a^4*b^4*c + 160*a^5*b^2*c^2 - 288*
a^6*c^3)*d + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*e)*f^2 - 2*((4*a^2*b^6*
c - 59*a^3*b^4*c^2 + 280*a^4*b^2*c^3 - 432*a^5*c^4)*d^2 + 5*(a^3*b^5*c - 8*
a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e + (a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*
e^2)*f + ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6*b^3*c^4 +
 512*a^7*b*c^5)*d + (a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*
c^5)*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*f)*
sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^4*c^2
 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e + 6*(
a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3*b^2*
c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^2*b^2
*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 -
64*a^9*c^5)))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^2 + 2*(a*b^4*c
 - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12*a^3*b*c^2)*e^2 + (a^3*
```

```
b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c^2)*d + 2*(3*a^3*b^2*c
 + 4*a^4*c^2)*e)*f - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c
^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^4
*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e +
 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3*
b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^2
*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^
4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4
))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4
*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^2 + 2*(a*b^4*
c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12*a^3*b*c^2)*e^2 + (a^3
*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c^2)*d + 2*(3*a^3*b^2*c
 + 4*a^4*c^2)*e)*f - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*
c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^
4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e
 + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3
*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^
2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c
^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^
4))*log(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^4 - (3*b^5*c^2 - 65*a*b
^3*c^3 + 324*a^2*b*c^4)*d^3*e - 3*(3*a*b^4*c^2 - 28*a^2*b^2*c^3)*d^2*e^2 -
(9*a^2*b^3*c^2 - 20*a^3*b*c^3)*d*e^3 - (3*a^3*b^2*c^2 + 4*a^4*c^3)*e^4 + (3
*a^5*b^2 + 4*a^6*c)*f^4 - ((a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2)*d + (a^4*b
^3 + 12*a^5*b*c)*e)*f^3 - 9*((a^2*b^4*c - 6*a^3*b^2*c^2 - 24*a^4*c^3)*d^2 +
 (a^3*b^3*c + 12*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 15*a*b^4*c^2 + 432*a^3*c^4
)*d^3 + 3*(a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d^2*e + 3*(a^2*b^4*c +
12*a^3*b^2*c^2)*d*e^2 + (a^3*b^3*c + 12*a^4*b*c^2)*e^3)*f)*x - 1/2*sqrt(1/2
)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 + 864*a^4*c^5)
*d^3 + 3*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*c^4)*d^2*e
+ 3*(a^2*b^6*c - 10*a^3*b^4*c^2 + 32*a^4*b^2*c^3 - 32*a^5*c^4)*d*e^2 + (a^3
*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*e^3 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*
a^7*c^2)*f^3 - ((a^3*b^6 - 26*a^4*b^4*c + 160*a^5*b^2*c^2 - 288*a^6*c^3)*d
+ (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*e)*f^2 - 2*((4*a^2*b^6*c - 59*a^3*
b^4*c^2 + 280*a^4*b^2*c^3 - 432*a^5*c^4)*d^2 + 5*(a^3*b^5*c - 8*a^4*b^3*c^2
 + 16*a^5*b*c^3)*d*e + (a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e^2)*f + ((
a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6*b^3*c^4 + 512*a^7*b*
c^5)*d + (a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*c^5)*e - 4*
(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*f)*sqrt((4*a^3
*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2
*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2
 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3*b^2*c - 27*a^4*
c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^
3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)
))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^2 + 2*(a*b^4*c - 6*a^2*b^
2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12*a^3*b*c^2)*e^2 + (a^3*b^3 + 12*a^
4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c^2)*d + 2*(3*a^3*b^2*c + 4*a^4*c^2
)*e)*f - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt((4
*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a
*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2
*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3*b^2*c - 27*
a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^2*b^2*c^2 -
9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*
c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))) - 2*(a*
b*e - 2*a^2*f - (b^2 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 -
4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.72 $\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=399

$$-\frac{x\left(cx^2\left(-abe-2a(cd-af)+b^2d\right)+a\left(\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{12a^2ce-ab^2e-4ab(af+}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}a^2\left(b^2-4ac\right)}$$

```
[Out] -(d/(a^2*x)) - (x*(a*((b^3*d)/a - b*(3*c*d + b*e) + a*(2*c*e + b*f)) + c*(b
^2*d - a*b*e - 2*a*(c*d - a*f))*x^2))/(2*a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a*f) + (3*b^3*d - a*b^2*e +
 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a*f) - (3*b^3
*d - a*b^2*e + 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a
*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 2.20293, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1669, 1664, 1166, 205}

$$-\frac{x\left(cx^2\left(-abe-2a(cd-af)+b^2d\right)+a\left(\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{12a^2ce-ab^2e-4ab(af+}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}a^2\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(d/(a^2*x)) - (x*(a*((b^3*d)/a - b*(3*c*d + b*e) + a*(2*c*e + b*f)) + c*(b
^2*d - a*b*e - 2*a*(c*d - a*f))*x^2))/(2*a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a*f) + (3*b^3*d - a*b^2*e +
 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a*f) - (3*b^3
*d - a*b^2*e + 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a
*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c\left(b^2d-abe-2a(cd-af)\right)x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-2\left(b^2-4ac\right)d+\frac{\left(b^3d\right.}{}}{}}{}$$

$$= -\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c\left(b^2d-abe-2a(cd-af)\right)x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int\left(\frac{2\left(-b^2+4ac\right)d}{ax^2}+\right.}{}$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c\left(b^2d-abe-2a(cd-af)\right)x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{3b^3d-ab^2}{}}{}$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c\left(b^2d-abe-2a(cd-af)\right)x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(3b^2d-\right.\right.}{}$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c\left(b^2d-abe-2a(cd-af)\right)x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\sqrt{c}\left(3b^2d\right.}{}$$

Mathematica [A] time = 1.47287, size = 444, normalized size = 1.11

$$\frac{-\frac{2x\left(b^2\left(cdx^2-ae\right)+ab\left(af-c\left(3d+ex^2\right)\right)+2ac\left(a\left(e+fx^2\right)-cdx^2\right)+b^3d\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(ab\left(e\sqrt{b^2-4ac}+4af+16cd\right)-2a\left(-5cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+6ac\right.\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*d)/x - (2*x*(b^3*d + b^2*(-(a*e) + c*d*x^2) + a*b*(a*f - c*(3*d + e*x^
2)) + 2*a*c*(-(c*d*x^2) + a*(e + f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3*d + b^2*(-3*Sqrt[b^2 - 4*a*c]*d + a*e) + a*b
*(16*c*d + Sqrt[b^2 - 4*a*c]*e + 4*a*f) - 2*a*(-5*c*Sqrt[b^2 - 4*a*c]*d + 6
*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
```

```
^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]
*Sqrt[c]*(3*b^3*d - b^2*(3*Sqrt[b^2 - 4*a*c]*d + a*e) + a*b*(-16*c*d + Sqrt
[b^2 - 4*a*c]*e - 4*a*f) + 2*a*(5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e - a*Sqrt[
b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

Maple [B] time = 0.045, size = 1575, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*e+4/a*c^2/(4
*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arcta
nh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d+1/4/a*c/(4*a*c-b^2)/(-
4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*b*d-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/
2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2
)-b)*c)^(1/2))*b^3*d-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))*b^3*d+5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*
arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d-5/2/a*c^2/(4*a*c-b^
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*d-3*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2))*e-3*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e-1/2/a/(c*
x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b*e-3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b*c*
d+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b^2*d+c/(c*x^4+b*x^2+a)/(4*a*c-
b^2)*x^3*f+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b*f-1/a/(c*x^4+b*x^2+a)*c^2/(4
*a*c-b^2)*x^3*d-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^2*e+1/2/a^2/(c*x^4+b*
x^2+a)/(4*a*c-b^2)*x*b^3*d-1/2*c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b
)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f+1/2*c/(4
*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*f+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*c*e-d/a^2/x+
c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*a
rctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*f+c/(4*a*c-b^2)/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*f-1/4/a*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b*e-3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctan
h(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*d+3/4/a^2*c/(4*a*c-b^2)
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*b^2*d+1/4/a*c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(abce - 2\,a^2cf - \left(3\,b^2c - 10\,ac^2\right)d\right)x^4 - \left(a^2bf + \left(3\,b^3 - 11\,abc\right)d - \left(ab^2 - 2\,a^2c\right)e\right)x^2 - 2\left(ab^2 - 4\,a^2c\right)d}{2\left(\left(a^2b^2c - 4\,a^3c^2\right)x^5 + \left(a^2b^3 - 4\,a^3bc\right)x^3 + \left(a^3b^2 - 4\,a^4c\right)x\right)} - \int \frac{a^2bf + (abc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a*b*c*e - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*d)*x^4 - (a^2*b*f + (3*b^3
 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2 - 2*(a*b^2 - 4*a^2*c)*d)/((a^2*b^
2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x) -
 1/2*integrate(-(a^2*b*f + (a*b*c*e - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*d)*x
^2 - (3*b^3 - 13*a*b*c)*d + (a*b^2 - 6*a^2*c)*e)/(c*x^4 + b*x^2 + a), x)/(a
^2*b^2 - 4*a^3*c)
```

Fricas [B] time = 79.0598, size = 28044, normalized size = 70.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a*b*c*e - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*d)*x^4 - 2*(a^2*b*f + (3
*b^3 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2 + sqrt(1/2)*((a^2*b^2*c - 4*a
^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-((9*
b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*
a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c +
60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3
*c - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f + (a^5*b^6
 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*
a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*
b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5
- 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^
2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*
b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*
c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^
3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 -
 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6
 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c
 + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-((189*b^6*c^3 - 1971*a*b^4*c^4 + 5625
*a^2*b^2*c^5 - 2500*a^3*c^6)*d^4 - (135*b^7*c^2 - 1323*a*b^5*c^3 + 2727*a^2
*b^3*c^4 + 2500*a^3*b*c^5)*d^3*e + 3*(45*a*b^6*c^2 - 558*a^2*b^4*c^3 + 1672
*a^3*b^2*c^4)*d^2*e^2 - (45*a^2*b^5*c^2 - 647*a^3*b^3*c^3 + 2268*a^4*b*c^4)
*d*e^3 + (5*a^3*b^4*c^2 - 81*a^4*b^2*c^3 + 324*a^5*c^4)*e^4 - (3*a^6*b^2*c
+ 4*a^7*c^2)*f^4 + ((27*a^4*b^4*c + 80*a^6*c^3)*d - (9*a^5*b^3*c - 20*a^6*b
*c^2)*e)*f^3 - 3*((27*a^2*b^6*c - 117*a^3*b^4*c^2 - 150*a^4*b^2*c^3 + 200*a
^5*c^4)*d^2 - (18*a^3*b^5*c - 123*a^4*b^3*c^2 - 100*a^5*b*c^3)*d*e + (3*a^4
*b^4*c - 28*a^5*b^2*c^2)*e^2)*f^2 + ((81*b^8*c - 945*a*b^6*c^2 + 3213*a^2*b
^4*c^3 - 3000*a^3*b^2*c^4 + 2000*a^4*c^5)*d^3 - 3*(27*a*b^7*c - 405*a^2*b^5
*c^2 + 1461*a^3*b^3*c^3 - 500*a^4*b*c^4)*d^2*e + 3*(9*a^2*b^6*c - 165*a^3*b
^4*c^2 + 692*a^4*b^2*c^3)*d*e^2 - (3*a^3*b^5*c - 65*a^4*b^3*c^2 + 324*a^5*b
*c^3)*e^3)*f)*x + 1/2*sqrt(1/2)*((27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2
- 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*d^3 - 3*(9*a*b^10
 - 177*a^2*b^8*c + 1285*a^3*b^6*c^2 - 4138*a^4*b^4*c^3 + 5216*a^5*b^2*c^4 -
 800*a^6*c^5)*d^2*e + 3*(3*a^2*b^9 - 64*a^3*b^7*c + 495*a^4*b^5*c^2 - 1656*
a^5*b^3*c^3 + 2032*a^6*b*c^4)*d*e^2 - (a^3*b^8 - 23*a^4*b^6*c + 190*a^5*b^4
*c^2 - 672*a^6*b^2*c^3 + 864*a^7*c^4)*e^3 - (a^6*b^5 - 8*a^7*b^3*c + 16*a^8
*b*c^2)*f^3 + 3*((3*a^4*b^7 - 25*a^5*b^5*c + 56*a^6*b^3*c^2 - 16*a^7*b*c^3)
*d - (a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*e)*f^2 - 3*((9*
```

```
a^2*b^9 - 105*a^3*b^7*c + 373*a^4*b^5*c^2 - 248*a^5*b^3*c^3 - 560*a^6*b*c^4
)*d^2 - 2*(3*a^3*b^8 - 40*a^4*b^6*c + 166*a^5*b^4*c^2 - 176*a^6*b^2*c^3 - 1
60*a^7*c^4)*d*e + (a^4*b^7 - 15*a^5*b^5*c + 72*a^6*b^3*c^2 - 112*a^7*b*c^3)
*e^2)*f - ((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3
+ 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*d - (a^6*b^9 - 20*a^7*b^7*c + 144*a^8*b
^5*c^2 - 448*a^9*b^3*c^3 + 512*a^10*b*c^4)*e - (a^7*b^8 - 8*a^8*b^6*c + 128
*a^10*b^2*c^3 - 256*a^11*c^4)*f)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 30
51*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^
2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*
b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c
 + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^
7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6
*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 -
4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*
a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c
- 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4
*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-((9*b^7 - 105*a*b^5*c + 385*a^2
*b^3*c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*a^2*b^4*c + 150*a^3*b^2*c^2
 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^
3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c - 12*a^4*b*c^2)*d - (a^3
*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^
2*c^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^
2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 119
7*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a
^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*
c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^
6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2
*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^
6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*
a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)
*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*
b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*
c^3))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3
 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-((9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 4
20*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c
^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b
*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4
*b^2*c - 24*a^5*c^2)*e)*f + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a
^8*c^3)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3
*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^
2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 -
 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 +
 (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^
7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3
- 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*
b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c -
65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^
5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64
*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(
(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d^4 - (135
*b^7*c^2 - 1323*a*b^5*c^3 + 2727*a^2*b^3*c^4 + 2500*a^3*b*c^5)*d^3*e + 3*(4
5*a*b^6*c^2 - 558*a^2*b^4*c^3 + 1672*a^3*b^2*c^4)*d^2*e^2 - (45*a^2*b^5*c^2
 - 647*a^3*b^3*c^3 + 2268*a^4*b*c^4)*d*e^3 + (5*a^3*b^4*c^2 - 81*a^4*b^2*c^
3 + 324*a^5*c^4)*e^4 - (3*a^6*b^2*c + 4*a^7*c^2)*f^4 + ((27*a^4*b^4*c + 80*
a^6*c^3)*d - (9*a^5*b^3*c - 20*a^6*b*c^2)*e)*f^3 - 3*((27*a^2*b^6*c - 117*a
^3*b^4*c^2 - 150*a^4*b^2*c^3 + 200*a^5*c^4)*d^2 - (18*a^3*b^5*c - 123*a^4*b
^3*c^2 - 100*a^5*b*c^3)*d*e + (3*a^4*b^4*c - 28*a^5*b^2*c^2)*e^2)*f^2 + ((8
1*b^8*c - 945*a*b^6*c^2 + 3213*a^2*b^4*c^3 - 3000*a^3*b^2*c^4 + 2000*a^4*c^
5)*d^3 - 3*(27*a*b^7*c - 405*a^2*b^5*c^2 + 1461*a^3*b^3*c^3 - 500*a^4*b*c^4
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)*d^2*e + 3*(9*a^2*b^6*c - 165*a^3*b^4*c^2 + 692*a^4*b^2*c^3)*d*e^2 - (3*a^
3*b^5*c - 65*a^4*b^3*c^2 + 324*a^5*b*c^3)*e^3)*f)*x - 1/2*sqrt(1/2)*((27*b^
11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4
 - 5200*a^5*b*c^5)*d^3 - 3*(9*a*b^10 - 177*a^2*b^8*c + 1285*a^3*b^6*c^2 - 4
138*a^4*b^4*c^3 + 5216*a^5*b^2*c^4 - 800*a^6*c^5)*d^2*e + 3*(3*a^2*b^9 - 64
*a^3*b^7*c + 495*a^4*b^5*c^2 - 1656*a^5*b^3*c^3 + 2032*a^6*b*c^4)*d*e^2 - (
a^3*b^8 - 23*a^4*b^6*c + 190*a^5*b^4*c^2 - 672*a^6*b^2*c^3 + 864*a^7*c^4)*e
^3 - (a^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*c^2)*f^3 + 3*((3*a^4*b^7 - 25*a^5*b^
5*c + 56*a^6*b^3*c^2 - 16*a^7*b*c^3)*d - (a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b
^2*c^2 - 32*a^8*c^3)*e)*f^2 - 3*((9*a^2*b^9 - 105*a^3*b^7*c + 373*a^4*b^5*c
^2 - 248*a^5*b^3*c^3 - 560*a^6*b*c^4)*d^2 - 2*(3*a^3*b^8 - 40*a^4*b^6*c + 1
66*a^5*b^4*c^2 - 176*a^6*b^2*c^3 - 160*a^7*c^4)*d*e + (a^4*b^7 - 15*a^5*b^5
*c + 72*a^6*b^3*c^2 - 112*a^7*b*c^3)*e^2)*f - ((3*a^5*b^10 - 55*a^6*b^8*c +
 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*d -
 (a^6*b^9 - 20*a^7*b^7*c + 144*a^8*b^5*c^2 - 448*a^9*b^3*c^3 + 512*a^10*b*c
^4)*e - (a^7*b^8 - 8*a^8*b^6*c + 128*a^10*b^2*c^3 - 256*a^11*c^4)*f)*sqrt((
a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625
*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*
c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d
^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18
*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 +
6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d
*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^
4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*
d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*
b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*s
qrt(-((9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*
b^6 - 40*a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3
*b^3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 1
3*a^3*b^3*c - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f +
 (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81
*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4
 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e +
6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3
*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c +
81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4
 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2
 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 1
25*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*
a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/
(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*
a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*
c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-((9*b^7
 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*a^2
*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c + 60*
a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c
 - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f - (a^5*b^6 -
12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81*b^8 - 918*a
*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b
^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6
 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 4
9*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*
e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2
*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*
e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*
d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22
*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c +
48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-((189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^
```

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2*b^2*c^5 - 2500*a^3*c^6)*d^4 - (135*b^7*c^2 - 1323*a*b^5*c^3 + 2727*a^2*b^
3*c^4 + 2500*a^3*b*c^5)*d^3*e + 3*(45*a*b^6*c^2 - 558*a^2*b^4*c^3 + 1672*a^
3*b^2*c^4)*d^2*e^2 - (45*a^2*b^5*c^2 - 647*a^3*b^3*c^3 + 2268*a^4*b*c^4)*d*
e^3 + (5*a^3*b^4*c^2 - 81*a^4*b^2*c^3 + 324*a^5*c^4)*e^4 - (3*a^6*b^2*c + 4
*a^7*c^2)*f^4 + ((27*a^4*b^4*c + 80*a^6*c^3)*d - (9*a^5*b^3*c - 20*a^6*b*c^
2)*e)*f^3 - 3*((27*a^2*b^6*c - 117*a^3*b^4*c^2 - 150*a^4*b^2*c^3 + 200*a^5*
c^4)*d^2 - (18*a^3*b^5*c - 123*a^4*b^3*c^2 - 100*a^5*b*c^3)*d*e + (3*a^4*b^
4*c - 28*a^5*b^2*c^2)*e^2)*f^2 + ((81*b^8*c - 945*a*b^6*c^2 + 3213*a^2*b^4*
c^3 - 3000*a^3*b^2*c^4 + 2000*a^4*c^5)*d^3 - 3*(27*a*b^7*c - 405*a^2*b^5*c^
2 + 1461*a^3*b^3*c^3 - 500*a^4*b*c^4)*d^2*e + 3*(9*a^2*b^6*c - 165*a^3*b^4*
c^2 + 692*a^4*b^2*c^3)*d*e^2 - (3*a^3*b^5*c - 65*a^4*b^3*c^2 + 324*a^5*b*c^
3)*e^3)*f)*x + 1/2*sqrt(1/2)*((27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 1
0549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*d^3 - 3*(9*a*b^10 -
177*a^2*b^8*c + 1285*a^3*b^6*c^2 - 4138*a^4*b^4*c^3 + 5216*a^5*b^2*c^4 - 80
0*a^6*c^5)*d^2*e + 3*(3*a^2*b^9 - 64*a^3*b^7*c + 495*a^4*b^5*c^2 - 1656*a^5
*b^3*c^3 + 2032*a^6*b*c^4)*d*e^2 - (a^3*b^8 - 23*a^4*b^6*c + 190*a^5*b^4*c^
2 - 672*a^6*b^2*c^3 + 864*a^7*c^4)*e^3 - (a^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*
c^2)*f^3 + 3*((3*a^4*b^7 - 25*a^5*b^5*c + 56*a^6*b^3*c^2 - 16*a^7*b*c^3)*d
- (a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*e)*f^2 - 3*((9*a^2
*b^9 - 105*a^3*b^7*c + 373*a^4*b^5*c^2 - 248*a^5*b^3*c^3 - 560*a^6*b*c^4)*d
^2 - 2*(3*a^3*b^8 - 40*a^4*b^6*c + 166*a^5*b^4*c^2 - 176*a^6*b^2*c^3 - 160*
a^7*c^4)*d*e + (a^4*b^7 - 15*a^5*b^5*c + 72*a^6*b^3*c^2 - 112*a^7*b*c^3)*e^
2)*f + ((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2
176*a^9*b^2*c^4 - 1280*a^10*c^5)*d - (a^6*b^9 - 20*a^7*b^7*c + 144*a^8*b^5*
c^2 - 448*a^9*b^3*c^3 + 512*a^10*b*c^4)*e - (a^7*b^8 - 8*a^8*b^6*c + 128*a^
10*b^2*c^3 - 256*a^11*c^4)*f)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*
a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b
^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4
*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c +
198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b
*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^
2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*(
(27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3
*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 1
5*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-((9*b^7 - 105*a*b^5*c + 385*a^2*b^
3*c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*a^2*b^4*c + 150*a^3*b^2*c^2 -
120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^3 +
 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c - 12*a^4*b*c^2)*d - (a^3*b^
4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c
^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 -
 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a
^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*
b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2
)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b
^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3
*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 -
 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4
*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*
e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2
*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3
))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 +
(a^3*b^2 - 4*a^4*c)*x)*sqrt(-((9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*
a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)
*d*e + (a^2*b^5 - 15*a^3*b^3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b*c)
*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^
2*c - 24*a^5*c^2)*e)*f - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*
c^3)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^
2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 -
```

550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75
*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a
^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c
)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4
*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4
*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*
a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b
^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^
13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-((18
9*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d^4 - (135*b^
7*c^2 - 1323*a*b^5*c^3 + 2727*a^2*b^3*c^4 + 2500*a^3*b*c^5)*d^3*e + 3*(45*a
*b^6*c^2 - 558*a^2*b^4*c^3 + 1672*a^3*b^2*c^4)*d^2*e^2 - (45*a^2*b^5*c^2 -
647*a^3*b^3*c^3 + 2268*a^4*b*c^4)*d*e^3 + (5*a^3*b^4*c^2 - 81*a^4*b^2*c^3 +
324*a^5*c^4)*e^4 - (3*a^6*b^2*c + 4*a^7*c^2)*f^4 + ((27*a^4*b^4*c + 80*a^6
*c^3)*d - (9*a^5*b^3*c - 20*a^6*b*c^2)*e)*f^3 - 3*((27*a^2*b^6*c - 117*a^3*
b^4*c^2 - 150*a^4*b^2*c^3 + 200*a^5*c^4)*d^2 - (18*a^3*b^5*c - 123*a^4*b^3*
c^2 - 100*a^5*b*c^3)*d*e + (3*a^4*b^4*c - 28*a^5*b^2*c^2)*e^2)*f^2 + ((81*b
^8*c - 945*a*b^6*c^2 + 3213*a^2*b^4*c^3 - 3000*a^3*b^2*c^4 + 2000*a^4*c^5)*
d^3 - 3*(27*a*b^7*c - 405*a^2*b^5*c^2 + 1461*a^3*b^3*c^3 - 500*a^4*b*c^4)*d
^2*e + 3*(9*a^2*b^6*c - 165*a^3*b^4*c^2 + 692*a^4*b^2*c^3)*d*e^2 - (3*a^3*b
^5*c - 65*a^4*b^3*c^2 + 324*a^5*b*c^3)*e^3)*f)*x - 1/2*sqrt(1/2)*((27*b^11
- 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 -
5200*a^5*b*c^5)*d^3 - 3*(9*a*b^10 - 177*a^2*b^8*c + 1285*a^3*b^6*c^2 - 4138
*a^4*b^4*c^3 + 5216*a^5*b^2*c^4 - 800*a^6*c^5)*d^2*e + 3*(3*a^2*b^9 - 64*a^
3*b^7*c + 495*a^4*b^5*c^2 - 1656*a^5*b^3*c^3 + 2032*a^6*b*c^4)*d*e^2 - (a^3
*b^8 - 23*a^4*b^6*c + 190*a^5*b^4*c^2 - 672*a^6*b^2*c^3 + 864*a^7*c^4)*e^3
- (a^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*c^2)*f^3 + 3*((3*a^4*b^7 - 25*a^5*b^5*c
+ 56*a^6*b^3*c^2 - 16*a^7*b*c^3)*d - (a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*
c^2 - 32*a^8*c^3)*e)*f^2 - 3*((9*a^2*b^9 - 105*a^3*b^7*c + 373*a^4*b^5*c^2
- 248*a^5*b^3*c^3 - 560*a^6*b*c^4)*d^2 - 2*(3*a^3*b^8 - 40*a^4*b^6*c + 166*
a^5*b^4*c^2 - 176*a^6*b^2*c^3 - 160*a^7*c^4)*d*e + (a^4*b^7 - 15*a^5*b^5*c
+ 72*a^6*b^3*c^2 - 112*a^7*b*c^3)*e^2)*f + ((3*a^5*b^10 - 55*a^6*b^8*c + 39
2*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*d - (a
^6*b^9 - 20*a^7*b^7*c + 144*a^8*b^5*c^2 - 448*a^9*b^3*c^3 + 512*a^10*b*c^4)
*e - (a^7*b^8 - 8*a^8*b^6*c + 128*a^10*b^2*c^3 - 256*a^11*c^4)*f)*sqrt((a^8
*f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^
4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3
)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*
e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^
5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*(
(9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e
+ (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b
^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2
e + 3(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c
)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt
(-((9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6
- 40*a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^
3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a
^3*b^3*c - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f - (a
^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81*b^
8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 -
4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(
9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^
3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*
a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 +
3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 -
3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*
a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4
*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^

```
10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6
*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - 4*(a*b^2 - 4*a^2*c)*d)/((a^2*b^2*
c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.73 $\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=575

$$\frac{x\left(a^2\left(\frac{b^4 d}{a^2}-\frac{b^2(be+4cd)}{a}-2acf+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\cdots\right.}{\cdots}$$

```
[Out] -d/(3*a^2*x^3) + (2*b*d - a*e)/(a^3*x) + (x*(a^2*((b^4*d)/a^2 + 2*c^2*d + 3
*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f - 2*a*c*f) + c*(b^3*d - a*b^2*e + 2*
a^2*c*e - a*b*(3*c*d - a*f))*x^2))/(2*a^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) + (Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*a^2*c*(14*c
*d + 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f) - a*b^2*(29*c*d + 3*Sqrt[b^2 - 4*a*c]*e
 - a*f) - a*b*(19*c*Sqrt[b^2 - 4*a*c]*d - 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f)
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(5*b^4*d - b^3*(
5*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + 2*a^2*c*(14*c*d - 5*Sqrt[b^2 - 4*a*c]*e -
6*a*f) - a*b^2*(29*c*d - 3*Sqrt[b^2 - 4*a*c]*e - a*f) + a*b*(19*c*Sqrt[b^2
- 4*a*c]*d + 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]])
```

Rubi [A] time = 9.90565, antiderivative size = 575, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1669, 1664, 1166, 205}

$$\frac{x\left(a^2\left(\frac{b^4 d}{a^2}-\frac{b^2(be+4cd)}{a}-2acf+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```


```
[Out] -d/(3*a^2*x^3) + (2*b*d - a*e)/(a^3*x) + (x*(a^2*((b^4*d)/a^2 + 2*c^2*d + 3
*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f - 2*a*c*f) + c*(b^3*d - a*b^2*e + 2*
a^2*c*e - a*b*(3*c*d - a*f))*x^2))/(2*a^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) + (Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*a^2*c*(14*c
*d + 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f) - a*b^2*(29*c*d + 3*Sqrt[b^2 - 4*a*c]*e
 - a*f) - a*b*(19*c*Sqrt[b^2 - 4*a*c]*d - 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f)
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(5*b^4*d - b^3*(
5*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + 2*a^2*c*(14*c*d - 5*Sqrt[b^2 - 4*a*c]*e -
6*a*f) - a*b^2*(29*c*d - 3*Sqrt[b^2 - 4*a*c]*e - a*f) + a*b*(19*c*Sqrt[b^2
- 4*a*c]*d + 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]])
```


Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
```

```
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c\left(b^3d-ab^2e+2a^2ce-ab(3cd-af\right.\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c\left(b^3d-ab^2e+2a^2ce-ab(3cd-af\right.\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c\left(b^3d-ab^2e\right.\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c\left(b^3d-ab^2e\right.\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c\left(b^3d-ab^2e\right.\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 1.96036, size = 548, normalized size = 0.95

$$\frac{\frac{6x\left(2a^2c\left(c\left(d+ex^2\right)-af\right)+ab^2\left(af-c\left(4d+ex^2\right)\right)+b^3\left(cdx^2-ae\right)+abc\left(3ae+afx^2-3cdx^2\right)+b^4d\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(2a^2c\left(5e\sqrt{b^2-4ac}-6af+14cd\right)+ab^2\left(\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*a*d)/x^3 + (24*b*d - 12*a*e)/x + (6*x*(b^4*d + b^3*(-(a*e) + c*d*x^2)
+ a*b*c*(3*a*e - 3*c*d*x^2 + a*f*x^2) + 2*a^2*c*(-(a*f) + c*(d + e*x^2)) +
a*b^2*(a*f - c*(4*d + e*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sq
rt[2]*Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*a^2*c*(14*
c*d + 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f) + a*b^2*(-29*c*d - 3*Sqrt[b^2 - 4*a*c]
*e + a*f) + a*b*(-19*c*Sqrt[b^2 - 4*a*c]*d + 16*a*c*e + a*Sqrt[b^2 - 4*a*c]
*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4*d + b^3*(
5*Sqrt[b^2 - 4*a*c]*d + 3*a*e) - a*b^2*(-29*c*d + 3*Sqrt[b^2 - 4*a*c]*e + a
*f) + 2*a^2*c*(-14*c*d + 5*Sqrt[b^2 - 4*a*c]*e + 6*a*f) + a*b*(-19*c*Sqrt[b
^2 - 4*a*c]*d - 16*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4
*a*c]]))/(12*a^3)
```

Maple [B] time = 0.048, size = 2180, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*e-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2
)*x*b^2*f-1/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*c^2*d+1/2/a^2/(c*x^4+b*x^2+a)/(
4*a*c-b^2)*x*b^3*e-1/2/a^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^4*d-3/4/a^2*c/(4
*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*e-29/4/a^2*c^2/(4*a*c-b
^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d+5/4/a^3*c/(4*a*c-b^2)/(-4*a
*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/
(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^4*d+5/4/a^3*c/(4*a*c-b^2)/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*b^4*d-29/4/a^2*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)
^(1/2)-b)*c)^(1/2))*b^2*d+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*b^2*f+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b
^2*f+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)
*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*e-3/4/a^2
*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*
arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*e+4/a*c^2/(4*a*c-
b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e+1/(c*x^4+b*x^2+a)/(4*a*c-b^2
)*x*c*f-1/a^2/x*e+2/a^3/x*b*d-1/3*d/a^2/x^3+7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2
)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2))*d-1/4/a*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*f+1/
4/a*c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*f-3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/(((
-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c
)^(1/2))*b^2*e-19/4/a^2*c^2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^
(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d+3/4/a^2*c/(
4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e+19/4/a^2*c^2/(4*a*c-b^2)*2^(1/2)/((b+(-
```

```
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*b*d+7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d+5/4/a^
3*c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2
)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*d-5/4/a^3*c/(4*a*c-b^2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*b^3*d-1/2/a^3/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b^3*d-1/2/a/(c*x^4+
b*x^2+a)*c/(4*a*c-b^2)*x^3*b*f+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b^
2*e+2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^2*c*d+3/2/a^2/(c*x^4+b*x^2+a)*c^2
/(4*a*c-b^2)*x^3*b*d+5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*
c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e-5/2/a*c^2/
(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e-3*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/
2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2
)-b)*c)^(1/2))*f-3*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f-
3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b*c*e
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.74 $\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$

Optimal. Leaf size=68

$$\frac{5x^8}{8}-\frac{9x^6}{2}+\frac{49x^4}{2}-\frac{293x^2}{2}+\frac{415x^2+414}{2(x^4+3x^2+2)}+2\log(x^2+1)+392\log(x^2+2)$$

```
[Out] (-293*x^2)/2 + (49*x^4)/2 - (9*x^6)/2 + (5*x^8)/8 + (414 + 415*x^2)/(2*(2 +
 3*x^2 + x^4)) + 2*Log[1 + x^2] + 392*Log[2 + x^2]
```

Rubi [A] time = 0.126049, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1663, 1660, 1657, 632, 31}

$$\frac{5x^8}{8}-\frac{9x^6}{2}+\frac{49x^4}{2}-\frac{293x^2}{2}+\frac{415x^2+414}{2(x^4+3x^2+2)}+2\log(x^2+1)+392\log(x^2+2)$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```


```
[Out] (-293*x^2)/2 + (49*x^4)/2 - (9*x^6)/2 + (5*x^8)/8 + (414 + 415*x^2)/(2*(2 +
 3*x^2 + x^4)) + 2*Log[1 + x^2] + 392*Log[2 + x^2]
```


Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9 \left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{x^4 \left(4+x+3x^2+5x^3\right)}{\left(2+3x+x^2\right)^2} \, dx, x, x^2\right) \\
&= \frac{414+415x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2} \text{Subst}\left(\int \frac{-206-105x+53x^2-27x^3+12x^4-5x^5}{2+3x+x^2} \, dx, x, x^2\right) \\
&= \frac{414+415x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2} \text{Subst}\left(\int \left(293-98x+27x^2-5x^3-\frac{4(198+197x)}{2+3x+x^2}\right) dx, x, x^2\right) \\
&= -\frac{293x^2}{2} + \frac{49x^4}{2} - \frac{9x^6}{2} + \frac{5x^8}{8} + \frac{414+415x^2}{2\left(2+3x^2+x^4\right)} + 2\,\text{Subst}\left(\int \frac{198+197x}{2+3x+x^2} \, dx, x, x^2\right) \\
&= -\frac{293x^2}{2} + \frac{49x^4}{2} - \frac{9x^6}{2} + \frac{5x^8}{8} + \frac{414+415x^2}{2\left(2+3x^2+x^4\right)} + 2\,\text{Subst}\left(\int \frac{1}{1+x} \, dx, x, x^2\right) + 392\,\text{S} \\
&= -\frac{293x^2}{2} + \frac{49x^4}{2} - \frac{9x^6}{2} + \frac{5x^8}{8} + \frac{414+415x^2}{2\left(2+3x^2+x^4\right)} + 2\log\left(1+x^2\right) + 392\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.035152, size = 62, normalized size = 0.91

$$\frac{1}{8}\left(5x^8-36x^6+196x^4-1172x^2+\frac{4\left(415x^2+414\right)}{x^4+3x^2+2}+16\log\left(x^2+1\right)+3136\log\left(x^2+2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (-1172*x^2 + 196*x^4 - 36*x^6 + 5*x^8 + (4*(414 + 415*x^2))/(2 + 3*x^2 + x^
4) + 16*Log[1 + x^2] + 3136*Log[2 + x^2])/8
```

Maple [A] time = 0.015, size = 56, normalized size = 0.8

$$\frac{5\,x^8}{8}-\frac{9\,x^6}{2}+\frac{49\,x^4}{2}-\frac{293\,x^2}{2}+392\,\ln\left(x^2+2\right)+208\,\left(x^2+2\right)^{-1}+2\,\ln\left(x^2+1\right)-\frac{1}{2\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5/8*x^8-9/2*x^6+49/2*x^4-293/2*x^2+392*ln(x^2+2)+208/(x^2+2)+2*ln(x^2+1)-1/
2/(x^2+1)
```

Maxima [A] time = 0.972301, size = 78, normalized size = 1.15

$$\frac{5}{8}\,x^{8}-\frac{9}{2}\,x^{6}+\frac{49}{2}\,x^{4}-\frac{293}{2}\,x^{2}+\frac{415\,x^{2}+414}{2\left(x^{4}+3\,x^{2}+2\right)}+392\,\log\left(x^{2}+2\right)+2\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/8*x^8 - 9/2*x^6 + 49/2*x^4 - 293/2*x^2 + 1/2*(415*x^2 + 414)/(x^4 + 3*x^2
 + 2) + 392*log(x^2 + 2) + 2*log(x^2 + 1)
```

Fricas [A] time = 1.9619, size = 220, normalized size = 3.24

$$\frac{5\,x^{12}-21\,x^{10}+98\,x^{8}-656\,x^{6}-3124\,x^{4}-684\,x^{2}+3136\left(x^{4}+3\,x^{2}+2\right)\log\left(x^{2}+2\right)+16\left(x^{4}+3\,x^{2}+2\right)\log\left(x^{2}+1\right)}{8\left(x^{4}+3\,x^{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(5*x^12 - 21*x^10 + 98*x^8 - 656*x^6 - 3124*x^4 - 684*x^2 + 3136*(x^4 +
 3*x^2 + 2)*log(x^2 + 2) + 16*(x^4 + 3*x^2 + 2)*log(x^2 + 1) + 1656)/(x^4 +
 3*x^2 + 2)
```

Sympy [A] time = 0.162926, size = 61, normalized size = 0.9

$$\frac{5x^{8}}{8}-\frac{9x^{6}}{2}+\frac{49x^{4}}{2}-\frac{293x^{2}}{2}+\frac{415x^{2}+414}{2x^{4}+6x^{2}+4}+2\log\left(x^{2}+1\right)+392\log\left(x^{2}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**8/8 - 9*x**6/2 + 49*x**4/2 - 293*x**2/2 + (415*x**2 + 414)/(2*x**4 + 6
*x**2 + 4) + 2*log(x**2 + 1) + 392*log(x**2 + 2)
```

Giac [A] time = 1.11876, size = 85, normalized size = 1.25

$$\frac{5}{8}\,x^{8}-\frac{9}{2}\,x^{6}+\frac{49}{2}\,x^{4}-\frac{293}{2}\,x^{2}-\frac{394\,x^{4}+767\,x^{2}+374}{2\left(x^{4}+3\,x^{2}+2\right)}+392\,\log\left(x^{2}+2\right)+2\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/8*x^8 - 9/2*x^6 + 49/2*x^4 - 293/2*x^2 - 1/2*(394*x^4 + 767*x^2 + 374)/(x
^4 + 3*x^2 + 2) + 392*log(x^2 + 2) + 2*log(x^2 + 1)
```

3.75 $\int \frac{x^7\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=61

$$\frac{5x^6}{6}-\frac{27x^4}{4}+49x^2-\frac{207x^2+206}{2\left(x^4+3x^2+2\right)}-\frac{5}{2}\log\left(x^2+1\right)-144\log\left(x^2+2\right)$$

```
[Out] 49*x^2 - (27*x^4)/4 + (5*x^6)/6 - (206 + 207*x^2)/(2*(2 + 3*x^2 + x^4)) - (
5*Log[1 + x^2])/2 - 144*Log[2 + x^2]
```

Rubi [A] time = 0.117539, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1663, 1660, 1657, 632, 31}

$$\frac{5x^6}{6}-\frac{27x^4}{4}+49x^2-\frac{207x^2+206}{2\left(x^4+3x^2+2\right)}-\frac{5}{2}\log\left(x^2+1\right)-144\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 49*x^2 - (27*x^4)/4 + (5*x^6)/6 - (206 + 207*x^2)/(2*(2 + 3*x^2 + x^4)) - (
5*Log[1 + x^2])/2 - 144*Log[2 + x^2]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^7\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3\left(4+x+3x^2+5x^3\right)}{\left(2+3x+x^2\right)^2}\,dx,x,x^2\right)$$
$$= -\frac{206+207x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{102+53x-27x^2+12x^3-5x^4}{2+3x+x^2}\,dx,x,x^2\right)$$
$$= -\frac{206+207x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\operatorname{Subst}\left(\int \left(-98+27x-5x^2+\frac{298+293x}{2+3x+x^2}\right)dx,x,x^2\right)$$
$$= 49x^2-\frac{27x^4}{4}+\frac{5x^6}{6}-\frac{206+207x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{298+293x}{2+3x+x^2}\,dx,x,x^2\right)$$
$$= 49x^2-\frac{27x^4}{4}+\frac{5x^6}{6}-\frac{206+207x^2}{2\left(2+3x^2+x^4\right)} - \frac{5}{2}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^2\right) - 144\operatorname{Subst}\left(\right.$$
$$= 49x^2-\frac{27x^4}{4}+\frac{5x^6}{6}-\frac{206+207x^2}{2\left(2+3x^2+x^4\right)} - \frac{5}{2}\log\left(1+x^2\right) - 144\log\left(2+x^2\right)$$

Mathematica [A] time = 0.0272733, size = 61, normalized size = 1.

$$\frac{5x^6}{6}-\frac{27x^4}{4}+49x^2+\frac{-207x^2-206}{2\left(x^4+3x^2+2\right)}-\frac{5}{2}\log\left(x^2+1\right)-144\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 49*x^2 - (27*x^4)/4 + (5*x^6)/6 + (-206 - 207*x^2)/(2*(2 + 3*x^2 + x^4)) -
(5*Log[1 + x^2])/2 - 144*Log[2 + x^2]
```

Maple [A] time = 0.016, size = 51, normalized size = 0.8

$$\frac{5\,x^6}{6}-\frac{27\,x^4}{4}+49\,x^2-144\,\ln\left(x^2+2\right)-104\,\left(x^2+2\right)^{-1}-\frac{5\,\ln\left(x^2+1\right)}{2}+\frac{1}{2\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5/6*x^6-27/4*x^4+49*x^2-144*ln(x^2+2)-104/(x^2+2)-5/2*ln(x^2+1)+1/2/(x^2+1)
```

Maxima [A] time = 1.01567, size = 72, normalized size = 1.18

$$\frac{5}{6}\,x^6-\frac{27}{4}\,x^4+49\,x^2-\frac{207\,x^2+206}{2\left(x^4+3\,x^2+2\right)}-144\,\log\left(x^2+2\right)-\frac{5}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/6*x^6 - 27/4*x^4 + 49*x^2 - 1/2*(207*x^2 + 206)/(x^4 + 3*x^2 + 2) - 144*l
og(x^2 + 2) - 5/2*log(x^2 + 1)
```

Fricas [A] time = 1.94918, size = 208, normalized size = 3.41

$$\frac{10\,x^{10} - 51\,x^{8} + 365\,x^{6} + 1602\,x^{4} - 66\,x^{2} - 1728\,\left(x^{4} + 3\,x^{2} + 2\right)\log\left(x^{2} + 2\right) - 30\,\left(x^{4} + 3\,x^{2} + 2\right)\log\left(x^{2} + 1\right) - 1236}{12\,\left(x^{4} + 3\,x^{2} + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(10*x^10 - 51*x^8 + 365*x^6 + 1602*x^4 - 66*x^2 - 1728*(x^4 + 3*x^2 +
2)*log(x^2 + 2) - 30*(x^4 + 3*x^2 + 2)*log(x^2 + 1) - 1236)/(x^4 + 3*x^2 +
2)
```

Sympy [A] time = 0.155932, size = 54, normalized size = 0.89

$$\frac{5x^{6}}{6} - \frac{27x^{4}}{4} + 49x^{2} - \frac{207x^{2} + 206}{2x^{4} + 6x^{2} + 4} - \frac{5\log\left(x^{2} + 1\right)}{2} - 144\log\left(x^{2} + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**6/6 - 27*x**4/4 + 49*x**2 - (207*x**2 + 206)/(2*x**4 + 6*x**2 + 4) - 5
*log(x**2 + 1)/2 - 144*log(x**2 + 2)
```

Giac [A] time = 1.11072, size = 78, normalized size = 1.28

$$\frac{5}{6}\,x^{6} - \frac{27}{4}\,x^{4} + 49\,x^{2} + \frac{293\,x^{4} + 465\,x^{2} + 174}{4\,\left(x^{4} + 3\,x^{2} + 2\right)} - 144\,\log\left(x^{2} + 2\right) - \frac{5}{2}\,\log\left(x^{2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/6*x^6 - 27/4*x^4 + 49*x^2 + 1/4*(293*x^4 + 465*x^2 + 174)/(x^4 + 3*x^2 +
2) - 144*log(x^2 + 2) - 5/2*log(x^2 + 1)
```

3.76 $\int \frac{x^5\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=54

$$\frac{5x^4}{4}-\frac{27x^2}{2}+\frac{103x^2+102}{2\left(x^4+3x^2+2\right)}+3\log\left(x^2+1\right)+46\log\left(x^2+2\right)$$

```
[Out] (-27*x^2)/2 + (5*x^4)/4 + (102 + 103*x^2)/(2*(2 + 3*x^2 + x^4)) + 3*Log[1 +
 x^2] + 46*Log[2 + x^2]
```

Rubi [A] time = 0.108365, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1663, 1660, 1657, 632, 31}

$$\frac{5x^4}{4}-\frac{27x^2}{2}+\frac{103x^2+102}{2\left(x^4+3x^2+2\right)}+3\log\left(x^2+1\right)+46\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (-27*x^2)/2 + (5*x^4)/4 + (102 + 103*x^2)/(2*(2 + 3*x^2 + x^4)) + 3*Log[1 +
 x^2] + 46*Log[2 + x^2]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2\left(4+x+3x^2+5x^3\right)}{\left(2+3x+x^2\right)^2}\,dx, x, x^2\right)\\
&= \frac{102+103x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\,\text{Subst}\left(\int \frac{-50-27x+12x^2-5x^3}{2+3x+x^2}\,dx, x, x^2\right)\\
&= \frac{102+103x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\,\text{Subst}\left(\int \left(27-5x-\frac{2(52+49x)}{2+3x+x^2}\right)dx, x, x^2\right)\\
&= -\frac{27x^2}{2}+\frac{5x^4}{4}+\frac{102+103x^2}{2\left(2+3x^2+x^4\right)} + \text{Subst}\left(\int \frac{52+49x}{2+3x+x^2}\,dx, x, x^2\right)\\
&= -\frac{27x^2}{2}+\frac{5x^4}{4}+\frac{102+103x^2}{2\left(2+3x^2+x^4\right)} + 3\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) + 46\,\text{Subst}\left(\int \frac{1}{2+x}\,d
\right.\\
&= -\frac{27x^2}{2}+\frac{5x^4}{4}+\frac{102+103x^2}{2\left(2+3x^2+x^4\right)} + 3\log\left(1+x^2\right) + 46\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0244659, size = 54, normalized size = 1.

$$\frac{5x^4}{4}-\frac{27x^2}{2}+\frac{103x^2+102}{2\left(x^4+3x^2+2\right)}+3\log\left(x^2+1\right)+46\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (-27*x^2)/2 + (5*x^4)/4 + (102 + 103*x^2)/(2*(2 + 3*x^2 + x^4)) + 3*Log[1 +
 x^2] + 46*Log[2 + x^2]
```

Maple [A] time = 0.014, size = 46, normalized size = 0.9

$$\frac{5\,x^4}{4}-\frac{27\,x^2}{2}+46\,\ln\left(x^2+2\right)+52\,\left(x^2+2\right)^{-1}+3\,\ln\left(x^2+1\right)-\frac{1}{2\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5/4*x^4-27/2*x^2+46*ln(x^2+2)+52/(x^2+2)+3*ln(x^2+1)-1/2/(x^2+1)
```

Maximा [A] time = 0.996348, size = 65, normalized size = 1.2

$$\frac{5}{4}\,x^4-\frac{27}{2}\,x^2+\frac{103\,x^2+102}{2\left(x^4+3\,x^2+2\right)}+46\,\log\left(x^2+2\right)+3\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/4*x^4 - 27/2*x^2 + 1/2*(103*x^2 + 102)/(x^4 + 3*x^2 + 2) + 46*log(x^2 + 2
) + 3*log(x^2 + 1)
```

Fricas [A] time = 1.96442, size = 186, normalized size = 3.44

$$\frac{5\,x^{8}-39\,x^{6}-152\,x^{4}+98\,x^{2}+184\left(x^{4}+3\,x^{2}+2\right)\log\left(x^{2}+2\right)+12\left(x^{4}+3\,x^{2}+2\right)\log\left(x^{2}+1\right)+204}{4\left(x^{4}+3\,x^{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(5*x^8 - 39*x^6 - 152*x^4 + 98*x^2 + 184*(x^4 + 3*x^2 + 2)*log(x^2 + 2)
 + 12*(x^4 + 3*x^2 + 2)*log(x^2 + 1) + 204)/(x^4 + 3*x^2 + 2)
```

Sympy [A] time = 0.154507, size = 48, normalized size = 0.89

$$\frac{5x^{4}}{4}-\frac{27x^{2}}{2}+\frac{103x^{2}+102}{2x^{4}+6x^{2}+4}+3\log\left(x^{2}+1\right)+46\log\left(x^{2}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**4/4 - 27*x**2/2 + (103*x**2 + 102)/(2*x**4 + 6*x**2 + 4) + 3*log(x**2
+ 1) + 46*log(x**2 + 2)
```

Giac [A] time = 1.13079, size = 72, normalized size = 1.33

$$\frac{5}{4}\,x^{4}-\frac{27}{2}\,x^{2}-\frac{49\,x^{4}+44\,x^{2}-4}{2\left(x^{4}+3\,x^{2}+2\right)}+46\,\log\left(x^{2}+2\right)+3\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/4*x^4 - 27/2*x^2 - 1/2*(49*x^4 + 44*x^2 - 4)/(x^4 + 3*x^2 + 2) + 46*log(x
^2 + 2) + 3*log(x^2 + 1)
```

3.77 $\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=49

$$\frac{5x^2}{2}-\frac{51x^2+50}{2\left(x^4+3x^2+2\right)}-\frac{7}{2}\log\left(x^2+1\right)-10\log\left(x^2+2\right)$$

```
[Out] (5*x^2)/2 - (50 + 51*x^2)/(2*(2 + 3*x^2 + x^4)) - (7*Log[1 + x^2])/2 - 10*L
og[2 + x^2]
```

Rubi [A] time = 0.0856682, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1663, 1660, 1657, 632, 31}

$$\frac{5x^2}{2}-\frac{51x^2+50}{2\left(x^4+3x^2+2\right)}-\frac{7}{2}\log\left(x^2+1\right)-10\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (5*x^2)/2 - (50 + 51*x^2)/(2*(2 + 3*x^2 + x^4)) - (7*Log[1 + x^2])/2 - 10*L
og[2 + x^2]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
```

```
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x\left(4+x+3x^2+5x^3\right)}{\left(2+3x+x^2\right)^2}\,dx, x, x^2\right)\\
&= -\frac{50+51x^2}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{24+12x-5x^2}{2+3x+x^2}\,dx, x, x^2\right)\\
&= -\frac{50+51x^2}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\operatorname{Subst}\left(\int \left(-5+\frac{34+27x}{2+3x+x^2}\right)dx, x, x^2\right)\\
&= \frac{5x^2}{2}-\frac{50+51x^2}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{34+27x}{2+3x+x^2}\,dx, x, x^2\right)\\
&= \frac{5x^2}{2}-\frac{50+51x^2}{2\left(2+3x^2+x^4\right)}-\frac{7}{2}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right)-10\operatorname{Subst}\left(\int \frac{1}{2+x}\,dx, x, x^2\right)\\
&= \frac{5x^2}{2}-\frac{50+51x^2}{2\left(2+3x^2+x^4\right)}-\frac{7}{2}\log\left(1+x^2\right)-10\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0220387, size = 49, normalized size = 1.

$$\frac{5x^2}{2}+\frac{-51x^2-50}{2\left(x^4+3x^2+2\right)}-\frac{7}{2}\log\left(x^2+1\right)-10\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (5*x^2)/2 + (-50 - 51*x^2)/(2*(2 + 3*x^2 + x^4)) - (7*Log[1 + x^2])/2 - 10*
Log[2 + x^2]
```

Maple [A] time = 0.013, size = 41, normalized size = 0.8

$$\frac{5\,x^2}{2}-10\,\ln\left(x^2+2\right)-26\,\left(x^2+2\right)^{-1}-\frac{7\,\ln\left(x^2+1\right)}{2}+\frac{1}{2\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5/2*x^2-10*ln(x^2+2)-26/(x^2+2)-7/2*ln(x^2+1)+1/2/(x^2+1)
```

Maxima [A] time = 1.02707, size = 58, normalized size = 1.18

$$\frac{5}{2}\,x^2-\frac{51\,x^2+50}{2\left(x^4+3\,x^2+2\right)}-10\,\log\left(x^2+2\right)-\frac{7}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/2*x^2 - 1/2*(51*x^2 + 50)/(x^4 + 3*x^2 + 2) - 10*log(x^2 + 2) - 7/2*log(x
^2 + 1)
```

Fricas [A] time = 1.97511, size = 169, normalized size = 3.45

$$\frac{5\,x^{6}+15\,x^{4}-41\,x^{2}-20\,\left(x^{4}+3\,x^{2}+2\right)\log\left(x^{2}+2\right)-7\,\left(x^{4}+3\,x^{2}+2\right)\log\left(x^{2}+1\right)-50}{2\,\left(x^{4}+3\,x^{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(5*x^6 + 15*x^4 - 41*x^2 - 20*(x^4 + 3*x^2 + 2)*log(x^2 + 2) - 7*(x^4 +
 3*x^2 + 2)*log(x^2 + 1) - 50)/(x^4 + 3*x^2 + 2)
```

Sympy [A] time = 0.156256, size = 42, normalized size = 0.86

$$\frac{5x^{2}}{2}-\frac{51x^{2}+50}{2x^{4}+6x^{2}+4}-\frac{7\log\left(x^{2}+1\right)}{2}-10\log\left(x^{2}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**2/2 - (51*x**2 + 50)/(2*x**4 + 6*x**2 + 4) - 7*log(x**2 + 1)/2 - 10*lo
g(x**2 + 2)
```

Giac [A] time = 1.0934, size = 61, normalized size = 1.24

$$\frac{5}{2}\,x^{2}-\frac{51\,x^{2}+50}{2\,\left(x^{2}+2\right)\left(x^{2}+1\right)}-10\,\log\left(x^{2}+2\right)-\frac{7}{2}\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/2*x^2 - 1/2*(51*x^2 + 50)/((x^2 + 2)*(x^2 + 1)) - 10*log(x^2 + 2) - 7/2*l
og(x^2 + 1)
```

3.78 $\int \frac{x\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2} \, dx$

Optimal. Leaf size=42

$$\frac{25x^2+24}{2\left(x^4+3x^2+2\right)}+4\log\left(x^2+1\right)-\frac{3}{2}\log\left(x^2+2\right)$$

```
[Out] (24 + 25*x^2)/(2*(2 + 3*x^2 + x^4)) + 4*Log[1 + x^2] - (3*Log[2 + x^2])/2
```

Rubi [A] time = 0.0491766, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1663, 1660, 632, 31}

$$\frac{25x^2+24}{2\left(x^4+3x^2+2\right)}+4\log\left(x^2+1\right)-\frac{3}{2}\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (24 + 25*x^2)/(2*(2 + 3*x^2 + x^4)) + 4*Log[1 + x^2] - (3*Log[2 + x^2])/2
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{4+x+3x^2+5x^3}{\left(2+3x+x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{24+25x^2}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{-13-5x}{2+3x+x^2}\,dx,x,x^2\right)\\
&= \frac{24+25x^2}{2\left(2+3x^2+x^4\right)}-\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{2+x}\,dx,x,x^2\right)+4\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^2\right)\\
&= \frac{24+25x^2}{2\left(2+3x^2+x^4\right)}+4\log\left(1+x^2\right)-\frac{3}{2}\log\left(2+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0170239, size = 42, normalized size = 1.

$$\frac{25x^2+24}{2\left(x^4+3x^2+2\right)}+4\log\left(x^2+1\right)-\frac{3}{2}\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] (24 + 25*x^2)/(2*(2 + 3*x^2 + x^4)) + 4*Log[1 + x^2] - (3*Log[2 + x^2])/2
```

Maple [A] time = 0.015, size = 36, normalized size = 0.9

$$-\frac{3\ln\left(x^2+2\right)}{2}+13\left(x^2+2\right)^{-1}+4\ln\left(x^2+1\right)-\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)

[Out] -3/2*ln(x^2+2)+13/(x^2+2)+4*ln(x^2+1)-1/2/(x^2+1)
```

Maxima [A] time = 0.965276, size = 51, normalized size = 1.21

$$\frac{25x^2+24}{2\left(x^4+3x^2+2\right)}-\frac{3}{2}\log\left(x^2+2\right)+4\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")

[Out] 1/2*(25*x^2 + 24)/(x^4 + 3*x^2 + 2) - 3/2*log(x^2 + 2) + 4*log(x^2 + 1)
```

Fricas [A] time = 2.00301, size = 144, normalized size = 3.43

$$\frac{25x^2-3\left(x^4+3x^2+2\right)\log\left(x^2+2\right)+8\left(x^4+3x^2+2\right)\log\left(x^2+1\right)+24}{2\left(x^4+3x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(25*x^2 - 3*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 8*(x^4 + 3*x^2 + 2)*log(x^
2 + 1) + 24)/(x^4 + 3*x^2 + 2)
```

Sympy [A] time = 0.150097, size = 36, normalized size = 0.86

$$\frac{25x^2+24}{2x^4+6x^2+4}+4\log\left(x^2+1\right)-\frac{3\log\left(x^2+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] (25*x**2 + 24)/(2*x**4 + 6*x**2 + 4) + 4*log(x**2 + 1) - 3*log(x**2 + 2)/2
```

Giac [A] time = 1.12106, size = 54, normalized size = 1.29

$$\frac{25\,x^2+24}{2\left(x^2+2\right)\left(x^2+1\right)}-\frac{3}{2}\,\log\left(x^2+2\right)+4\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(25*x^2 + 24)/((x^2 + 2)*(x^2 + 1)) - 3/2*log(x^2 + 2) + 4*log(x^2 + 1)
```

3.79 $\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2} dx$

Optimal. Leaf size=44

$$-\frac{12x^2+11}{2\left(x^4+3x^2+2\right)}-\frac{9}{2}\log\left(x^2+1\right)+4\log\left(x^2+2\right)+\log(x)$$

```
[Out] -(11 + 12*x^2)/(2*(2 + 3*x^2 + x^4)) + Log[x] - (9*Log[1 + x^2])/2 + 4*Log[
2 + x^2]
```

Rubi [A] time = 0.0766938, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1663, 1646, 800}

$$-\frac{12x^2+11}{2\left(x^4+3x^2+2\right)}-\frac{9}{2}\log\left(x^2+1\right)+4\log\left(x^2+2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -(11 + 12*x^2)/(2*(2 + 3*x^2 + x^4)) + Log[x] - (9*Log[1 + x^2])/2 + 4*Log[
2 + x^2]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x\left(2+3x+x^2\right)^2}\,dx, x, x^2\right)$$
$$= -\frac{11+12x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{-2+7x}{x\left(2+3x+x^2\right)}\,dx, x, x^2\right)$$
$$= -\frac{11+12x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{1}{x}+\frac{9}{1+x}-\frac{8}{2+x}\right)dx, x, x^2\right)$$
$$= -\frac{11+12x^2}{2\left(2+3x^2+x^4\right)} + \log(x) - \frac{9}{2}\log\left(1+x^2\right) + 4\log\left(2+x^2\right)$$

Mathematica [A] time = 0.0215405, size = 44, normalized size = 1.

$$\frac{-12x^2-11}{2\left(x^4+3x^2+2\right)} - \frac{9}{2}\log\left(x^2+1\right) + 4\log\left(x^2+2\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-11 - 12*x^2)/(2*(2 + 3*x^2 + x^4)) + Log[x] - (9*Log[1 + x^2])/2 + 4*Log[
2 + x^2]
```

Maple [A] time = 0.017, size = 38, normalized size = 0.9

$$4\ln\left(x^2+2\right) - \frac{13}{2x^2+4} - \frac{9\ln\left(x^2+1\right)}{2} + \frac{1}{2x^2+2} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x)
```

```
[Out] 4*ln(x^2+2)-13/2/(x^2+2)-9/2*ln(x^2+1)+1/2/(x^2+1)+ln(x)
```

Maxima [A] time = 1.0193, size = 59, normalized size = 1.34

$$-\frac{12x^2+11}{2\left(x^4+3x^2+2\right)} + 4\log\left(x^2+2\right) - \frac{9}{2}\log\left(x^2+1\right) + \frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(12*x^2 + 11)/(x^4 + 3*x^2 + 2) + 4*log(x^2 + 2) - 9/2*log(x^2 + 1) +
1/2*log(x^2)
```

Fricas [A] time = 1.74922, size = 185, normalized size = 4.2

$$-\frac{12\,x^2-8\left(x^4+3\,x^2+2\right)\log\left(x^2+2\right)+9\left(x^4+3\,x^2+2\right)\log\left(x^2+1\right)-2\left(x^4+3\,x^2+2\right)\log\left(x\right)+11}{2\left(x^4+3\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(12*x^2 - 8*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 9*(x^4 + 3*x^2 + 2)*log(x
^2 + 1) - 2*(x^4 + 3*x^2 + 2)*log(x) + 11)/(x^4 + 3*x^2 + 2)
```

Sympy [A] time = 0.16426, size = 39, normalized size = 0.89

$$-\frac{12x^2+11}{2x^4+6x^2+4}+\log\left(x\right)-\frac{9\log\left(x^2+1\right)}{2}+4\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x/(x**4+3*x**2+2)**2,x)
```

```
[Out] -(12*x**2 + 11)/(2*x**4 + 6*x**2 + 4) + log(x) - 9*log(x**2 + 1)/2 + 4*log(
x**2 + 2)
```

Giac [A] time = 1.08408, size = 63, normalized size = 1.43

$$\frac{x^4-21\,x^2-20}{4\left(x^4+3\,x^2+2\right)}+4\,\log\left(x^2+2\right)-\frac{9}{2}\,\log\left(x^2+1\right)+\frac{1}{2}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(x^4 - 21*x^2 - 20)/(x^4 + 3*x^2 + 2) + 4*log(x^2 + 2) - 9/2*log(x^2 +
1) + 1/2*log(x^2)
```

3.80 $\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=55

$$\frac{11x^2+9}{4\left(x^4+3x^2+2\right)}-\frac{1}{2x^2}+5\log\left(x^2+1\right)-\frac{29}{8}\log\left(x^2+2\right)-\frac{11\log(x)}{4}$$

```
[Out] -1/(2*x^2) + (9 + 11*x^2)/(4*(2 + 3*x^2 + x^4)) - (11*Log[x])/4 + 5*Log[1 +
 x^2] - (29*Log[2 + x^2])/8
```

Rubi [A] time = 0.104323, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1663, 1646, 1628}

$$\frac{11x^2+9}{4\left(x^4+3x^2+2\right)}-\frac{1}{2x^2}+5\log\left(x^2+1\right)-\frac{29}{8}\log\left(x^2+2\right)-\frac{11\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/(2*x^2) + (9 + 11*x^2)/(4*(2 + 3*x^2 + x^4)) - (11*Log[x])/4 + 5*Log[1 +
 x^2] - (29*Log[2 + x^2])/8
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^2\left(2+3x+x^2\right)^2}\,dx, x, x^2\right)$$
$$= \frac{9+11x^2}{4\left(2+3x^2+x^4\right)} - \frac{1}{2}\text{Subst}\left(\int \frac{-2+\frac{5x}{2}-\frac{11x^2}{2}}{x^2\left(2+3x+x^2\right)}\,dx, x, x^2\right)$$
$$= \frac{9+11x^2}{4\left(2+3x^2+x^4\right)} - \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^2}+\frac{11}{4x}-\frac{10}{1+x}+\frac{29}{4(2+x)}\right)dx, x, x^2\right)$$
$$= -\frac{1}{2x^2} + \frac{9+11x^2}{4\left(2+3x^2+x^4\right)} - \frac{11\log(x)}{4} + 5\log\left(1+x^2\right) - \frac{29}{8}\log\left(2+x^2\right)$$

Mathematica [A] time = 0.0251518, size = 50, normalized size = 0.91

$$\frac{1}{8}\left(\frac{22x^2+18}{x^4+3x^2+2} - \frac{4}{x^2} + 40\log\left(x^2+1\right) - 29\log\left(x^2+2\right) - 22\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-4/x^2 + (18 + 22*x^2)/(2 + 3*x^2 + x^4) - 22*Log[x] + 40*Log[1 + x^2] - 2
9*Log[2 + x^2])/8
```

Maple [A] time = 0.017, size = 45, normalized size = 0.8

$$-\frac{29\ln\left(x^2+2\right)}{8} + \frac{13}{4x^2+8} + 5\ln\left(x^2+1\right) - \frac{1}{2x^2+2} - \frac{1}{2x^2} - \frac{11\ln(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x)
```

```
[Out] -29/8*ln(x^2+2)+13/4/(x^2+2)+5*ln(x^2+1)-1/2/(x^2+1)-1/2/x^2-11/4*ln(x)
```

Maxima [A] time = 1.00124, size = 72, normalized size = 1.31

$$\frac{9x^4+3x^2-4}{4\left(x^6+3x^4+2x^2\right)} - \frac{29}{8}\log\left(x^2+2\right) + 5\log\left(x^2+1\right) - \frac{11}{8}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^4 + 3*x^2 - 4)/(x^6 + 3*x^4 + 2*x^2) - 29/8*log(x^2 + 2) + 5*log(x
^2 + 1) - 11/8*log(x^2)
```

Fricas [A] time = 1.85876, size = 219, normalized size = 3.98

$$\frac{18\,x^4+6\,x^2-29\left(x^6+3\,x^4+2\,x^2\right)\log\left(x^2+2\right)+40\left(x^6+3\,x^4+2\,x^2\right)\log\left(x^2+1\right)-22\left(x^6+3\,x^4+2\,x^2\right)\log\left(x\right)-8}{8\left(x^6+3\,x^4+2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(18*x^4 + 6*x^2 - 29*(x^6 + 3*x^4 + 2*x^2)*log(x^2 + 2) + 40*(x^6 + 3*x
^4 + 2*x^2)*log(x^2 + 1) - 22*(x^6 + 3*x^4 + 2*x^2)*log(x) - 8)/(x^6 + 3*x^
4 + 2*x^2)
```

Sympy [A] time = 0.190424, size = 51, normalized size = 0.93

$$\frac{9x^4+3x^2-4}{4x^6+12x^4+8x^2}-\frac{11\log\left(x\right)}{4}+5\log\left(x^2+1\right)-\frac{29\log\left(x^2+2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**3/(x**4+3*x**2+2)**2,x)
```

```
[Out] (9*x**4 + 3*x**2 - 4)/(4*x**6 + 12*x**4 + 8*x**2) - 11*log(x)/4 + 5*log(x**
2 + 1) - 29*log(x**2 + 2)/8
```

Giac [A] time = 1.1098, size = 72, normalized size = 1.31

$$\frac{9\,x^4+3\,x^2-4}{4\left(x^6+3\,x^4+2\,x^2\right)}-\frac{29}{8}\,\log\left(x^2+2\right)+5\,\log\left(x^2+1\right)-\frac{11}{8}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(9*x^4 + 3*x^2 - 4)/(x^6 + 3*x^4 + 2*x^2) - 29/8*log(x^2 + 2) + 5*log(x
^2 + 1) - 11/8*log(x^2)
```

3.81 $\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=64

$$-\frac{9x^2+5}{8\left(x^4+3x^2+2\right)}+\frac{11}{8x^2}-\frac{1}{4x^4}-\frac{11}{2}\log\left(x^2+1\right)+\frac{21}{8}\log\left(x^2+2\right)+\frac{23\log(x)}{4}$$

```
[Out] -1/(4*x^4) + 11/(8*x^2) - (5 + 9*x^2)/(8*(2 + 3*x^2 + x^4)) + (23*Log[x])/4
 - (11*Log[1 + x^2])/2 + (21*Log[2 + x^2])/8
```

Rubi [A] time = 0.110765, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1663, 1646, 1628}

$$-\frac{9x^2+5}{8\left(x^4+3x^2+2\right)}+\frac{11}{8x^2}-\frac{1}{4x^4}-\frac{11}{2}\log\left(x^2+1\right)+\frac{21}{8}\log\left(x^2+2\right)+\frac{23\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/(4*x^4) + 11/(8*x^2) - (5 + 9*x^2)/(8*(2 + 3*x^2 + x^4)) + (23*Log[x])/4
 - (11*Log[1 + x^2])/2 + (21*Log[2 + x^2])/8
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^3\left(2+3x+x^2\right)^2}\,dx,x,x^2\right)\\
&= -\frac{5+9x^2}{8\left(2+3x^2+x^4\right)}-\frac{1}{2}\,\text{Subst}\left(\int \frac{-2+\frac{5x}{2}-\frac{17x^2}{4}+\frac{9x^3}{4}}{x^3\left(2+3x+x^2\right)}\,dx,x,x^2\right)\\
&= -\frac{5+9x^2}{8\left(2+3x^2+x^4\right)}-\frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{1}{x^3}+\frac{11}{4x^2}-\frac{23}{4x}+\frac{11}{1+x}-\frac{21}{4(2+x)}\right)dx,x,x^2\right)\\
&= -\frac{1}{4x^4}+\frac{11}{8x^2}-\frac{5+9x^2}{8\left(2+3x^2+x^4\right)}+\frac{23\log(x)}{4}-\frac{11}{2}\log\left(1+x^2\right)+\frac{21}{8}\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0293778, size = 56, normalized size = 0.88

$$\frac{1}{8}\left(-\frac{9x^2+5}{x^4+3x^2+2}+\frac{11}{x^2}-\frac{2}{x^4}-44\log\left(x^2+1\right)+21\log\left(x^2+2\right)+46\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-2/x^4 + 11/x^2 - (5 + 9*x^2)/(2 + 3*x^2 + x^4) + 46*Log[x] - 44*Log[1 + x
^2] + 21*Log[2 + x^2])/8
```

Maple [A] time = 0.018, size = 50, normalized size = 0.8

$$\frac{21\ln\left(x^2+2\right)}{8}-\frac{13}{8x^2+16}-\frac{11\ln\left(x^2+1\right)}{2}+\frac{1}{2x^2+2}-\frac{1}{4x^4}+\frac{11}{8x^2}+\frac{23\ln(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x)
```

```
[Out] 21/8*ln(x^2+2)-13/8/(x^2+2)-11/2*ln(x^2+1)+1/2/(x^2+1)-1/4/x^4+11/8/x^2+23/
4*ln(x)
```

Maxima [A] time = 0.952735, size = 76, normalized size = 1.19

$$\frac{x^6+13x^4+8x^2-2}{4\left(x^8+3x^6+2x^4\right)}+\frac{21}{8}\log\left(x^2+2\right)-\frac{11}{2}\log\left(x^2+1\right)+\frac{23}{8}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(x^6 + 13*x^4 + 8*x^2 - 2)/(x^8 + 3*x^6 + 2*x^4) + 21/8*log(x^2 + 2) -
11/2*log(x^2 + 1) + 23/8*log(x^2)
```

Fricas [A] time = 1.7272, size = 231, normalized size = 3.61

$$\frac{2x^6+26x^4+16x^2+21\left(x^8+3x^6+2x^4\right)\log\left(x^2+2\right)-44\left(x^8+3x^6+2x^4\right)\log\left(x^2+1\right)+46\left(x^8+3x^6+2x^4\right)\log\left(x\right)}{8\left(x^8+3x^6+2x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^6 + 26*x^4 + 16*x^2 + 21*(x^8 + 3*x^6 + 2*x^4)*log(x^2 + 2) - 44*(
x^8 + 3*x^6 + 2*x^4)*log(x^2 + 1) + 46*(x^8 + 3*x^6 + 2*x^4)*log(x) - 4)/(x
^8 + 3*x^6 + 2*x^4)
```

Sympy [A] time = 0.204195, size = 56, normalized size = 0.88

$$\frac{23\log\left(x\right)}{4}-\frac{11\log\left(x^2+1\right)}{2}+\frac{21\log\left(x^2+2\right)}{8}+\frac{x^6+13x^4+8x^2-2}{4x^8+12x^6+8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**5/(x**4+3*x**2+2)**2,x)
```

```
[Out] 23*log(x)/4 - 11*log(x**2 + 1)/2 + 21*log(x**2 + 2)/8 + (x**6 + 13*x**4 + 8
*x**2 - 2)/(4*x**8 + 12*x**6 + 8*x**4)
```

Giac [A] time = 1.09415, size = 89, normalized size = 1.39

$$\frac{23x^4+51x^2+36}{16\left(x^4+3x^2+2\right)}-\frac{69x^4-22x^2+4}{16x^4}+\frac{21}{8}\log\left(x^2+2\right)-\frac{11}{2}\log\left(x^2+1\right)+\frac{23}{8}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 1/16*(23*x^4 + 51*x^2 + 36)/(x^4 + 3*x^2 + 2) - 1/16*(69*x^4 - 22*x^2 + 4)/
x^4 + 21/8*log(x^2 + 2) - 11/2*log(x^2 + 1) + 23/8*log(x^2)
```

3.82 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=70

$$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-\frac{(207x^2+206)\,x}{2\,(x^4+3x^2+2)}-293x+\frac{9}{2}\tan^{-1}(x)+340\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] -293*x + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7 - (x*(206 + 207*x^2))/(2*(2 +
3*x^2 + x^4)) + (9*ArcTan[x])/2 + 340*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rubi [A] time = 0.0845314, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1668, 1676, 1166, 203}

$$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-\frac{(207x^2+206)\,x}{2\,(x^4+3x^2+2)}-293x+\frac{9}{2}\tan^{-1}(x)+340\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -293*x + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7 - (x*(206 + 207*x^2))/(2*(2 +
3*x^2 + x^4)) + (9*ArcTan[x])/2 + 340*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx &= -\frac{x\left(206+207x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \frac{-412-6x^2+212x^4-108x^6+48x^8-20x^{10}}{2+3x^2+x^4}\,dx\\
&= -\frac{x\left(206+207x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \left(1172-392x^2+108x^4-20x^6-\frac{2\left(1378+1369x^2\right)}{2+3x^2+x^4}\right)dx\\
&= -293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}-\frac{x\left(206+207x^2\right)}{2\left(2+3x^2+x^4\right)}+\frac{1}{2}\int \frac{1378+1369x^2}{2+3x^2+x^4}\,dx\\
&= -293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}-\frac{x\left(206+207x^2\right)}{2\left(2+3x^2+x^4\right)}+\frac{9}{2}\int \frac{1}{1+x^2}\,dx+680\int \frac{1}{2+x^2}\,dx\\
&= -293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}-\frac{x\left(206+207x^2\right)}{2\left(2+3x^2+x^4\right)}+\frac{9}{2}\tan^{-1}(x)+340\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.045937, size = 71, normalized size = 1.01

$$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}+\frac{-207x^3-206x}{2\left(x^4+3x^2+2\right)}-293x+\frac{9}{2}\tan^{-1}(x)+340\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -293*x + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7 + (-206*x - 207*x^3)/(2*(2 + 3
*x^2 + x^4)) + (9*ArcTan[x])/2 + 340*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Maple [A] time = 0.013, size = 56, normalized size = 0.8

$$\frac{5\,x^7}{7}-\frac{27\,x^5}{5}+\frac{98\,x^3}{3}-293\,x-104\,\frac{x}{x^2+2}+340\,\arctan\left(1/2\,x\sqrt{2}\right)\sqrt{2}+\frac{x}{2\,x^2+2}+\frac{9\,\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5/7*x^7-27/5*x^5+98/3*x^3-293*x-104*x/(x^2+2)+340*arctan(1/2*x*2^(1/2))*2^(
1/2)+1/2*x/(x^2+1)+9/2*arctan(x)
```

Maxima [A] time = 1.46827, size = 78, normalized size = 1.11

$$\frac{5}{7}\,x^7-\frac{27}{5}\,x^5+\frac{98}{3}\,x^3+340\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-293\,x-\frac{207\,x^3+206\,x}{2\left(x^4+3\,x^2+2\right)}+\frac{9}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/7*x^7 - 27/5*x^5 + 98/3*x^3 + 340*sqrt(2)*arctan(1/2*sqrt(2)*x) - 293*x -
 1/2*(207*x^3 + 206*x)/(x^4 + 3*x^2 + 2) + 9/2*arctan(x)
```

Fricas [A] time = 1.84306, size = 247, normalized size = 3.53

$$\frac{150\,x^{11} - 684\,x^{9} + 3758\,x^{7} - 43218\,x^{5} - 192605\,x^{3} + 71400\,\sqrt{2}\left(x^{4} + 3\,x^{2} + 2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 945\,\left(x^{4} + 3\,x^{2} + 2\right)a}{210\,\left(x^{4} + 3\,x^{2} + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/210*(150*x^11 - 684*x^9 + 3758*x^7 - 43218*x^5 - 192605*x^3 + 71400*sqrt(
2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) + 945*(x^4 + 3*x^2 + 2)*arctan(x
) - 144690*x)/(x^4 + 3*x^2 + 2)
```

Sympy [A] time = 0.186366, size = 66, normalized size = 0.94

$$\frac{5x^{7}}{7} - \frac{27x^{5}}{5} + \frac{98x^{3}}{3} - 293x - \frac{207x^{3} + 206x}{2x^{4} + 6x^{2} + 4} + \frac{9\,\mathrm{atan}\,(x)}{2} + 340\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**7/7 - 27*x**5/5 + 98*x**3/3 - 293*x - (207*x**3 + 206*x)/(2*x**4 + 6*x
**2 + 4) + 9*atan(x)/2 + 340*sqrt(2)*atan(sqrt(2)*x/2)
```

Giac [A] time = 1.08255, size = 78, normalized size = 1.11

$$\frac{5}{7}\,x^{7} - \frac{27}{5}\,x^{5} + \frac{98}{3}\,x^{3} + 340\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 293\,x - \frac{207\,x^{3} + 206\,x}{2\left(x^{4} + 3\,x^{2} + 2\right)} + \frac{9}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/7*x^7 - 27/5*x^5 + 98/3*x^3 + 340*sqrt(2)*arctan(1/2*sqrt(2)*x) - 293*x -
 1/2*(207*x^3 + 206*x)/(x^4 + 3*x^2 + 2) + 9/2*arctan(x)
```

3.83 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=57

$$x^5-9x^3+\frac{(103x^2+102)\,x}{2\,(x^4+3x^2+2)}+98x-\frac{11}{2}\tan^{-1}(x)-118\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 98*x - 9*x^3 + x^5 + (x*(102 + 103*x^2))/(2*(2 + 3*x^2 + x^4)) - (11*ArcTan
[x])/2 - 118*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rubi [A] time = 0.0821456, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1668, 1676, 1166, 203}

$$x^5-9x^3+\frac{(103x^2+102)\,x}{2\,(x^4+3x^2+2)}+98x-\frac{11}{2}\tan^{-1}(x)-118\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 98*x - 9*x^3 + x^5 + (x*(102 + 103*x^2))/(2*(2 + 3*x^2 + x^4)) - (11*ArcTan
[x])/2 - 118*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx &= \frac{x\left(102+103x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \frac{204+6x^2-108x^4+48x^6-20x^8}{2+3x^2+x^4}\,dx\\
&= \frac{x\left(102+103x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \left(-392+108x^2-20x^4+\frac{2\left(494+483x^2\right)}{2+3x^2+x^4}\right)dx\\
&= 98x-9x^3+x^5+\frac{x\left(102+103x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\int \frac{494+483x^2}{2+3x^2+x^4}\,dx\\
&= 98x-9x^3+x^5+\frac{x\left(102+103x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{11}{2}\int \frac{1}{1+x^2}\,dx-236\int \frac{1}{2+x^2}\,dx\\
&= 98x-9x^3+x^5+\frac{x\left(102+103x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{11}{2}\tan^{-1}(x)-118\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0479191, size = 58, normalized size = 1.02

$$x^5-9x^3+\frac{103x^3+102x}{2\left(x^4+3x^2+2\right)}+98x-\frac{11}{2}\tan^{-1}(x)-118\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 98*x - 9*x^3 + x^5 + (102*x + 103*x^3)/(2*(2 + 3*x^2 + x^4)) - (11*ArcTan[x
])/2 - 118*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Maple [A] time = 0.013, size = 49, normalized size = 0.9

$$x^5-9\,x^3+98\,x+52\,\frac{x}{x^2+2}-118\,\arctan\left(1/2\,x\sqrt{2}\right)\sqrt{2}-\frac{x}{2\,x^2+2}-\frac{11\,\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] x^5-9*x^3+98*x+52*x/(x^2+2)-118*arctan(1/2*x*2^(1/2))*2^(1/2)-1/2*x/(x^2+1)
-11/2*arctan(x)
```

Maxima [A] time = 1.48617, size = 69, normalized size = 1.21

$$x^5-9\,x^3-118\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+98\,x+\frac{103\,x^3+102\,x}{2\left(x^4+3\,x^2+2\right)}-\frac{11}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] x^5 - 9*x^3 - 118*sqrt(2)*arctan(1/2*sqrt(2)*x) + 98*x + 1/2*(103*x^3 + 102
*x)/(x^4 + 3*x^2 + 2) - 11/2*arctan(x)
```

Fricas [A] time = 1.75625, size = 209, normalized size = 3.67

$$\frac{2\,x^9-12\,x^7+146\,x^5+655\,x^3-236\,\sqrt{2}\left(x^4+3\,x^2+2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-11\,\left(x^4+3\,x^2+2\right)\arctan\left(x\right)+494\,x}{2\,\left(x^4+3\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^9 - 12*x^7 + 146*x^5 + 655*x^3 - 236*sqrt(2)*(x^4 + 3*x^2 + 2)*arc
tan(1/2*sqrt(2)*x) - 11*(x^4 + 3*x^2 + 2)*arctan(x) + 494*x)/(x^4 + 3*x^2 +
 2)
```

Sympy [A] time = 0.184926, size = 54, normalized size = 0.95

$$x^5-9x^3+98x+\frac{103x^3+102x}{2x^4+6x^2+4}-\frac{11\,\mathrm{atan}\,(x)}{2}-118\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] x**5 - 9*x**3 + 98*x + (103*x**3 + 102*x)/(2*x**4 + 6*x**2 + 4) - 11*atan(x
)/2 - 118*sqrt(2)*atan(sqrt(2)*x/2)
```

Giac [A] time = 1.08387, size = 69, normalized size = 1.21

$$x^5-9\,x^3-118\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+98\,x+\frac{103\,x^3+102\,x}{2\,\left(x^4+3\,x^2+2\right)}-\frac{11}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] x^5 - 9*x^3 - 118*sqrt(2)*arctan(1/2*sqrt(2)*x) + 98*x + 1/2*(103*x^3 + 102
*x)/(x^4 + 3*x^2 + 2) - 11/2*arctan(x)
```

3.84 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=56

$$\frac{5x^3}{3}-\frac{(51x^2+50)\,x}{2\,(x^4+3x^2+2)}-27x+\frac{13}{2}\tan^{-1}(x)+33\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] -27*x + (5*x^3)/3 - (x*(50 + 51*x^2))/(2*(2 + 3*x^2 + x^4)) + (13*ArcTan[x]
)/2 + 33*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rubi [A] time = 0.0727921, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1668, 1676, 1166, 203}

$$\frac{5x^3}{3}-\frac{(51x^2+50)\,x}{2\,(x^4+3x^2+2)}-27x+\frac{13}{2}\tan^{-1}(x)+33\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -27*x + (5*x^3)/3 - (x*(50 + 51*x^2))/(2*(2 + 3*x^2 + x^4)) + (13*ArcTan[x]
)/2 + 33*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx &= -\frac{x\left(50+51x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \frac{-100-6x^2+48x^4-20x^6}{2+3x^2+x^4}\,dx\\
&= -\frac{x\left(50+51x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \left(108-20x^2-\frac{2\left(158+145x^2\right)}{2+3x^2+x^4}\right)dx\\
&= -27x+\frac{5x^3}{3}-\frac{x\left(50+51x^2\right)}{2\left(2+3x^2+x^4\right)}+\frac{1}{2}\int \frac{158+145x^2}{2+3x^2+x^4}\,dx\\
&= -27x+\frac{5x^3}{3}-\frac{x\left(50+51x^2\right)}{2\left(2+3x^2+x^4\right)}+\frac{13}{2}\int \frac{1}{1+x^2}\,dx+66\int \frac{1}{2+x^2}\,dx\\
&= -27x+\frac{5x^3}{3}-\frac{x\left(50+51x^2\right)}{2\left(2+3x^2+x^4\right)}+\frac{13}{2}\tan^{-1}(x)+33\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0427317, size = 57, normalized size = 1.02

$$\frac{5x^3}{3}+\frac{-51x^3-50x}{2\left(x^4+3x^2+2\right)}-27x+\frac{13}{2}\tan^{-1}(x)+33\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -27*x + (5*x^3)/3 + (-50*x - 51*x^3)/(2*(2 + 3*x^2 + x^4)) + (13*ArcTan[x])
/2 + 33*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Maple [A] time = 0.01, size = 46, normalized size = 0.8

$$\frac{5\,x^3}{3}-27\,x-26\,\frac{x}{x^2+2}+33\,\arctan\left(1/2\,x\sqrt{2}\right)\sqrt{2}+\frac{x}{2\,x^2+2}+\frac{13\,\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5/3*x^3-27*x-26*x/(x^2+2)+33*arctan(1/2*x*2^(1/2))*2^(1/2)+1/2*x/(x^2+1)+13
/2*arctan(x)
```

Maxima [A] time = 1.48765, size = 65, normalized size = 1.16

$$\frac{5}{3}\,x^3+33\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-27\,x-\frac{51\,x^3+50\,x}{2\left(x^4+3\,x^2+2\right)}+\frac{13}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 + 33*sqrt(2)*arctan(1/2*sqrt(2)*x) - 27*x - 1/2*(51*x^3 + 50*x)/(x^
4 + 3*x^2 + 2) + 13/2*arctan(x)
```

Fricas [A] time = 1.90295, size = 198, normalized size = 3.54

$$\frac{10\,x^{7}-132\,x^{5}-619\,x^{3}+198\,\sqrt{2}\left(x^{4}+3\,x^{2}+2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+39\,\left(x^{4}+3\,x^{2}+2\right)\arctan\left(x\right)-474\,x}{6\,\left(x^{4}+3\,x^{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(10*x^7 - 132*x^5 - 619*x^3 + 198*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*
sqrt(2)*x) + 39*(x^4 + 3*x^2 + 2)*arctan(x) - 474*x)/(x^4 + 3*x^2 + 2)
```

Sympy [A] time = 0.189747, size = 53, normalized size = 0.95

$$\frac{5x^{3}}{3}-27x-\frac{51x^{3}+50x}{2x^{4}+6x^{2}+4}+\frac{13\,\mathrm{atan}\,(x)}{2}+33\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**3/3 - 27*x - (51*x**3 + 50*x)/(2*x**4 + 6*x**2 + 4) + 13*atan(x)/2 + 3
3*sqrt(2)*atan(sqrt(2)*x/2)
```

Giac [A] time = 1.14651, size = 65, normalized size = 1.16

$$\frac{5}{3}\,x^{3}+33\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-27\,x-\frac{51\,x^{3}+50\,x}{2\,\left(x^{4}+3\,x^{2}+2\right)}+\frac{13}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/3*x^3 + 33*sqrt(2)*arctan(1/2*sqrt(2)*x) - 27*x - 1/2*(51*x^3 + 50*x)/(x^
4 + 3*x^2 + 2) + 13/2*arctan(x)
```

3.85 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$

Optimal. Leaf size=49

$$\frac{(25x^2+24)x}{2(x^4+3x^2+2)}+5x-\frac{15}{2}\tan^{-1}(x)-\frac{7\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 5*x + (x*(24 + 25*x^2))/(2*(2 + 3*x^2 + x^4)) - (15*ArcTan[x])/2 - (7*ArcTa
n[x/Sqrt[2]])/Sqrt[2]
```

Rubi [A] time = 0.0663581, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1668, 1676, 1166, 203}

$$\frac{(25x^2+24)x}{2(x^4+3x^2+2)}+5x-\frac{15}{2}\tan^{-1}(x)-\frac{7\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (x*(24 + 25*x^2))/(2*(2 + 3*x^2 + x^4)) - (15*ArcTan[x])/2 - (7*ArcTa
n[x/Sqrt[2]])/Sqrt[2]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx &= \frac{x(24+25x^2)}{2(2+3x^2+x^4)} - \frac{1}{4}\int \frac{48-2x^2-20x^4}{2+3x^2+x^4}\,dx \\
&= \frac{x(24+25x^2)}{2(2+3x^2+x^4)} - \frac{1}{4}\int \left(-20+\frac{2(44+29x^2)}{2+3x^2+x^4}\right)dx \\
&= 5x + \frac{x(24+25x^2)}{2(2+3x^2+x^4)} - \frac{1}{2}\int \frac{44+29x^2}{2+3x^2+x^4}\,dx \\
&= 5x + \frac{x(24+25x^2)}{2(2+3x^2+x^4)} - 7\int \frac{1}{2+x^2}\,dx - \frac{15}{2}\int \frac{1}{1+x^2}\,dx \\
&= 5x + \frac{x(24+25x^2)}{2(2+3x^2+x^4)} - \frac{15}{2}\tan^{-1}(x) - \frac{7\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.038853, size = 50, normalized size = 1.02

$$\frac{25x^3+24x}{2(x^4+3x^2+2)} + 5x - \frac{15}{2}\tan^{-1}(x) - \frac{7\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (24*x + 25*x^3)/(2*(2 + 3*x^2 + x^4)) - (15*ArcTan[x])/2 - (7*ArcTan[
x/Sqrt[2]])/Sqrt[2]
```

Maple [A] time = 0.01, size = 41, normalized size = 0.8

$$5x + 13\frac{x}{x^2+2} - \frac{7\sqrt{2}}{2}\arctan\left(\frac{x\sqrt{2}}{2}\right) - \frac{x}{2x^2+2} - \frac{15\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5*x+13*x/(x^2+2)-7/2*arctan(1/2*x*2^(1/2))*2^(1/2)-1/2*x/(x^2+1)-15/2*arcta
n(x)
```

Maxima [A] time = 1.48706, size = 58, normalized size = 1.18

$$-\frac{7}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 5x + \frac{25x^3+24x}{2(x^4+3x^2+2)} - \frac{15}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -7/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x + 1/2*(25*x^3 + 24*x)/(x^4 + 3*x^2
 + 2) - 15/2*arctan(x)
```

Fricas [A] time = 2.13975, size = 180, normalized size = 3.67

$$\frac{10\,x^5 + 55\,x^3 - 7\,\sqrt{2}\left(x^4 + 3\,x^2 + 2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 15\,\left(x^4 + 3\,x^2 + 2\right)\arctan\left(x\right) + 44\,x}{2\,\left(x^4 + 3\,x^2 + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(10*x^5 + 55*x^3 - 7*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) -
15*(x^4 + 3*x^2 + 2)*arctan(x) + 44*x)/(x^4 + 3*x^2 + 2)
```

Sympy [A] time = 0.185744, size = 48, normalized size = 0.98

$$5x + \frac{25x^3 + 24x}{2x^4 + 6x^2 + 4} - \frac{15\,\mathrm{atan}\,(x)}{2} - \frac{7\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x + (25*x**3 + 24*x)/(2*x**4 + 6*x**2 + 4) - 15*atan(x)/2 - 7*sqrt(2)*ata
n(sqrt(2)*x/2)/2
```

Giac [A] time = 1.10117, size = 58, normalized size = 1.18

$$-\frac{7}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 5\,x + \frac{25\,x^3 + 24\,x}{2\,\left(x^4 + 3\,x^2 + 2\right)} - \frac{15}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] -7/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x + 1/2*(25*x^3 + 24*x)/(x^4 + 3*x^2
 + 2) - 15/2*arctan(x)
```

3.86 $\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^2} \, dx$

Optimal. Leaf size=48

$$-\frac{x\left(12x^2+11\right)}{2\left(x^4+3x^2+2\right)}+\frac{17}{2}\tan^{-1}(x)-\frac{19\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] -(x*(11 + 12*x^2))/(2*(2 + 3*x^2 + x^4)) + (17*ArcTan[x])/2 - (19*ArcTan[x/
Sqrt[2]])/(2*Sqrt[2])
```

Rubi [A] time = 0.0280516, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1678, 1166, 203}

$$-\frac{x\left(12x^2+11\right)}{2\left(x^4+3x^2+2\right)}+\frac{17}{2}\tan^{-1}(x)-\frac{19\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -(x*(11 + 12*x^2))/(2*(2 + 3*x^2 + x^4)) + (17*ArcTan[x])/2 - (19*ArcTan[x/
Sqrt[2]])/(2*Sqrt[2])
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^2}\,dx = -\frac{x\left(11+12x^2\right)}{2\left(2+3x^2+x^4\right)} - \frac{1}{4}\int \frac{-30+4x^2}{2+3x^2+x^4}\,dx$$
$$= -\frac{x\left(11+12x^2\right)}{2\left(2+3x^2+x^4\right)} + \frac{17}{2}\int \frac{1}{1+x^2}\,dx - \frac{19}{2}\int \frac{1}{2+x^2}\,dx$$
$$= -\frac{x\left(11+12x^2\right)}{2\left(2+3x^2+x^4\right)} + \frac{17}{2}\tan^{-1}(x) - \frac{19\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Mathematica [A] time = 0.0405299, size = 46, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2x\left(12x^2+11\right)}{x^4+3x^2+2} + 34\tan^{-1}(x) - 19\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] ((-2*x*(11 + 12*x^2))/(2 + 3*x^2 + x^4) + 34*ArcTan[x] - 19*Sqrt[2]*ArcTan[
x/Sqrt[2]])/4
```

Maple [A] time = 0.011, size = 38, normalized size = 0.8

$$-\frac{13\,x}{2\,x^2+4} - \frac{19\,\sqrt{2}}{4}\arctan\left(\frac{x\sqrt{2}}{2}\right) + \frac{x}{2\,x^2+2} + \frac{17\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] -13/2*x/(x^2+2)-19/4*arctan(1/2*x*2^(1/2))*2^(1/2)+1/2*x/(x^2+1)+17/2*arcta
n(x)
```

Maxima [A] time = 1.49892, size = 54, normalized size = 1.12

$$-\frac{19}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{12\,x^3+11\,x}{2\left(x^4+3\,x^2+2\right)} + \frac{17}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -19/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/2*(12*x^3 + 11*x)/(x^4 + 3*x^2 + 2)
 + 17/2*arctan(x)
```

Fricas [A] time = 2.05986, size = 170, normalized size = 3.54

$$-\frac{24\,x^3+19\,\sqrt{2}\left(x^4+3\,x^2+2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-34\,\left(x^4+3\,x^2+2\right)\arctan\left(x\right)+22\,x}{4\,\left(x^4+3\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(24*x^3 + 19*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) - 34*(x^4
 + 3*x^2 + 2)*arctan(x) + 22*x)/(x^4 + 3*x^2 + 2)
```

Sympy [A] time = 0.181602, size = 44, normalized size = 0.92

$$-\frac{12x^3+11x}{2x^4+6x^2+4}+\frac{17\,\mathrm{atan}\left(x\right)}{2}-\frac{19\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] -(12*x**3 + 11*x)/(2*x**4 + 6*x**2 + 4) + 17*atan(x)/2 - 19*sqrt(2)*atan(sq
rt(2)*x/2)/4
```

Giac [A] time = 1.08092, size = 54, normalized size = 1.12

$$-\frac{19}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-\frac{12\,x^3+11\,x}{2\,\left(x^4+3\,x^2+2\right)}+\frac{17}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] -19/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/2*(12*x^3 + 11*x)/(x^4 + 3*x^2 + 2)
 + 17/2*arctan(x)
```

3.87 $\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=53

$$\frac{x(11x^2+9)}{4(x^4+3x^2+2)}-\frac{1}{x}-\frac{19}{2}\tan^{-1}(x)+\frac{45\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] -x^(-1) + (x*(9 + 11*x^2))/(4*(2 + 3*x^2 + x^4)) - (19*ArcTan[x])/2 + (45*A
rcTan[x/Sqrt[2]])/(4*Sqrt[2])
```

Rubi [A] time = 0.0729781, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1669, 1664, 203}

$$\frac{x(11x^2+9)}{4(x^4+3x^2+2)}-\frac{1}{x}-\frac{19}{2}\tan^{-1}(x)+\frac{45\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -x^(-1) + (x*(9 + 11*x^2))/(4*(2 + 3*x^2 + x^4)) - (19*ArcTan[x])/2 + (45*A
rcTan[x/Sqrt[2]])/(4*Sqrt[2])
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx=\frac{x\left(9+11x^2\right)}{4\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \frac{-8+19x^2-11x^4}{x^2\left(2+3x^2+x^4\right)}\,dx$$
$$=\frac{x\left(9+11x^2\right)}{4\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \left(-\frac{4}{x^2}+\frac{38}{1+x^2}-\frac{45}{2+x^2}\right)dx$$
$$=-\frac{1}{x}+\frac{x\left(9+11x^2\right)}{4\left(2+3x^2+x^4\right)}-\frac{19}{2}\int \frac{1}{1+x^2}\,dx+\frac{45}{4}\int \frac{1}{2+x^2}\,dx$$
$$=-\frac{1}{x}+\frac{x\left(9+11x^2\right)}{4\left(2+3x^2+x^4\right)}-\frac{19}{2}\tan^{-1}(x)+\frac{45\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Mathematica [A] time = 0.0493476, size = 51, normalized size = 0.96

$$\frac{1}{8}\left(\frac{2x\left(11x^2+9\right)}{x^4+3x^2+2}-\frac{8}{x}-76\tan^{-1}(x)+45\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-8/x + (2*x*(9 + 11*x^2))/(2 + 3*x^2 + x^4) - 76*ArcTan[x] + 45*Sqrt[2]*Ar
cTan[x/Sqrt[2]])/8
```

Maple [A] time = 0.012, size = 43, normalized size = 0.8

$$\frac{13\,x}{4\,x^2+8}+\frac{45\,\sqrt{2}}{8}\arctan\left(\frac{x\sqrt{2}}{2}\right)-\frac{x}{2\,x^2+2}-\frac{19\,\arctan(x)}{2}-x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x)
```

```
[Out] 13/4*x/(x^2+2)+45/8*arctan(1/2*x*2^(1/2))*2^(1/2)-1/2*x/(x^2+1)-19/2*arctan
(x)-1/x
```

Maxima [A] time = 1.45781, size = 61, normalized size = 1.15

$$\frac{45}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{7\,x^4-3\,x^2-8}{4\left(x^5+3\,x^3+2\,x\right)}-\frac{19}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 45/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(7*x^4 - 3*x^2 - 8)/(x^5 + 3*x^3 +
 2*x) - 19/2*arctan(x)
```

Fricas [A] time = 2.17958, size = 185, normalized size = 3.49

$$\frac{14\,x^4+45\,\sqrt{2}\left(x^5+3\,x^3+2\,x\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-6\,x^2-76\,\left(x^5+3\,x^3+2\,x\right)\arctan\left(x\right)-16}{8\,\left(x^5+3\,x^3+2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(14*x^4 + 45*sqrt(2)*(x^5 + 3*x^3 + 2*x)*arctan(1/2*sqrt(2)*x) - 6*x^2
- 76*(x^5 + 3*x^3 + 2*x)*arctan(x) - 16)/(x^5 + 3*x^3 + 2*x)
```

Sympy [A] time = 0.201047, size = 49, normalized size = 0.92

$$\frac{7x^4-3x^2-8}{4x^5+12x^3+8x}-\frac{19\operatorname{atan}\left(x\right)}{2}+\frac{45\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+3*x**2+2)**2,x)
```

```
[Out] (7*x**4 - 3*x**2 - 8)/(4*x**5 + 12*x**3 + 8*x) - 19*atan(x)/2 + 45*sqrt(2)*
atan(sqrt(2)*x/2)/8
```

Giac [A] time = 1.12253, size = 61, normalized size = 1.15

$$\frac{45}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+\frac{7\,x^4-3\,x^2-8}{4\,\left(x^5+3\,x^3+2\,x\right)}-\frac{19}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 45/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(7*x^4 - 3*x^2 - 8)/(x^5 + 3*x^3 +
 2*x) - 19/2*arctan(x)
```

3.88 $\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=62

$$-\frac{x\left(9x^2+5\right)}{8\left(x^4+3x^2+2\right)}-\frac{1}{3x^3}+\frac{11}{4x}+\frac{21}{2}\tan^{-1}(x)-\frac{71\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] -1/(3*x^3) + 11/(4*x) - (x*(5 + 9*x^2))/(8*(2 + 3*x^2 + x^4)) + (21*ArcTan[
x])/2 - (71*ArcTan[x/Sqrt[2]])/(8*Sqrt[2])
```

Rubi [A] time = 0.0842406, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1669, 1664, 203}

$$-\frac{x\left(9x^2+5\right)}{8\left(x^4+3x^2+2\right)}-\frac{1}{3x^3}+\frac{11}{4x}+\frac{21}{2}\tan^{-1}(x)-\frac{71\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^2),x]
```


```
[Out] -1/(3*x^3) + 11/(4*x) - (x*(5 + 9*x^2))/(8*(2 + 3*x^2 + x^4)) + (21*ArcTan[
x])/2 - (71*ArcTan[x/Sqrt[2]])/(8*Sqrt[2])
```


Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^2}dx = -\frac{x\left(5+9x^2\right)}{8\left(2+3x^2+x^4\right)} - \frac{1}{4}\int \frac{-8+10x^2-\frac{39x^4}{2}+\frac{9x^6}{2}}{x^4\left(2+3x^2+x^4\right)}dx$$

$$= -\frac{x\left(5+9x^2\right)}{8\left(2+3x^2+x^4\right)} - \frac{1}{4}\int \left(-\frac{4}{x^4}+\frac{11}{x^2}-\frac{42}{1+x^2}+\frac{71}{2\left(2+x^2\right)}\right)dx$$

$$= -\frac{1}{3x^3}+\frac{11}{4x}-\frac{x\left(5+9x^2\right)}{8\left(2+3x^2+x^4\right)}-\frac{71}{8}\int \frac{1}{2+x^2}dx+\frac{21}{2}\int \frac{1}{1+x^2}dx$$

$$= -\frac{1}{3x^3}+\frac{11}{4x}-\frac{x\left(5+9x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{21}{2}\tan^{-1}(x)-\frac{71\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Mathematica [A] time = 0.0533273, size = 56, normalized size = 0.9

$$\frac{1}{48}\left(-\frac{6x\left(9x^2+5\right)}{x^4+3x^2+2}-\frac{16}{x^3}+\frac{132}{x}+504\tan^{-1}(x)-213\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-16/x^3 + 132/x - (6*x*(5 + 9*x^2))/(2 + 3*x^2 + x^4) + 504*ArcTan[x] - 21
3*Sqrt[2]*ArcTan[x/Sqrt[2]])/48
```

Maple [A] time = 0.014, size = 48, normalized size = 0.8

$$-\frac{13x}{8x^2+16}-\frac{71\sqrt{2}}{16}\arctan\left(\frac{x\sqrt{2}}{2}\right)+\frac{x}{2x^2+2}+\frac{21\arctan(x)}{2}-\frac{1}{3x^3}+\frac{11}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x)
```

```
[Out] -13/8*x/(x^2+2)-71/16*arctan(1/2*x*2^(1/2))*2^(1/2)+1/2*x/(x^2+1)+21/2*arct
an(x)-1/3/x^3+11/4/x
```

Maxima [A] time = 1.47063, size = 70, normalized size = 1.13

$$-\frac{71}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{39x^6+175x^4+108x^2-16}{24\left(x^7+3x^5+2x^3\right)}+\frac{21}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -71/16*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/24*(39*x^6 + 175*x^4 + 108*x^2 - 1
6)/(x^7 + 3*x^5 + 2*x^3) + 21/2*arctan(x)
```

Fricas [A] time = 1.58719, size = 213, normalized size = 3.44

$$\frac{78\,x^{6} + 350\,x^{4} - 213\,\sqrt{2}\left(x^{7} + 3\,x^{5} + 2\,x^{3}\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 216\,x^{2} + 504\,\left(x^{7} + 3\,x^{5} + 2\,x^{3}\right)\arctan\left(x\right) - 32}{48\,\left(x^{7} + 3\,x^{5} + 2\,x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(78*x^6 + 350*x^4 - 213*sqrt(2)*(x^7 + 3*x^5 + 2*x^3)*arctan(1/2*sqrt(
2)*x) + 216*x^2 + 504*(x^7 + 3*x^5 + 2*x^3)*arctan(x) - 32)/(x^7 + 3*x^5 +
2*x^3)
```

Sympy [A] time = 0.225061, size = 56, normalized size = 0.9

$$\frac{21\,\operatorname{atan}\left(x\right)}{2} - \frac{71\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{16} + \frac{39x^{6} + 175x^{4} + 108x^{2} - 16}{24x^{7} + 72x^{5} + 48x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+3*x**2+2)**2,x)
```

```
[Out] 21*atan(x)/2 - 71*sqrt(2)*atan(sqrt(2)*x/2)/16 + (39*x**6 + 175*x**4 + 108*
x**2 - 16)/(24*x**7 + 72*x**5 + 48*x**3)
```

Giac [A] time = 1.11528, size = 70, normalized size = 1.13

$$-\frac{71}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - \frac{9\,x^{3} + 5\,x}{8\,\left(x^{4} + 3\,x^{2} + 2\right)} + \frac{33\,x^{2} - 4}{12\,x^{3}} + \frac{21}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] -71/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(9*x^3 + 5*x)/(x^4 + 3*x^2 + 2)
+ 1/12*(33*x^2 - 4)/x^3 + 21/2*arctan(x)
```

3.89 $\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=69

$$-\frac{x\left(3-5x^2\right)}{16\left(x^4+3x^2+2\right)}+\frac{11}{12x^3}-\frac{1}{5x^5}-\frac{23}{4x}-\frac{23}{2}\tan^{-1}(x)+\frac{97\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

```
[Out] -1/(5*x^5) + 11/(12*x^3) - 23/(4*x) - (x*(3 - 5*x^2))/(16*(2 + 3*x^2 + x^4)
) - (23*ArcTan[x])/2 + (97*ArcTan[x/Sqrt[2]])/(16*Sqrt[2])
```

Rubi [A] time = 0.0904842, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1669, 1664, 203}

$$-\frac{x\left(3-5x^2\right)}{16\left(x^4+3x^2+2\right)}+\frac{11}{12x^3}-\frac{1}{5x^5}-\frac{23}{4x}-\frac{23}{2}\tan^{-1}(x)+\frac{97\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/(5*x^5) + 11/(12*x^3) - 23/(4*x) - (x*(3 - 5*x^2))/(16*(2 + 3*x^2 + x^4)
) - (23*ArcTan[x])/2 + (97*ArcTan[x/Sqrt[2]])/(16*Sqrt[2])
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx = -\frac{x\left(3-5x^2\right)}{16\left(2+3x^2+x^4\right)} - \frac{1}{4}\int \frac{-8+10x^2-17x^4+\frac{39x^6}{4}-\frac{5x^8}{4}}{x^6\left(2+3x^2+x^4\right)}\,dx$$
$$= -\frac{x\left(3-5x^2\right)}{16\left(2+3x^2+x^4\right)} - \frac{1}{4}\int \left(-\frac{4}{x^6}+\frac{11}{x^4}-\frac{23}{x^2}+\frac{46}{1+x^2}-\frac{97}{4\left(2+x^2\right)}\right)dx$$
$$= -\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{23}{4x}-\frac{x\left(3-5x^2\right)}{16\left(2+3x^2+x^4\right)}+\frac{97}{16}\int \frac{1}{2+x^2}\,dx-\frac{23}{2}\int \frac{1}{1+x^2}\,dx$$
$$= -\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{23}{4x}-\frac{x\left(3-5x^2\right)}{16\left(2+3x^2+x^4\right)}-\frac{23}{2}\tan^{-1}(x)+\frac{97\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

Mathematica [A] time = 0.0595521, size = 61, normalized size = 0.88

$$\frac{1}{480}\left(\frac{30x\left(5x^2-3\right)}{x^4+3x^2+2}+\frac{440}{x^3}-\frac{96}{x^5}-\frac{2760}{x}-5520\tan^{-1}(x)+1455\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-96/x^5 + 440/x^3 - 2760/x + (30*x*(-3 + 5*x^2))/(2 + 3*x^2 + x^4) - 5520*
ArcTan[x] + 1455*Sqrt[2]*ArcTan[x/Sqrt[2]])/480
```

Maple [A] time = 0.016, size = 53, normalized size = 0.8

$$\frac{13\,x}{16\,x^2+32}+\frac{97\,\sqrt{2}}{32}\arctan\left(\frac{x\sqrt{2}}{2}\right)-\frac{x}{2\,x^2+2}-\frac{23\,\arctan(x)}{2}-\frac{1}{5\,x^5}+\frac{11}{12\,x^3}-\frac{23}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x)
```

```
[Out] 13/16*x/(x^2+2)+97/32*arctan(1/2*x*2^(1/2))*2^(1/2)-1/2*x/(x^2+1)-23/2*arct
an(x)-1/5/x^5+11/12/x^3-23/4/x
```

Maxima [A] time = 1.46622, size = 77, normalized size = 1.12

$$\frac{97}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1305\,x^8+3965\,x^6+2148\,x^4-296\,x^2+96}{240\left(x^9+3\,x^7+2\,x^5\right)}-\frac{23}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 97/32*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/240*(1305*x^8 + 3965*x^6 + 2148*x^4
 - 296*x^2 + 96)/(x^9 + 3*x^7 + 2*x^5) - 23/2*arctan(x)
```

Fricas [A] time = 1.53035, size = 239, normalized size = 3.46

$$-\frac{2610\,x^{8}+7930\,x^{6}+4296\,x^{4}-1455\,\sqrt{2}\left(x^{9}+3\,x^{7}+2\,x^{5}\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-592\,x^{2}+5520\,\left(x^{9}+3\,x^{7}+2\,x^{5}\right)\arctan\left(x\right)}{480\,\left(x^{9}+3\,x^{7}+2\,x^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] -1/480*(2610*x^8 + 7930*x^6 + 4296*x^4 - 1455*sqrt(2)*(x^9 + 3*x^7 + 2*x^5)
*arctan(1/2*sqrt(2)*x) - 592*x^2 + 5520*(x^9 + 3*x^7 + 2*x^5)*arctan(x) + 1
92)/(x^9 + 3*x^7 + 2*x^5)
```

Sympy [A] time = 0.24267, size = 61, normalized size = 0.88

$$-\frac{23\operatorname{atan}\left(x\right)}{2}+\frac{97\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{32}-\frac{1305x^{8}+3965x^{6}+2148x^{4}-296x^{2}+96}{240x^{9}+720x^{7}+480x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+3*x**2+2)**2,x)
```

```
[Out] -23*atan(x)/2 + 97*sqrt(2)*atan(sqrt(2)*x/2)/32 - (1305*x**8 + 3965*x**6 +
2148*x**4 - 296*x**2 + 96)/(240*x**9 + 720*x**7 + 480*x**5)
```

Giac [A] time = 1.14244, size = 77, normalized size = 1.12

$$\frac{97}{32}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+\frac{5\,x^{3}-3\,x}{16\,\left(x^{4}+3\,x^{2}+2\right)}-\frac{345\,x^{4}-55\,x^{2}+12}{60\,x^{5}}-\frac{23}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 97/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(5*x^3 - 3*x)/(x^4 + 3*x^2 + 2)
- 1/60*(345*x^4 - 55*x^2 + 12)/x^5 - 23/2*arctan(x)
```

3.90 $\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2} \, dx$

Optimal. Leaf size=76

$$\frac{x\left(3x^2+19\right)}{32\left(x^4+3x^2+2\right)}-\frac{23}{12x^3}+\frac{11}{20x^5}-\frac{1}{7x^7}+\frac{137}{16x}+\frac{25}{2}\tan^{-1}(x)-\frac{123\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

```
[Out] -1/(7*x^7) + 11/(20*x^5) - 23/(12*x^3) + 137/(16*x) + (x*(19 + 3*x^2))/(32*
(2 + 3*x^2 + x^4)) + (25*ArcTan[x])/2 - (123*ArcTan[x/Sqrt[2]])/(32*Sqrt[2]
)
```

Rubi [A] time = 0.0999244, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1669, 1664, 203}

$$\frac{x\left(3x^2+19\right)}{32\left(x^4+3x^2+2\right)}-\frac{23}{12x^3}+\frac{11}{20x^5}-\frac{1}{7x^7}+\frac{137}{16x}+\frac{25}{2}\tan^{-1}(x)-\frac{123\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^8*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/(7*x^7) + 11/(20*x^5) - 23/(12*x^3) + 137/(16*x) + (x*(19 + 3*x^2))/(32*
(2 + 3*x^2 + x^4)) + (25*ArcTan[x])/2 - (123*ArcTan[x/Sqrt[2]])/(32*Sqrt[2]
)
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx = \frac{x\left(19+3x^2\right)}{32\left(2+3x^2+x^4\right)} - \frac{1}{4}\int \frac{-8+10x^2-17x^4+\frac{21x^6}{2}-\frac{39x^8}{8}-\frac{3x^{10}}{8}}{x^8\left(2+3x^2+x^4\right)}\,dx$$
$$= \frac{x\left(19+3x^2\right)}{32\left(2+3x^2+x^4\right)} - \frac{1}{4}\int \left(-\frac{4}{x^8}+\frac{11}{x^6}-\frac{23}{x^4}+\frac{137}{4x^2}-\frac{50}{1+x^2}+\frac{123}{8\left(2+x^2\right)}\right)dx$$
$$= -\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}+\frac{x\left(19+3x^2\right)}{32\left(2+3x^2+x^4\right)}-\frac{123}{32}\int \frac{1}{2+x^2}\,dx+\frac{25}{2}\int \frac{1}{1+x^2}\,dx$$
$$= -\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}+\frac{x\left(19+3x^2\right)}{32\left(2+3x^2+x^4\right)}+\frac{25}{2}\tan^{-1}(x)-\frac{123\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Mathematica [A] time = 0.057174, size = 77, normalized size = 1.01

$$\frac{3x^3+19x}{32\left(x^4+3x^2+2\right)}-\frac{23}{12x^3}+\frac{11}{20x^5}-\frac{1}{7x^7}+\frac{137}{16x}+\frac{25}{2}\tan^{-1}(x)-\frac{123\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^8*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/(7*x^7) + 11/(20*x^5) - 23/(12*x^3) + 137/(16*x) + (19*x + 3*x^3)/(32*(2
 + 3*x^2 + x^4)) + (25*ArcTan[x])/2 - (123*ArcTan[x/Sqrt[2]])/(32*Sqrt[2])
```

Maple [A] time = 0.013, size = 58, normalized size = 0.8

$$-\frac{13\,x}{32\,x^2+64}-\frac{123\,\sqrt{2}}{64}\arctan\left(\frac{x\sqrt{2}}{2}\right)+\frac{x}{2\,x^2+2}+\frac{25\arctan(x)}{2}-\frac{1}{7\,x^7}+\frac{11}{20\,x^5}-\frac{23}{12\,x^3}+\frac{137}{16\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x)
```

```
[Out] -13/32*x/(x^2+2)-123/64*arctan(1/2*x*2^(1/2))*2^(1/2)+1/2*x/(x^2+1)+25/2*ar
ctan(x)-1/7/x^7+11/20/x^5-23/12/x^3+137/16/x
```

Maxima [A] time = 1.53116, size = 84, normalized size = 1.11

$$-\frac{123}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{29085\,x^{10}+81865\,x^8+40068\,x^6-7816\,x^4+2256\,x^2-960}{3360\left(x^{11}+3\,x^9+2\,x^7\right)}+\frac{25}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -123/64*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/3360*(29085*x^10 + 81865*x^8 + 40
068*x^6 - 7816*x^4 + 2256*x^2 - 960)/(x^11 + 3*x^9 + 2*x^7) + 25/2*arctan(x
)
```

Fricas [A] time = 1.56454, size = 271, normalized size = 3.57

$$\frac{58170\,x^{10} + 163730\,x^{8} + 80136\,x^{6} - 15632\,x^{4} - 12915\,\sqrt{2}\left(x^{11} + 3\,x^{9} + 2\,x^{7}\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 4512\,x^{2} + 84000\,\left(x^{11}\right.}{6720\,\left(x^{11} + 3\,x^{9} + 2\,x^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/6720*(58170*x^10 + 163730*x^8 + 80136*x^6 - 15632*x^4 - 12915*sqrt(2)*(x^
11 + 3*x^9 + 2*x^7)*arctan(1/2*sqrt(2)*x) + 4512*x^2 + 84000*(x^11 + 3*x^9
+ 2*x^7)*arctan(x) - 1920)/(x^11 + 3*x^9 + 2*x^7)
```

Sympy [A] time = 0.263308, size = 66, normalized size = 0.87

$$\frac{25\operatorname{atan}(x)}{2} - \frac{123\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{64} + \frac{29085x^{10} + 81865x^{8} + 40068x^{6} - 7816x^{4} + 2256x^{2} - 960}{3360x^{11} + 10080x^{9} + 6720x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**8/(x**4+3*x**2+2)**2,x)
```

```
[Out] 25*atan(x)/2 - 123*sqrt(2)*atan(sqrt(2)*x/2)/64 + (29085*x**10 + 81865*x**8
 + 40068*x**6 - 7816*x**4 + 2256*x**2 - 960)/(3360*x**11 + 10080*x**9 + 672
0*x**7)
```

Giac [A] time = 1.07734, size = 84, normalized size = 1.11

$$-\frac{123}{64}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + \frac{3\,x^{3} + 19\,x}{32\,\left(x^{4} + 3\,x^{2} + 2\right)} + \frac{14385\,x^{6} - 3220\,x^{4} + 924\,x^{2} - 240}{1680\,x^{7}} + \frac{25}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] -123/64*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/32*(3*x^3 + 19*x)/(x^4 + 3*x^2 +
2) + 1/1680*(14385*x^6 - 3220*x^4 + 924*x^2 - 240)/x^7 + 25/2*arctan(x)
```

3.91 $\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

Optimal. Leaf size=81

$$x^5-14x^3+\frac{(1669x^2+824)\,x}{8\,(x^4+3x^2+2)}+\frac{(415x^2+414)\,x}{4\,(x^4+3x^2+2)^2}+214x+\frac{477}{8}\tan^{-1}(x)-351\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 214*x - 14*x^3 + x^5 + (x*(414 + 415*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(82
4 + 1669*x^2))/(8*(2 + 3*x^2 + x^4)) + (477*ArcTan[x])/8 - 351*Sqrt[2]*ArcT
an[x/Sqrt[2]]
```

Rubi [A] time = 0.112492, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1668, 1678, 1676, 1166, 203}

$$x^5-14x^3+\frac{(1669x^2+824)\,x}{8\,(x^4+3x^2+2)}+\frac{(415x^2+414)\,x}{4\,(x^4+3x^2+2)^2}+214x+\frac{477}{8}\tan^{-1}(x)-351\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] 214*x - 14*x^3 + x^5 + (x*(414 + 415*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(82
4 + 1669*x^2))/(8*(2 + 3*x^2 + x^4)) + (477*ArcTan[x])/8 - 351*Sqrt[2]*ArcT
an[x/Sqrt[2]]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{10}\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^3}\,dx = \frac{x\left(414+415x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{1}{8}\int \frac{828-2478x^2-840x^4+424x^6-216x^8+96x^{10}-40x^{12}}{\left(2+3x^2+x^4\right)^2}\,d$$

$$= \frac{x\left(414+415x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(824+1669x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \frac{-4952-2700x^2+3136x^4-864x^6}{2+3x^2+x^4}$$

$$= \frac{x\left(414+415x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(824+1669x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \left(6848-1344x^2+160x^4-\frac{36\left(518+\right.}{2+3x}\right.$$

$$= 214x-14x^3+x^5+\frac{x\left(414+415x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(824+1669x^2\right)}{8\left(2+3x^2+x^4\right)}-\frac{9}{8}\int \frac{518+571x^2}{2+3x^2+x^4}\,dx$$

$$= 214x-14x^3+x^5+\frac{x\left(414+415x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(824+1669x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{477}{8}\int \frac{1}{1+x^2}\,dx-70$$

$$= 214x-14x^3+x^5+\frac{x\left(414+415x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(824+1669x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{477}{8}\tan^{-1}(x)-351\sqrt{2}$$

Mathematica [A] time = 0.0603729, size = 71, normalized size = 0.88

$$\frac{x\left(8x^{12}-64x^{10}+1144x^8+10581x^6+26775x^4+26736x^2+9324\right)}{8\left(x^4+3x^2+2\right)^2}+\frac{477}{8}\tan^{-1}(x)-351\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] (x*(9324 + 26736*x^2 + 26775*x^4 + 10581*x^6 + 1144*x^8 - 64*x^10 + 8*x^12)
)/(8*(2 + 3*x^2 + x^4)^2) + (477*ArcTan[x])/8 - 351*Sqrt[2]*ArcTan[x/Sqrt[2
]]
```

Maple [A] time = 0.013, size = 64, normalized size = 0.8

$$x^5 - 14x^3 + 214x - 16\frac{1}{(x^2+2)^2}\left(-\frac{105x^3}{8} - \frac{79x}{4}\right) - 351\arctan\left(1/2x\sqrt{2}\right)\sqrt{2} + \frac{1}{(x^2+1)^2}\left(-\frac{11x^3}{8} - \frac{13x}{8}\right) + \frac{477\,\mathrm{arct}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

```
[Out] x^5-14*x^3+214*x-16*(-105/8*x^3-79/4*x)/(x^2+2)^2-351*arctan(1/2*x*2^(1/2))
*2^(1/2)+(-11/8*x^3-13/8*x)/(x^2+1)^2+477/8*arctan(x)
```

Maxima [A] time = 1.49385, size = 96, normalized size = 1.19

$$x^5 - 14x^3 - 351\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 214x + \frac{1669x^7 + 5831x^5 + 6640x^3 + 2476x}{8\left(x^8 + 6x^6 + 13x^4 + 12x^2 + 4\right)} + \frac{477}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] x^5 - 14*x^3 - 351*sqrt(2)*arctan(1/2*sqrt(2)*x) + 214*x + 1/8*(1669*x^7 +
5831*x^5 + 6640*x^3 + 2476*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 477/8*a
rctan(x)
```

Fricas [A] time = 1.58518, size = 325, normalized size = 4.01

$$\frac{8x^{13} - 64x^{11} + 1144x^9 + 10581x^7 + 26775x^5 + 26736x^3 - 2808\sqrt{2}\left(x^8 + 6x^6 + 13x^4 + 12x^2 + 4\right)\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 4}{8\left(x^8 + 6x^6 + 13x^4 + 12x^2 + 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*x^13 - 64*x^11 + 1144*x^9 + 10581*x^7 + 26775*x^5 + 26736*x^3 - 2808
*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) + 477*(x
^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(x) + 9324*x)/(x^8 + 6*x^6 + 13*x^4
 + 12*x^2 + 4)
```

Sympy [A] time = 0.240526, size = 75, normalized size = 0.93

$$x^5 - 14x^3 + 214x + \frac{1669x^7 + 5831x^5 + 6640x^3 + 2476x}{8x^8 + 48x^6 + 104x^4 + 96x^2 + 32} + \frac{477\,\mathrm{atan}(x)}{8} - 351\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] x**5 - 14*x**3 + 214*x + (1669*x**7 + 5831*x**5 + 6640*x**3 + 2476*x)/(8*x*
*8 + 48*x**6 + 104*x**4 + 96*x**2 + 32) + 477*atan(x)/8 - 351*sqrt(2)*atan(
```

```
sqrt(2)*x/2)
```

Giac [A] time = 1.09603, size = 82, normalized size = 1.01

$$x^5 - 14x^3 - 351\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 214x + \frac{1669x^7 + 5831x^5 + 6640x^3 + 2476x}{8\left(x^4 + 3x^2 + 2\right)^2} + \frac{477}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] x^5 - 14*x^3 - 351*sqrt(2)*arctan(1/2*sqrt(2)*x) + 214*x + 1/8*(1669*x^7 +
5831*x^5 + 6640*x^3 + 2476*x)/(x^4 + 3*x^2 + 2)^2 + 477/8*arctan(x)
```

3.92 $\int \frac{x^8\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^3} dx$

Optimal. Leaf size=80

$$\frac{5x^3}{3}+\frac{\left(24-409x^2\right)x}{8\left(x^4+3x^2+2\right)}-\frac{\left(207x^2+206\right)x}{4\left(x^4+3x^2+2\right)^2}-42x-\frac{449}{8}\tan^{-1}(x)+\frac{219\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -42*x + (5*x^3)/3 - (x*(206 + 207*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(24 -
409*x^2))/(8*(2 + 3*x^2 + x^4)) - (449*ArcTan[x])/8 + (219*ArcTan[x/Sqrt[2]
])/Sqrt[2]
```

Rubi [A] time = 0.100481, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1668, 1678, 1676, 1166, 203}

$$\frac{5x^3}{3}+\frac{\left(24-409x^2\right)x}{8\left(x^4+3x^2+2\right)}-\frac{\left(207x^2+206\right)x}{4\left(x^4+3x^2+2\right)^2}-42x-\frac{449}{8}\tan^{-1}(x)+\frac{219\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] -42*x + (5*x^3)/3 - (x*(206 + 207*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(24 -
409*x^2))/(8*(2 + 3*x^2 + x^4)) - (449*ArcTan[x])/8 + (219*ArcTan[x/Sqrt[2]
])/Sqrt[2]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^3}\,dx &= -\frac{x\left(206+207x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{1}{8}\int \frac{-412+1230x^2+424x^4-216x^6+96x^8-40x^{10}}{\left(2+3x^2+x^4\right)^2}\,dx \\
&= -\frac{x\left(206+207x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(24-409x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \frac{728+1500x^2-864x^4+160x^6}{2+3x^2+x^4}\,dx \\
&= -\frac{x\left(206+207x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(24-409x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \left(-1344+160x^2+\frac{4\left(854+1303x^2\right)}{2+3x^2+x^4}\right. \\
&= -42x+\frac{5x^3}{3}-\frac{x\left(206+207x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(24-409x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{1}{8}\int \frac{854+1303x^2}{2+3x^2+x^4}\,dx \\
&= -42x+\frac{5x^3}{3}-\frac{x\left(206+207x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(24-409x^2\right)}{8\left(2+3x^2+x^4\right)}-\frac{449}{8}\int \frac{1}{1+x^2}\,dx+219\int \frac{1}{2+\cdots} \\
&= -42x+\frac{5x^3}{3}-\frac{x\left(206+207x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(24-409x^2\right)}{8\left(2+3x^2+x^4\right)}-\frac{449}{8}\tan^{-1}(x)+\frac{219\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.0552934, size = 66, normalized size = 0.82

$$\frac{x\left(40x^{10}-768x^8-6755x^6-16233x^4-15416x^2-5124\right)}{24\left(x^4+3x^2+2\right)^2}-\frac{449}{8}\tan^{-1}(x)+\frac{219\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]

[Out] (x*(-5124 - 15416*x^2 - 16233*x^4 - 6755*x^6 - 768*x^8 + 40*x^10))/(24*(2 +
 3*x^2 + x^4)^2) - (449*ArcTan[x])/8 + (219*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Maple [A] time = 0.011, size = 62, normalized size = 0.8

$$\frac{5x^3}{3}-42x+16\frac{1}{\left(x^2+2\right)^2}\left(-\frac{53x^3}{16}-\frac{27x}{8}\right)+\frac{219\sqrt{2}}{2}\arctan\left(\frac{x\sqrt{2}}{2}\right)-\frac{1}{\left(x^2+1\right)^2}\left(-\frac{15x^3}{8}-\frac{17x}{8}\right)-\frac{449\arctan(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

```
[Out] 5/3*x^3-42*x+16*(-53/16*x^3-27/8*x)/(x^2+2)^2+219/2*arctan(1/2*x*2^(1/2))*2
^(1/2)-(-15/8*x^3-17/8*x)/(x^2+1)^2-449/8*arctan(x)
```

Maxima [A] time = 1.49531, size = 92, normalized size = 1.15

$$\frac{5}{3}x^3+\frac{219}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-42x-\frac{409x^7+1203x^5+1160x^3+364x}{8\left(x^8+6x^6+13x^4+12x^2+4\right)}-\frac{449}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 + 219/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 42*x - 1/8*(409*x^7 + 1203*
x^5 + 1160*x^3 + 364*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 449/8*arctan(
x)
```

Fricas [A] time = 1.53826, size = 313, normalized size = 3.91

$$\frac{40x^{11}-768x^9-6755x^7-16233x^5-15416x^3+2628\sqrt{2}\left(x^8+6x^6+13x^4+12x^2+4\right)\arctan\left(\frac{1}{2}\sqrt{2}x\right)-1347\left(x^8+6\right.}{24\left(x^8+6x^6+13x^4+12x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(40*x^11 - 768*x^9 - 6755*x^7 - 16233*x^5 - 15416*x^3 + 2628*sqrt(2)*(
x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 1347*(x^8 + 6*x^
6 + 13*x^4 + 12*x^2 + 4)*arctan(x) - 5124*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2
 + 4)
```

Sympy [A] time = 0.238198, size = 75, normalized size = 0.94

$$\frac{5x^3}{3}-42x-\frac{409x^7+1203x^5+1160x^3+364x}{8x^8+48x^6+104x^4+96x^2+32}-\frac{449\operatorname{atan}(x)}{8}+\frac{219\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] 5*x**3/3 - 42*x - (409*x**7 + 1203*x**5 + 1160*x**3 + 364*x)/(8*x**8 + 48*x
**6 + 104*x**4 + 96*x**2 + 32) - 449*atan(x)/8 + 219*sqrt(2)*atan(sqrt(2)*x
/2)/2
```

Giac [A] time = 1.11295, size = 78, normalized size = 0.98

$$\frac{5}{3}x^3 + \frac{219}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 42x - \frac{409x^7 + 1203x^5 + 1160x^3 + 364x}{8\left(x^4 + 3x^2 + 2\right)^2} - \frac{449}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] 5/3*x^3 + 219/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 42*x - 1/8*(409*x^7 + 1203*
x^5 + 1160*x^3 + 364*x)/(x^4 + 3*x^2 + 2)^2 - 449/8*arctan(x)
```

3.93 $\int \frac{x^6\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=75

$$-\frac{\left(15x^2+244\right)x}{8\left(x^4+3x^2+2\right)}+\frac{\left(103x^2+102\right)x}{4\left(x^4+3x^2+2\right)^2}+5x+\frac{413}{8}\tan^{-1}(x)-\frac{191\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] 5*x + (x*(102 + 103*x^2))/(4*(2 + 3*x^2 + x^4)^2) - (x*(244 + 15*x^2))/(8*(
2 + 3*x^2 + x^4)) + (413*ArcTan[x])/8 - (191*ArcTan[x/Sqrt[2]])/(2*Sqrt[2])
```

Rubi [A] time = 0.0913924, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1668, 1678, 1676, 1166, 203}

$$-\frac{\left(15x^2+244\right)x}{8\left(x^4+3x^2+2\right)}+\frac{\left(103x^2+102\right)x}{4\left(x^4+3x^2+2\right)^2}+5x+\frac{413}{8}\tan^{-1}(x)-\frac{191\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```


```
[Out] 5*x + (x*(102 + 103*x^2))/(4*(2 + 3*x^2 + x^4)^2) - (x*(244 + 15*x^2))/(8*(
2 + 3*x^2 + x^4)) + (413*ArcTan[x])/8 - (191*ArcTan[x/Sqrt[2]])/(2*Sqrt[2])
```


Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
```

```
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^3}\,dx &= \frac{x\left(102+103x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{1}{8}\int \frac{204-606x^2-216x^4+96x^6-40x^8}{\left(2+3x^2+x^4\right)^2}\,dx\\
&= \frac{x\left(102+103x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{x\left(244+15x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \frac{568-924x^2+160x^4}{2+3x^2+x^4}\,dx\\
&= \frac{x\left(102+103x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{x\left(244+15x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \left(160+\frac{4\left(62-351x^2\right)}{2+3x^2+x^4}\right)dx\\
&= 5x+\frac{x\left(102+103x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{x\left(244+15x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{1}{8}\int \frac{62-351x^2}{2+3x^2+x^4}\,dx\\
&= 5x+\frac{x\left(102+103x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{x\left(244+15x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{413}{8}\int \frac{1}{1+x^2}\,dx-\frac{191}{2}\int \frac{1}{2+x^2}\,dx\\
&= 5x+\frac{x\left(102+103x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{x\left(244+15x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{413}{8}\tan^{-1}(x)-\frac{191\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.061187, size = 60, normalized size = 0.8

$$\frac{1}{8}\left(\frac{x\left(40x^8+225x^6+231x^4-76x^2-124\right)}{\left(x^4+3x^2+2\right)^2}+413\tan^{-1}(x)-382\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] ((x*(-124 - 76*x^2 + 231*x^4 + 225*x^6 + 40*x^8))/(2 + 3*x^2 + x^4)^2 + 413
*ArcTan[x] - 382*Sqrt[2]*ArcTan[x/Sqrt[2]])/8
```

Maple [A] time = 0.011, size = 56, normalized size = 0.8

$$5\,x-16\,\frac{1}{\left(x^2+2\right)^2}\left(-1/32\,x^3+\frac{25\,x}{16}\right)-\frac{191\,\sqrt{2}}{4}\arctan\left(\frac{x\sqrt{2}}{2}\right)+\frac{1}{\left(x^2+1\right)^2}\left(-\frac{19\,x^3}{8}-\frac{21\,x}{8}\right)+\frac{413\,\arctan(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

```
[Out] 5*x-16*(-1/32*x^3+25/16*x)/(x^2+2)^2-191/4*arctan(1/2*x*2^(1/2))*2^(1/2)+(-
19/8*x^3-21/8*x)/(x^2+1)^2+413/8*arctan(x)
```

Maxima [A] time = 1.4955, size = 85, normalized size = 1.13

$$-\frac{191}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+5\,x-\frac{15\,x^{7}+289\,x^{5}+556\,x^{3}+284\,x}{8\left(x^{8}+6\,x^{6}+13\,x^{4}+12\,x^{2}+4\right)}+\frac{413}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] -191/4*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x - 1/8*(15*x^7 + 289*x^5 + 556*x^
3 + 284*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 413/8*arctan(x)
```

Fricas [A] time = 1.62588, size = 285, normalized size = 3.8

$$\frac{40\,x^{9}+225\,x^{7}+231\,x^{5}-76\,x^{3}-382\,\sqrt{2}\left(x^{8}+6\,x^{6}+13\,x^{4}+12\,x^{2}+4\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+413\,\left(x^{8}+6\,x^{6}+13\,x^{4}+12\,x^{2}}{8\left(x^{8}+6\,x^{6}+13\,x^{4}+12\,x^{2}+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(40*x^9 + 225*x^7 + 231*x^5 - 76*x^3 - 382*sqrt(2)*(x^8 + 6*x^6 + 13*x^
4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) + 413*(x^8 + 6*x^6 + 13*x^4 + 12*x^2
+ 4)*arctan(x) - 124*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A] time = 0.240573, size = 68, normalized size = 0.91

$$5x-\frac{15x^{7}+289x^{5}+556x^{3}+284x}{8x^{8}+48x^{6}+104x^{4}+96x^{2}+32}+\frac{413\operatorname{atan}\left(x\right)}{8}-\frac{191\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] 5*x - (15*x**7 + 289*x**5 + 556*x**3 + 284*x)/(8*x**8 + 48*x**6 + 104*x**4
+ 96*x**2 + 32) + 413*atan(x)/8 - 191*sqrt(2)*atan(sqrt(2)*x/2)/4
```

Giac [A] time = 1.08504, size = 72, normalized size = 0.96

$$-\frac{191}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+5\,x-\frac{15\,x^{7}+289\,x^{5}+556\,x^{3}+284\,x}{8\left(x^{4}+3\,x^{2}+2\right)^{2}}+\frac{413}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] -191/4*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x - 1/8*(15*x^7 + 289*x^5 + 556*x^
3 + 284*x)/(x^4 + 3*x^2 + 2)^2 + 413/8*arctan(x)
```

3.94 $\int \frac{x^4\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=72

$$-\frac{x\left(51x^2+50\right)}{4\left(x^4+3x^2+2\right)^2}+\frac{x\left(125x^2+254\right)}{8\left(x^4+3x^2+2\right)}-\frac{369}{8}\tan^{-1}(x)+\frac{267\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] -(x*(50 + 51*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(254 + 125*x^2))/(8*(2 + 3*
x^2 + x^4)) - (369*ArcTan[x])/8 + (267*ArcTan[x/Sqrt[2]])/(4*Sqrt[2])
```

Rubi [A] time = 0.0678667, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1668, 1678, 1166, 203}

$$-\frac{x\left(51x^2+50\right)}{4\left(x^4+3x^2+2\right)^2}+\frac{x\left(125x^2+254\right)}{8\left(x^4+3x^2+2\right)}-\frac{369}{8}\tan^{-1}(x)+\frac{267\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] -(x*(50 + 51*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(254 + 125*x^2))/(8*(2 + 3*
x^2 + x^4)) - (369*ArcTan[x])/8 + (267*ArcTan[x/Sqrt[2]])/(4*Sqrt[2])
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^3}\,dx &= -\frac{x\left(50+51x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{1}{8}\int \frac{-100+294x^2+96x^4-40x^6}{\left(2+3x^2+x^4\right)^2}\,dx\\ &= -\frac{x\left(50+51x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(254+125x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \frac{-816+660x^2}{2+3x^2+x^4}\,dx\\ &= -\frac{x\left(50+51x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(254+125x^2\right)}{8\left(2+3x^2+x^4\right)}-\frac{369}{8}\int \frac{1}{1+x^2}\,dx+\frac{267}{4}\int \frac{1}{2+x^2}\,dx\\ &= -\frac{x\left(50+51x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(254+125x^2\right)}{8\left(2+3x^2+x^4\right)}-\frac{369}{8}\tan^{-1}(x)+\frac{267\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0605023, size = 55, normalized size = 0.76

$$\frac{1}{8}\left(\frac{x\left(125x^6+629x^4+910x^2+408\right)}{\left(x^4+3x^2+2\right)^2}-369\tan^{-1}(x)+267\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] ((x*(408 + 910*x^2 + 629*x^4 + 125*x^6))/(2 + 3*x^2 + x^4)^2 - 369*ArcTan[x
] + 267*Sqrt[2]*ArcTan[x/Sqrt[2]])/8
```

Maple [A] time = 0.012, size = 54, normalized size = 0.8

$$2\,\frac{1}{\left(x^2+2\right)^2}\left(\frac{51\,x^3}{8}+\frac{77\,x}{4}\right)+\frac{267\,\sqrt{2}}{8}\arctan\left(\frac{x\sqrt{2}}{2}\right)-\frac{1}{\left(x^2+1\right)^2}\left(-\frac{23\,x^3}{8}-\frac{25\,x}{8}\right)-\frac{369\,\arctan(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

```
[Out] 2*(51/8*x^3+77/4*x)/(x^2+2)^2+267/8*arctan(1/2*x*2^(1/2))*2^(1/2)-(-23/8*x^
3-25/8*x)/(x^2+1)^2-369/8*arctan(x)
```

Maxima [A] time = 1.49012, size = 81, normalized size = 1.12

$$\frac{267}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{125\,x^7+629\,x^5+910\,x^3+408\,x}{8\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)}-\frac{369}{8}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] 267/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 40
8*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 369/8*arctan(x)
```

Fricas [A] time = 1.64032, size = 274, normalized size = 3.81

$$\frac{125\,x^7+629\,x^5+910\,x^3+267\,\sqrt{2}\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)\arctan\left(\frac{1}{2}\sqrt{2}x\right)-369\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)\mathrm{a}}{8\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 267*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x
^2 + 4)*arctan(1/2*sqrt(2)*x) - 369*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arc
tan(x) + 408*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A] time = 0.236653, size = 65, normalized size = 0.9

$$\frac{125x^7+629x^5+910x^3+408x}{8x^8+48x^6+104x^4+96x^2+32}-\frac{369\,\mathrm{atan}\,(x)}{8}+\frac{267\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] (125*x**7 + 629*x**5 + 910*x**3 + 408*x)/(8*x**8 + 48*x**6 + 104*x**4 + 96*
x**2 + 32) - 369*atan(x)/8 + 267*sqrt(2)*atan(sqrt(2)*x/2)/8
```

Giac [A] time = 1.10008, size = 68, normalized size = 0.94

$$\frac{267}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{125\,x^7+629\,x^5+910\,x^3+408\,x}{8\left(x^4+3\,x^2+2\right)^2}-\frac{369}{8}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] 267/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 40
8*x)/(x^4 + 3*x^2 + 2)^2 - 369/8*arctan(x)
```

3.95 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

Optimal. Leaf size=72

$$\frac{x(25x^2+24)}{4(x^4+3x^2+2)^2}-\frac{x(130x^2+211)}{8(x^4+3x^2+2)}+\frac{317}{8}\tan^{-1}(x)-\frac{447\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] (x*(24 + 25*x^2))/(4*(2 + 3*x^2 + x^4)^2) - (x*(211 + 130*x^2))/(8*(2 + 3*x
^2 + x^4)) + (317*ArcTan[x])/8 - (447*ArcTan[x/Sqrt[2]])/(8*Sqrt[2])
```

Rubi [A] time = 0.0655951, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1668, 1678, 1166, 203}

$$\frac{x(25x^2+24)}{4(x^4+3x^2+2)^2}-\frac{x(130x^2+211)}{8(x^4+3x^2+2)}+\frac{317}{8}\tan^{-1}(x)-\frac{447\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] (x*(24 + 25*x^2))/(4*(2 + 3*x^2 + x^4)^2) - (x*(211 + 130*x^2))/(8*(2 + 3*x
^2 + x^4)) + (317*ArcTan[x])/8 - (447*ArcTan[x/Sqrt[2]])/(8*Sqrt[2])
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^3}\,dx &= \frac{x\left(24+25x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{1}{8}\int \frac{48-154x^2-40x^4}{\left(2+3x^2+x^4\right)^2}\,dx\\ &= \frac{x\left(24+25x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{x\left(211+130x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \frac{748-520x^2}{2+3x^2+x^4}\,dx\\ &= \frac{x\left(24+25x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{x\left(211+130x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{317}{8}\int \frac{1}{1+x^2}\,dx-\frac{447}{8}\int \frac{1}{2+x^2}\,dx\\ &= \frac{x\left(24+25x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{x\left(211+130x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{317}{8}\tan^{-1}(x)-\frac{447\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0627458, size = 56, normalized size = 0.78

$$\frac{1}{16}\left(-\frac{2x\left(130x^6+601x^4+843x^2+374\right)}{\left(x^4+3x^2+2\right)^2}+634\tan^{-1}(x)-447\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] ((-2*x*(374 + 843*x^2 + 601*x^4 + 130*x^6))/(2 + 3*x^2 + x^4)^2 + 634*ArcTa
n[x] - 447*Sqrt[2]*ArcTan[x/Sqrt[2]])/16
```

Maple [A] time = 0.012, size = 53, normalized size = 0.7

$$-\frac{1}{\left(x^2+2\right)^2}\left(\frac{103\,x^3}{8}+\frac{129\,x}{4}\right)-\frac{447\,\sqrt{2}}{16}\arctan\left(\frac{x\sqrt{2}}{2}\right)+\frac{1}{\left(x^2+1\right)^2}\left(-\frac{27\,x^3}{8}-\frac{29\,x}{8}\right)+\frac{317\arctan(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

```
[Out] -(103/8*x^3+129/4*x)/(x^2+2)^2-447/16*arctan(1/2*x*2^(1/2))*2^(1/2)+(-27/8*
x^3-29/8*x)/(x^2+1)^2+317/8*arctan(x)
```

Maxima [A] time = 1.49742, size = 81, normalized size = 1.12

$$-\frac{447}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{130\,x^7+601\,x^5+843\,x^3+374\,x}{8\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)}+\frac{317}{8}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] -447/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(130*x^7 + 601*x^5 + 843*x^3 +
374*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 317/8*arctan(x)
```

Fricas [A] time = 1.52068, size = 279, normalized size = 3.88

$$-\frac{260\,x^7+1202\,x^5+1686\,x^3+447\,\sqrt{2}\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)\arctan\left(\frac{1}{2}\sqrt{2}x\right)-634\left(x^8+6\,x^6+13\,x^4+12\,x^2\right.}{16\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(260*x^7 + 1202*x^5 + 1686*x^3 + 447*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 +
12*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 634*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
*arctan(x) + 748*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A] time = 0.236459, size = 65, normalized size = 0.9

$$-\frac{130x^7+601x^5+843x^3+374x}{8x^8+48x^6+104x^4+96x^2+32}+\frac{317\,\mathrm{atan}\,(x)}{8}-\frac{447\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] -(130*x**7 + 601*x**5 + 843*x**3 + 374*x)/(8*x**8 + 48*x**6 + 104*x**4 + 96
*x**2 + 32) + 317*atan(x)/8 - 447*sqrt(2)*atan(sqrt(2)*x/2)/16
```

Giac [A] time = 1.10467, size = 68, normalized size = 0.94

$$-\frac{447}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{130\,x^7+601\,x^5+843\,x^3+374\,x}{8\left(x^4+3\,x^2+2\right)^2}+\frac{317}{8}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] -447/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(130*x^7 + 601*x^5 + 843*x^3 +
374*x)/(x^4 + 3*x^2 + 2)^2 + 317/8*arctan(x)
```

3.96 $\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^3} \, dx$

Optimal. Leaf size=72

$$-\frac{x\left(12x^2+11\right)}{4\left(x^4+3x^2+2\right)^2}+\frac{x\left(217x^2+335\right)}{16\left(x^4+3x^2+2\right)}-\frac{257}{8}\tan^{-1}(x)+\frac{731\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

```
[Out] -(x*(11 + 12*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(335 + 217*x^2))/(16*(2 + 3
*x^2 + x^4)) - (257*ArcTan[x])/8 + (731*ArcTan[x/Sqrt[2]])/(16*Sqrt[2])
```

Rubi [A] time = 0.037454, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1678, 1178, 1166, 203}

$$-\frac{x\left(12x^2+11\right)}{4\left(x^4+3x^2+2\right)^2}+\frac{x\left(217x^2+335\right)}{16\left(x^4+3x^2+2\right)}-\frac{257}{8}\tan^{-1}(x)+\frac{731\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] -(x*(11 + 12*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(335 + 217*x^2))/(16*(2 + 3
*x^2 + x^4)) - (257*ArcTan[x])/8 + (731*ArcTan[x/Sqrt[2]])/(16*Sqrt[2])
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^3}\,dx &= -\frac{x\left(11+12x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{1}{8}\int \frac{-38+80x^2}{\left(2+3x^2+x^4\right)^2}\,dx\\
&= -\frac{x\left(11+12x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(335+217x^2\right)}{16\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \frac{-594+434x^2}{2+3x^2+x^4}\,dx\\
&= -\frac{x\left(11+12x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(335+217x^2\right)}{16\left(2+3x^2+x^4\right)}-\frac{257}{8}\int \frac{1}{1+x^2}\,dx+\frac{731}{16}\int \frac{1}{2+x^2}\,dx\\
&= -\frac{x\left(11+12x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(335+217x^2\right)}{16\left(2+3x^2+x^4\right)}-\frac{257}{8}\tan^{-1}(x)+\frac{731\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0595557, size = 56, normalized size = 0.78

$$\frac{1}{32}\left(\frac{2x\left(217x^6+986x^4+1391x^2+626\right)}{\left(x^4+3x^2+2\right)^2}-1028\tan^{-1}(x)+731\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] ((2*x*(626 + 1391*x^2 + 986*x^4 + 217*x^6))/(2 + 3*x^2 + x^4)^2 - 1028*ArcT
an[x] + 731*Sqrt[2]*ArcTan[x/Sqrt[2]])/32
```

Maple [A] time = 0.013, size = 53, normalized size = 0.7

$$\frac{1}{\left(x^2+2\right)^2}\left(\frac{155\,x^3}{16}+\frac{181\,x}{8}\right)+\frac{731\,\sqrt{2}}{32}\arctan\left(\frac{x\sqrt{2}}{2}\right)-\frac{1}{\left(x^2+1\right)^2}\left(-\frac{31\,x^3}{8}-\frac{33\,x}{8}\right)-\frac{257\arctan(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

```
[Out] (155/16*x^3+181/8*x)/(x^2+2)^2+731/32*arctan(1/2*x*2^(1/2))*2^(1/2)-(-31/8*
x^3-33/8*x)/(x^2+1)^2-257/8*arctan(x)
```

Maxima [A] time = 1.51475, size = 81, normalized size = 1.12

$$\frac{731}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{217\,x^7+986\,x^5+1391\,x^3+626\,x}{16\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)}-\frac{257}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] 731/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(217*x^7 + 986*x^5 + 1391*x^3 +
 626*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 257/8*arctan(x)
```

Fricas [A] time = 1.57343, size = 281, normalized size = 3.9

$$\frac{434\,x^7 + 1972\,x^5 + 2782\,x^3 + 731\,\sqrt{2}\left(x^8 + 6\,x^6 + 13\,x^4 + 12\,x^2 + 4\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 1028\,\left(x^8 + 6\,x^6 + 13\,x^4 + 12\,x^2 + 4\right.}{32\,\left(x^8 + 6\,x^6 + 13\,x^4 + 12\,x^2 + 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(434*x^7 + 1972*x^5 + 2782*x^3 + 731*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 1
2*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 1028*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
*arctan(x) + 1252*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A] time = 0.231106, size = 65, normalized size = 0.9

$$\frac{217x^7 + 986x^5 + 1391x^3 + 626x}{16x^8 + 96x^6 + 208x^4 + 192x^2 + 64} - \frac{257\,\mathrm{atan}\,(x)}{8} + \frac{731\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] (217*x**7 + 986*x**5 + 1391*x**3 + 626*x)/(16*x**8 + 96*x**6 + 208*x**4 + 1
92*x**2 + 64) - 257*atan(x)/8 + 731*sqrt(2)*atan(sqrt(2)*x/2)/32
```

Giac [A] time = 1.1064, size = 68, normalized size = 0.94

$$\frac{731}{32}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + \frac{217\,x^7 + 986\,x^5 + 1391\,x^3 + 626\,x}{16\,\left(x^4 + 3\,x^2 + 2\right)^2} - \frac{257}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] 731/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(217*x^7 + 986*x^5 + 1391*x^3 +
 626*x)/(x^4 + 3*x^2 + 2)^2 - 257/8*arctan(x)
```

3.97 $\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3} \, dx$

Optimal. Leaf size=79

$$\frac{x\left(11x^2+9\right)}{8\left(x^4+3x^2+2\right)^2}-\frac{x\left(347x^2+547\right)}{32\left(x^4+3x^2+2\right)}-\frac{1}{2x}+\frac{189}{8}\tan^{-1}(x)-\frac{1119\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

```
[Out] -1/(2*x) + (x*(9 + 11*x^2))/(8*(2 + 3*x^2 + x^4)^2) - (x*(547 + 347*x^2))/(
32*(2 + 3*x^2 + x^4)) + (189*ArcTan[x])/8 - (1119*ArcTan[x/Sqrt[2]])/(32*Sq
rt[2])
```

Rubi [A] time = 0.103181, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1669, 1664, 203}

$$\frac{x\left(11x^2+9\right)}{8\left(x^4+3x^2+2\right)^2}-\frac{x\left(347x^2+547\right)}{32\left(x^4+3x^2+2\right)}-\frac{1}{2x}+\frac{189}{8}\tan^{-1}(x)-\frac{1119\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] -1/(2*x) + (x*(9 + 11*x^2))/(8*(2 + 3*x^2 + x^4)^2) - (x*(547 + 347*x^2))/(
32*(2 + 3*x^2 + x^4)) + (189*ArcTan[x])/8 - (1119*ArcTan[x/Sqrt[2]])/(32*Sq
rt[2])
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx &= \frac{x\left(9+11x^2\right)}{8\left(2+3x^2+x^4\right)^2}-\frac{1}{8}\int \frac{-16+29x^2-55x^4}{x^2\left(2+3x^2+x^4\right)^2}\,dx\\ &= \frac{x\left(9+11x^2\right)}{8\left(2+3x^2+x^4\right)^2}-\frac{x\left(547+347x^2\right)}{32\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \frac{32+441x^2-347x^4}{x^2\left(2+3x^2+x^4\right)}\,dx\\ &= \frac{x\left(9+11x^2\right)}{8\left(2+3x^2+x^4\right)^2}-\frac{x\left(547+347x^2\right)}{32\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \left(\frac{16}{x^2}+\frac{756}{1+x^2}-\frac{1119}{2+x^2}\right)dx\\ &= -\frac{1}{2x}+\frac{x\left(9+11x^2\right)}{8\left(2+3x^2+x^4\right)^2}-\frac{x\left(547+347x^2\right)}{32\left(2+3x^2+x^4\right)}+\frac{189}{8}\int \frac{1}{1+x^2}\,dx-\frac{1119}{32}\int \frac{1}{2+x^2}\,dx\\ &= -\frac{1}{2x}+\frac{x\left(9+11x^2\right)}{8\left(2+3x^2+x^4\right)^2}-\frac{x\left(547+347x^2\right)}{32\left(2+3x^2+x^4\right)}+\frac{189}{8}\tan^{-1}(x)-\frac{1119\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0678454, size = 63, normalized size = 0.8

$$\frac{1}{64}\left(-\frac{2\left(363x^8+1684x^6+2499x^4+1250x^2+64\right)}{x\left(x^4+3x^2+2\right)^2}+1512\tan^{-1}(x)-1119\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] ((-2*(64 + 1250*x^2 + 2499*x^4 + 1684*x^6 + 363*x^8))/(x*(2 + 3*x^2 + x^4)^
2) + 1512*ArcTan[x] - 1119*Sqrt[2]*ArcTan[x/Sqrt[2]])/64
```

Maple [A] time = 0.014, size = 58, normalized size = 0.7

$$-\frac{1}{2\left(x^2+2\right)^2}\left(\frac{207\,x^3}{16}+\frac{233\,x}{8}\right)-\frac{1119\sqrt{2}}{64}\arctan\left(\frac{x\sqrt{2}}{2}\right)+\frac{1}{\left(x^2+1\right)^2}\left(-\frac{35\,x^3}{8}-\frac{37\,x}{8}\right)+\frac{189\arctan(x)}{8}-\frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x)
```

```
[Out] -1/2*(207/16*x^3+233/8*x)/(x^2+2)^2-1119/64*arctan(1/2*x*2^(1/2))*2^(1/2)+(
-35/8*x^3-37/8*x)/(x^2+1)^2+189/8*arctan(x)-1/2/x
```

Maxima [A] time = 1.48621, size = 88, normalized size = 1.11

$$-\frac{1119}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{363\,x^8+1684\,x^6+2499\,x^4+1250\,x^2+64}{32\left(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x\right)}+\frac{189}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] -1119/64*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/32*(363*x^8 + 1684*x^6 + 2499*x^
4 + 1250*x^2 + 64)/(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x) + 189/8*arctan(x)
```

Fricas [A] time = 1.56684, size = 302, normalized size = 3.82

$$-\frac{726\,x^8+3368\,x^6+4998\,x^4+1119\,\sqrt{2}\left(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+2500\,x^2-1512\left(x^9+6\,x^7\right.}{64\left(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(726*x^8 + 3368*x^6 + 4998*x^4 + 1119*sqrt(2)*(x^9 + 6*x^7 + 13*x^5 +
 12*x^3 + 4*x)*arctan(1/2*sqrt(2)*x) + 2500*x^2 - 1512*(x^9 + 6*x^7 + 13*x^
5 + 12*x^3 + 4*x)*arctan(x) + 128)/(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x)
```

Sympy [A] time = 0.260194, size = 70, normalized size = 0.89

$$-\frac{363x^8+1684x^6+2499x^4+1250x^2+64}{32x^9+192x^7+416x^5+384x^3+128x}+\frac{189\operatorname{atan}(x)}{8}-\frac{1119\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+3*x**2+2)**3,x)
```

```
[Out] -(363*x**8 + 1684*x**6 + 2499*x**4 + 1250*x**2 + 64)/(32*x**9 + 192*x**7 +
416*x**5 + 384*x**3 + 128*x) + 189*atan(x)/8 - 1119*sqrt(2)*atan(sqrt(2)*x/
2)/64
```

Giac [A] time = 1.12745, size = 74, normalized size = 0.94

$$-\frac{1119}{64}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-\frac{347\,x^7+1588\,x^5+2291\,x^3+1058\,x}{32\left(x^4+3\,x^2+2\right)^2}-\frac{1}{2\,x}+\frac{189}{8}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] -1119/64*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/32*(347*x^7 + 1588*x^5 + 2291*x^
3 + 1058*x)/(x^4 + 3*x^2 + 2)^2 - 1/2/x + 189/8*arctan(x)
```

3.98 $\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=86

$$-\frac{x\left(9x^2+5\right)}{16\left(x^4+3x^2+2\right)^2}+\frac{x\left(571x^2+951\right)}{64\left(x^4+3x^2+2\right)}-\frac{1}{6x^3}+\frac{17}{8x}-\frac{113}{8}\tan^{-1}(x)+\frac{1611\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{64\sqrt{2}}$$

```
[Out] -1/(6*x^3) + 17/(8*x) - (x*(5 + 9*x^2))/(16*(2 + 3*x^2 + x^4)^2) + (x*(951
+ 571*x^2))/(64*(2 + 3*x^2 + x^4)) - (113*ArcTan[x])/8 + (1611*ArcTan[x/Sqr
t[2]])/(64*Sqrt[2])
```

Rubi [A] time = 0.118885, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1669, 1664, 203}

$$-\frac{x\left(9x^2+5\right)}{16\left(x^4+3x^2+2\right)^2}+\frac{x\left(571x^2+951\right)}{64\left(x^4+3x^2+2\right)}-\frac{1}{6x^3}+\frac{17}{8x}-\frac{113}{8}\tan^{-1}(x)+\frac{1611\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] -1/(6*x^3) + 17/(8*x) - (x*(5 + 9*x^2))/(16*(2 + 3*x^2 + x^4)^2) + (x*(951
+ 571*x^2))/(64*(2 + 3*x^2 + x^4)) - (113*ArcTan[x])/8 + (1611*ArcTan[x/Sqr
t[2]])/(64*Sqrt[2])
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3}\,dx = -\frac{x\left(5+9x^2\right)}{16\left(2+3x^2+x^4\right)^2} - \frac{1}{8}\int \frac{-16+20x^2-\frac{73x^4}{2}+\frac{45x^6}{2}}{x^4\left(2+3x^2+x^4\right)^2}\,dx$$

$$= -\frac{x\left(5+9x^2\right)}{16\left(2+3x^2+x^4\right)^2} + \frac{x\left(951+571x^2\right)}{64\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \frac{32-88x^2-\frac{573x^4}{2}+\frac{571x^6}{2}}{x^4\left(2+3x^2+x^4\right)}\,dx$$

$$= -\frac{x\left(5+9x^2\right)}{16\left(2+3x^2+x^4\right)^2} + \frac{x\left(951+571x^2\right)}{64\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \left(\frac{16}{x^4}-\frac{68}{x^2}-\frac{452}{1+x^2}+\frac{1611}{2\left(2+x^2\right)}\right)dx$$

$$= -\frac{1}{6x^3}+\frac{17}{8x}-\frac{x\left(5+9x^2\right)}{16\left(2+3x^2+x^4\right)^2} + \frac{x\left(951+571x^2\right)}{64\left(2+3x^2+x^4\right)} - \frac{113}{8}\int \frac{1}{1+x^2}\,dx + \frac{1611}{64}\int \frac{1}{2+x}$$

$$= -\frac{1}{6x^3}+\frac{17}{8x}-\frac{x\left(5+9x^2\right)}{16\left(2+3x^2+x^4\right)^2} + \frac{x\left(951+571x^2\right)}{64\left(2+3x^2+x^4\right)} - \frac{113}{8}\tan^{-1}(x) + \frac{1611\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{64\sqrt{2}}$$

Mathematica [A] time = 0.0603144, size = 78, normalized size = 0.91

$$\frac{1}{384}\left(-\frac{24x\left(9x^2+5\right)}{\left(x^4+3x^2+2\right)^2}+\frac{6x\left(571x^2+951\right)}{x^4+3x^2+2}-\frac{64}{x^3}+\frac{816}{x}-5424\tan^{-1}(x)+4833\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] (-64/x^3 + 816/x - (24*x*(5 + 9*x^2))/(2 + 3*x^2 + x^4)^2 + (6*x*(951 + 571
*x^2))/(2 + 3*x^2 + x^4) - 5424*ArcTan[x] + 4833*Sqrt[2]*ArcTan[x/Sqrt[2]])
/384
```

Maple [A] time = 0.016, size = 64, normalized size = 0.7

$$\frac{1}{8\left(x^2+2\right)^2}\left(\frac{259\,x^3}{8}+\frac{285\,x}{4}\right)+\frac{1611\,\sqrt{2}}{128}\arctan\left(\frac{x\sqrt{2}}{2}\right)-\frac{1}{\left(x^2+1\right)^2}\left(-\frac{39\,x^3}{8}-\frac{41\,x}{8}\right)-\frac{113\,\arctan(x)}{8}-\frac{1}{6\,x^3}+\frac{17}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x)
```

```
[Out] 1/8*(259/8*x^3+285/4*x)/(x^2+2)^2+1611/128*arctan(1/2*x*2^(1/2))*2^(1/2)-(-
39/8*x^3-41/8*x)/(x^2+1)^2-113/8*arctan(x)-1/6/x^3+17/8/x
```

Maxima [A] time = 1.54805, size = 97, normalized size = 1.13

$$\frac{1611}{128}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{2121\,x^{10}+10408\,x^8+16989\,x^6+10126\,x^4+1248\,x^2-128}{192\left(x^{11}+6\,x^9+13\,x^7+12\,x^5+4\,x^3\right)}-\frac{113}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] 1611/128*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/192*(2121*x^10 + 10408*x^8 + 169
89*x^6 + 10126*x^4 + 1248*x^2 - 128)/(x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*x^
3) - 113/8*arctan(x)
```

Fricas [A] time = 1.54364, size = 336, normalized size = 3.91

$$\frac{4242\,x^{10} + 20816\,x^{8} + 33978\,x^{6} + 20252\,x^{4} + 4833\,\sqrt{2}\left(x^{11} + 6\,x^{9} + 13\,x^{7} + 12\,x^{5} + 4\,x^{3}\right)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 2496\,x^{2} - 54}{384\,\left(x^{11} + 6\,x^{9} + 13\,x^{7} + 12\,x^{5} + 4\,x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(4242*x^10 + 20816*x^8 + 33978*x^6 + 20252*x^4 + 4833*sqrt(2)*(x^11 +
 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3)*arctan(1/2*sqrt(2)*x) + 2496*x^2 - 5424*(
x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3)*arctan(x) - 256)/(x^11 + 6*x^9 + 13
*x^7 + 12*x^5 + 4*x^3)
```

Sympy [A] time = 0.277716, size = 76, normalized size = 0.88

$$-\frac{113\operatorname{atan}{\left(x \right)}}{8} + \frac{1611\sqrt{2}\operatorname{atan}{\left(\frac{\sqrt{2}x}{2} \right)}}{128} + \frac{2121x^{10} + 10408x^{8} + 16989x^{6} + 10126x^{4} + 1248x^{2} - 128}{192x^{11} + 1152x^{9} + 2496x^{7} + 2304x^{5} + 768x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+3*x**2+2)**3,x)
```

```
[Out] -113*atan(x)/8 + 1611*sqrt(2)*atan(sqrt(2)*x/2)/128 + (2121*x**10 + 10408*x
**8 + 16989*x**6 + 10126*x**4 + 1248*x**2 - 128)/(192*x**11 + 1152*x**9 + 2
496*x**7 + 2304*x**5 + 768*x**3)
```

Giac [A] time = 1.11011, size = 84, normalized size = 0.98

$$\frac{1611}{128}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + \frac{571\,x^{7} + 2664\,x^{5} + 3959\,x^{3} + 1882\,x}{64\,\left(x^{4} + 3\,x^{2} + 2\right)^{2}} + \frac{51\,x^{2} - 4}{24\,x^{3}} - \frac{113}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] 1611/128*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/64*(571*x^7 + 2664*x^5 + 3959*x^
3 + 1882*x)/(x^4 + 3*x^2 + 2)^2 + 1/24*(51*x^2 - 4)/x^3 - 113/8*arctan(x)
```

3.99 $\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=93

$$-\frac{x\left(3-5x^2\right)}{32\left(x^4+3x^2+2\right)^2}-\frac{x\left(999x^2+1771\right)}{128\left(x^4+3x^2+2\right)}+\frac{17}{24x^3}-\frac{1}{10x^5}-\frac{93}{16x}+\frac{29}{8}\tan^{-1}(x)-\frac{2207\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{128\sqrt{2}}$$

```
[Out] -1/(10*x^5) + 17/(24*x^3) - 93/(16*x) - (x*(3 - 5*x^2))/(32*(2 + 3*x^2 + x^
4)^2) - (x*(1771 + 999*x^2))/(128*(2 + 3*x^2 + x^4)) + (29*ArcTan[x])/8 - (
2207*ArcTan[x/Sqrt[2]])/(128*Sqrt[2])
```

Rubi [A] time = 0.134206, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1669, 1664, 203}

$$-\frac{x\left(3-5x^2\right)}{32\left(x^4+3x^2+2\right)^2}-\frac{x\left(999x^2+1771\right)}{128\left(x^4+3x^2+2\right)}+\frac{17}{24x^3}-\frac{1}{10x^5}-\frac{93}{16x}+\frac{29}{8}\tan^{-1}(x)-\frac{2207\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] -1/(10*x^5) + 17/(24*x^3) - 93/(16*x) - (x*(3 - 5*x^2))/(32*(2 + 3*x^2 + x^
4)^2) - (x*(1771 + 999*x^2))/(128*(2 + 3*x^2 + x^4)) + (29*ArcTan[x])/8 - (
2207*ArcTan[x/Sqrt[2]])/(128*Sqrt[2])
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^3}\,dx = -\frac{x\left(3-5x^2\right)}{32\left(2+3x^2+x^4\right)^2} - \frac{1}{8}\int \frac{-16+20x^2-34x^4+\frac{81x^6}{4}-\frac{25x^8}{4}}{x^6\left(2+3x^2+x^4\right)^2}\,dx$$

$$= -\frac{x\left(3-5x^2\right)}{32\left(2+3x^2+x^4\right)^2} - \frac{x\left(1771+999x^2\right)}{128\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \frac{32-88x^2+184x^4+\frac{681x^6}{4}-\frac{999x^8}{4}}{x^6\left(2+3x^2+x^4\right)}\,dx$$

$$= -\frac{x\left(3-5x^2\right)}{32\left(2+3x^2+x^4\right)^2} - \frac{x\left(1771+999x^2\right)}{128\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \left(\frac{16}{x^6}-\frac{68}{x^4}+\frac{186}{x^2}+\frac{116}{1+x^2}-\frac{2207}{4\left(2+x^2\right)}\right) d$$

$$= -\frac{1}{10x^5}+\frac{17}{24x^3}-\frac{93}{16x}-\frac{x\left(3-5x^2\right)}{32\left(2+3x^2+x^4\right)^2}-\frac{x\left(1771+999x^2\right)}{128\left(2+3x^2+x^4\right)}+\frac{29}{8}\int \frac{1}{1+x^2}\,dx-\frac{2207}{128}$$

$$= -\frac{1}{10x^5}+\frac{17}{24x^3}-\frac{93}{16x}-\frac{x\left(3-5x^2\right)}{32\left(2+3x^2+x^4\right)^2}-\frac{x\left(1771+999x^2\right)}{128\left(2+3x^2+x^4\right)}+\frac{29}{8}\tan^{-1}(x)-\frac{2207\tan^{-}}{128\sqrt{}}$$

Mathematica [A] time = 0.0788035, size = 73, normalized size = 0.78

$$\frac{-\frac{2\left(26145x^{12}+137120x^{10}+246477x^8+170702x^6+30816x^4-3136x^2+768\right)}{x^5\left(x^4+3x^2+2\right)^2}+13920\tan^{-1}(x)-33105\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{3840}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] ((-2*(768 - 3136*x^2 + 30816*x^4 + 170702*x^6 + 246477*x^8 + 137120*x^10 +
26145*x^12))/(x^5*(2 + 3*x^2 + x^4)^2) + 13920*ArcTan[x] - 33105*Sqrt[2]*Ar
cTan[x/Sqrt[2]])/3840
```

Maple [A] time = 0.015, size = 68, normalized size = 0.7

$$-\frac{1}{16\left(x^2+2\right)^2}\left(\frac{311x^3}{8}+\frac{337x}{4}\right)-\frac{2207\sqrt{2}}{256}\arctan\left(\frac{x\sqrt{2}}{2}\right)+\frac{1}{\left(x^2+1\right)^2}\left(-\frac{43x^3}{8}-\frac{45x}{8}\right)+\frac{29\arctan(x)}{8}-\frac{1}{10x^5}+\frac{17}{24x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x)
```

```
[Out] -1/16*(311/8*x^3+337/4*x)/(x^2+2)^2-2207/256*arctan(1/2*x*2^(1/2))*2^(1/2)+
(-43/8*x^3-45/8*x)/(x^2+1)^2+29/8*arctan(x)-1/10/x^5+17/24/x^3-93/16/x
```

Maxima [A] time = 1.47447, size = 104, normalized size = 1.12

$$-\frac{2207}{256}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{26145x^{12}+137120x^{10}+246477x^8+170702x^6+30816x^4-3136x^2+768}{1920\left(x^{13}+6x^{11}+13x^9+12x^7+4x^5\right)}+\frac{29}{8}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] -2207/256*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/1920*(26145*x^12 + 137120*x^10
+ 246477*x^8 + 170702*x^6 + 30816*x^4 - 3136*x^2 + 768)/(x^13 + 6*x^11 + 13
*x^9 + 12*x^7 + 4*x^5) + 29/8*arctan(x)
```

Fricas [A] time = 1.56934, size = 370, normalized size = 3.98

$$-\frac{52290\,x^{12}+274240\,x^{10}+492954\,x^{8}+341404\,x^{6}+61632\,x^{4}+33105\,\sqrt{2}\left(x^{13}+6\,x^{11}+13\,x^{9}+12\,x^{7}+4\,x^{5}\right)\arctan}{3840\left(x^{13}+6\,x^{11}+13\,x^{9}+12\,x^{7}+4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] -1/3840*(52290*x^12 + 274240*x^10 + 492954*x^8 + 341404*x^6 + 61632*x^4 + 3
3105*sqrt(2)*(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)*arctan(1/2*sqrt(2)*x
) - 6272*x^2 - 13920*(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)*arctan(x) +
1536)/(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)
```

Sympy [A] time = 0.300799, size = 82, normalized size = 0.88

$$\frac{29\operatorname{atan}(x)}{8}-\frac{2207\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{256}-\frac{26145x^{12}+137120x^{10}+246477x^{8}+170702x^{6}+30816x^{4}-3136x^{2}+768}{1920x^{13}+11520x^{11}+24960x^{9}+23040x^{7}+7680x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+3*x**2+2)**3,x)
```

```
[Out] 29*atan(x)/8 - 2207*sqrt(2)*atan(sqrt(2)*x/2)/256 - (26145*x**12 + 137120*x
**10 + 246477*x**8 + 170702*x**6 + 30816*x**4 - 3136*x**2 + 768)/(1920*x**1
3 + 11520*x**11 + 24960*x**9 + 23040*x**7 + 7680*x**5)
```

Giac [A] time = 1.12132, size = 90, normalized size = 0.97

$$-\frac{2207}{256}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{999\,x^{7}+4768\,x^{5}+7291\,x^{3}+3554\,x}{128\left(x^{4}+3\,x^{2}+2\right)^{2}}-\frac{1395\,x^{4}-170\,x^{2}+24}{240\,x^{5}}+\frac{29}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] -2207/256*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/128*(999*x^7 + 4768*x^5 + 7291*
x^3 + 3554*x)/(x^4 + 3*x^2 + 2)^2 - 1/240*(1395*x^4 - 170*x^2 + 24)/x^5 + 2
9/8*arctan(x)
```

3.100 $\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=86

$$\frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2-\frac{25(7x^2+15)}{8(x^4+2x^2+3)}-\frac{183}{4}\log(x^4+2x^2+3)+\frac{201\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] 19*x^2 + (19*x^4)/4 - (17*x^6)/6 + (5*x^8)/8 - (25*(15 + 7*x^2))/(8*(3 + 2*
x^2 + x^4)) + (201*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) - (183*Log[3 + 2*
x^2 + x^4])/4
```

Rubi [A] time = 0.135313, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1663, 1660, 1657, 634, 618, 204, 628}

$$\frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2-\frac{25(7x^2+15)}{8(x^4+2x^2+3)}-\frac{183}{4}\log(x^4+2x^2+3)+\frac{201\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 19*x^2 + (19*x^4)/4 - (17*x^6)/6 + (5*x^8)/8 - (25*(15 + 7*x^2))/(8*(3 + 2*
x^2 + x^4)) + (201*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) - (183*Log[3 + 2*
x^2 + x^4])/4
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^9\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4\left(4+x+3x^2+5x^3\right)}{\left(3+2x+x^2\right)^2}\,dx, x, x^2\right)\\
&= -\frac{25\left(15+7x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\,\text{Subst}\left(\int \frac{-150-400x+200x^2-56x^4+40x^5}{3+2x+x^2}\,dx, x, x^2\right)\\
&= -\frac{25\left(15+7x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\,\text{Subst}\left(\int \left(304+152x-136x^2+40x^3-\frac{6(177+244x)}{3+2x+x^2}\right) dx\right.\\
&= 19x^2+\frac{19x^4}{4}-\frac{17x^6}{6}+\frac{5x^8}{8}-\frac{25\left(15+7x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{3}{8}\,\text{Subst}\left(\int \frac{177+244x}{3+2x+x^2}\,dx, x, x^2\right)\\
&= 19x^2+\frac{19x^4}{4}-\frac{17x^6}{6}+\frac{5x^8}{8}-\frac{25\left(15+7x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{201}{8}\,\text{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx, x, x^2\right.\\
&= 19x^2+\frac{19x^4}{4}-\frac{17x^6}{6}+\frac{5x^8}{8}-\frac{25\left(15+7x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{183}{4}\log\left(3+2x^2+x^4\right)-\frac{201}{4}\,\text{Subst}\\
&= 19x^2+\frac{19x^4}{4}-\frac{17x^6}{6}+\frac{5x^8}{8}-\frac{25\left(15+7x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{201\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{183}{4}\log\left(3+2x\right.
\end{aligned}
$$

Mathematica [A] time = 0.0472626, size = 78, normalized size = 0.91

$$\frac{1}{48}\left(30x^8-136x^6+228x^4+912x^2-\frac{150\left(7x^2+15\right)}{x^4+2x^2+3}-2196\log\left(x^4+2x^2+3\right)+603\sqrt{2}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (912*x^2 + 228*x^4 - 136*x^6 + 30*x^8 - (150*(15 + 7*x^2))/(3 + 2*x^2 + x^4
) + 603*Sqrt[2]*ArcTan[(1 + x^2)/Sqrt[2]] - 2196*Log[3 + 2*x^2 + x^4])/48
```

Maple [A] time = 0.011, size = 74, normalized size = 0.9

$$\frac{5 x^{8}}{8}-\frac{17 x^{6}}{6}+\frac{19 x^{4}}{4}+19 x^{2}-\frac{1}{2 x^{4}+4 x^{2}+6}\left(\frac{175 x^{2}}{4}+\frac{375}{4}\right)-\frac{183 \ln \left(x^{4}+2 x^{2}+3\right)}{4}+\frac{201 \sqrt{2}}{16} \arctan \left(\frac{\left(2 x^{2}+2\right) \sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5/8*x^8-17/6*x^6+19/4*x^4+19*x^2-1/2*(175/4*x^2+375/4)/(x^4+2*x^2+3)-183/4*
ln(x^4+2*x^2+3)+201/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Maxima [A] time = 1.47227, size = 96, normalized size = 1.12

$$\frac{5}{8} x^{8}-\frac{17}{6} x^{6}+\frac{19}{4} x^{4}+19 x^{2}+\frac{201}{16} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2}\left(x^{2}+1\right)\right)-\frac{25\left(7 x^{2}+15\right)}{8\left(x^{4}+2 x^{2}+3\right)}-\frac{183}{4} \log \left(x^{4}+2 x^{2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/8*x^8 - 17/6*x^6 + 19/4*x^4 + 19*x^2 + 201/16*sqrt(2)*arctan(1/2*sqrt(2)*
(x^2 + 1)) - 25/8*(7*x^2 + 15)/(x^4 + 2*x^2 + 3) - 183/4*log(x^4 + 2*x^2 +
3)
```

Fricas [A] time = 1.50026, size = 270, normalized size = 3.14

$$\frac{30 x^{12}-76 x^{10}+46 x^{8}+960 x^{6}+2508 x^{4}+603 \sqrt{2}\left(x^{4}+2 x^{2}+3\right) \arctan \left(\frac{1}{2} \sqrt{2}\left(x^{2}+1\right)\right)+1686 x^{2}-2196\left(x^{4}+2 x^{2}+\right.}{48\left(x^{4}+2 x^{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(30*x^12 - 76*x^10 + 46*x^8 + 960*x^6 + 2508*x^4 + 603*sqrt(2)*(x^4 +
2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1686*x^2 - 2196*(x^4 + 2*x^2 + 3
)*log(x^4 + 2*x^2 + 3) - 2250)/(x^4 + 2*x^2 + 3)
```

Sympy [A] time = 0.171973, size = 85, normalized size = 0.99

$$\frac{5x^{8}}{8}-\frac{17x^{6}}{6}+\frac{19x^{4}}{4}+19x^{2}-\frac{175x^{2}+375}{8x^{4}+16x^{2}+24}-\frac{183 \log \left(x^{4}+2x^{2}+3\right)}{4}+\frac{201 \sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x^{2}}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**8/8 - 17*x**6/6 + 19*x**4/4 + 19*x**2 - (175*x**2 + 375)/(8*x**4 + 16*
x**2 + 24) - 183*log(x**4 + 2*x**2 + 3)/4 + 201*sqrt(2)*atan(sqrt(2)*x**2/2
 + sqrt(2)/2)/16
```

Giac [A] time = 1.13086, size = 103, normalized size = 1.2

$$\frac{5}{8}x^8 - \frac{17}{6}x^6 + \frac{19}{4}x^4 + 19x^2 + \frac{201}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) + \frac{366x^4 + 557x^2 + 723}{8(x^4+2x^2+3)} - \frac{183}{4}\log\left(x^4+2x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/8*x^8 - 17/6*x^6 + 19/4*x^4 + 19*x^2 + 201/16*sqrt(2)*arctan(1/2*sqrt(2)*
(x^2 + 1)) + 1/8*(366*x^4 + 557*x^2 + 723)/(x^4 + 2*x^2 + 3) - 183/4*log(x^
4 + 2*x^2 + 3)
```

3.101 $\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=81

$$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}+\frac{25(5x^2+3)}{8(x^4+2x^2+3)}+\frac{19}{2}\log(x^4+2x^2+3)-\frac{455\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] (19*x^2)/2 - (17*x^4)/4 + (5*x^6)/6 + (25*(3 + 5*x^2))/(8*(3 + 2*x^2 + x^4)
) - (455*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (19*Log[3 + 2*x^2 + x^4])
/2
```

Rubi [A] time = 0.127361, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1663, 1660, 1657, 634, 618, 204, 628}

$$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}+\frac{25(5x^2+3)}{8(x^4+2x^2+3)}+\frac{19}{2}\log(x^4+2x^2+3)-\frac{455\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (19*x^2)/2 - (17*x^4)/4 + (5*x^6)/6 + (25*(3 + 5*x^2))/(8*(3 + 2*x^2 + x^4)
) - (455*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (19*Log[3 + 2*x^2 + x^4])
/2
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3\left(4+x+3x^2+5x^3\right)}{\left(3+2x+x^2\right)^2}\,dx, x, x^2\right)\\
&= \frac{25\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\text{Subst}\left(\int \frac{-150+200x-56x^3+40x^4}{3+2x+x^2}\,dx, x, x^2\right)\\
&= \frac{25\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\text{Subst}\left(\int \left(152-136x+40x^2-\frac{2(303-152x)}{3+2x+x^2}\right)dx, x, x^2\right)\\
&= \frac{19x^2}{2}-\frac{17x^4}{4}+\frac{5x^6}{6}+\frac{25\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{8}\text{Subst}\left(\int \frac{303-152x}{3+2x+x^2}\,dx, x, x^2\right)\\
&= \frac{19x^2}{2}-\frac{17x^4}{4}+\frac{5x^6}{6}+\frac{25\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{19}{2}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx, x, x^2\right)-\frac{455}{8}\\
&= \frac{19x^2}{2}-\frac{17x^4}{4}+\frac{5x^6}{6}+\frac{25\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{19}{2}\log\left(3+2x^2+x^4\right)+\frac{455}{4}\text{Subst}\left(\int \frac{}{-8}\right.\\
&= \frac{19x^2}{2}-\frac{17x^4}{4}+\frac{5x^6}{6}+\frac{25\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{455\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{19}{2}\log\left(3+2x^2+x^4\right)
\end{aligned}
$$

Mathematica [A] time = 0.0313759, size = 73, normalized size = 0.9

$$
\frac{1}{48}\left(40x^6-204x^4+456x^2+\frac{150\left(5x^2+3\right)}{x^4+2x^2+3}+456\log\left(x^4+2x^2+3\right)-1365\sqrt{2}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (456*x^2 - 204*x^4 + 40*x^6 + (150*(3 + 5*x^2))/(3 + 2*x^2 + x^4) - 1365*Sq
rt[2]*ArcTan[(1 + x^2)/Sqrt[2]] + 456*Log[3 + 2*x^2 + x^4])/48
```

Maple [A] time = 0.01, size = 69, normalized size = 0.9

$$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}+\frac{1}{2x^4+4x^2+6}\left(\frac{125x^2}{4}+\frac{75}{4}\right)+\frac{19\ln\left(x^4+2x^2+3\right)}{2}-\frac{455\sqrt{2}}{16}\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5/6*x^6-17/4*x^4+19/2*x^2+1/2*(125/4*x^2+75/4)/(x^4+2*x^2+3)+19/2*ln(x^4+2*
x^2+3)-455/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Maxima [A] time = 1.49928, size = 89, normalized size = 1.1

$$\frac{5}{6}x^6-\frac{17}{4}x^4+\frac{19}{2}x^2-\frac{455}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{25\left(5x^2+3\right)}{8\left(x^4+2x^2+3\right)}+\frac{19}{2}\log\left(x^4+2x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/6*x^6 - 17/4*x^4 + 19/2*x^2 - 455/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)
) + 25/8*(5*x^2 + 3)/(x^4 + 2*x^2 + 3) + 19/2*log(x^4 + 2*x^2 + 3)
```

Fricas [A] time = 1.54505, size = 255, normalized size = 3.15

$$\frac{40x^{10}-124x^8+168x^6+300x^4-1365\sqrt{2}\left(x^4+2x^2+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+2118x^2+456\left(x^4+2x^2+3\right)\log\left(x\right.}{48\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(40*x^10 - 124*x^8 + 168*x^6 + 300*x^4 - 1365*sqrt(2)*(x^4 + 2*x^2 + 3
)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 2118*x^2 + 456*(x^4 + 2*x^2 + 3)*log(x^4
+ 2*x^2 + 3) + 450)/(x^4 + 2*x^2 + 3)
```

Sympy [A] time = 0.168, size = 80, normalized size = 0.99

$$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}+\frac{125x^2+75}{8x^4+16x^2+24}+\frac{19\log\left(x^4+2x^2+3\right)}{2}-\frac{455\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**6/6 - 17*x**4/4 + 19*x**2/2 + (125*x**2 + 75)/(8*x**4 + 16*x**2 + 24)
+ 19*log(x**4 + 2*x**2 + 3)/2 - 455*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2
```

```
)/16
```

Giac [A] time = 1.12135, size = 96, normalized size = 1.19

$$\frac{5}{6}x^6 - \frac{17}{4}x^4 + \frac{19}{2}x^2 - \frac{455}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right) - \frac{76x^4+27x^2+153}{8\left(x^4+2x^2+3\right)} + \frac{19}{2}\log\left(x^4+2x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/6*x^6 - 17/4*x^4 + 19/2*x^2 - 455/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)
) - 1/8*(76*x^4 + 27*x^2 + 153)/(x^4 + 2*x^2 + 3) + 19/2*log(x^4 + 2*x^2 +
3)
```

3.102 $\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=74

$$\frac{5x^4}{4}-\frac{17x^2}{2}+\frac{25(3-x^2)}{8(x^4+2x^2+3)}+\frac{19}{4}\log\left(x^4+2x^2+3\right)+\frac{203\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] (-17*x^2)/2 + (5*x^4)/4 + (25*(3 - x^2))/(8*(3 + 2*x^2 + x^4)) + (203*ArcTa
n[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (19*Log[3 + 2*x^2 + x^4])/4
```

Rubi [A] time = 0.120869, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1663, 1660, 1657, 634, 618, 204, 628}

$$\frac{5x^4}{4}-\frac{17x^2}{2}+\frac{25(3-x^2)}{8(x^4+2x^2+3)}+\frac{19}{4}\log\left(x^4+2x^2+3\right)+\frac{203\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (-17*x^2)/2 + (5*x^4)/4 + (25*(3 - x^2))/(8*(3 + 2*x^2 + x^4)) + (203*ArcTa
n[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (19*Log[3 + 2*x^2 + x^4])/4
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2\left(4+x+3x^2+5x^3\right)}{\left(3+2x+x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{25\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\operatorname{Subst}\left(\int \frac{150-56x^2+40x^3}{3+2x+x^2}\,dx,x,x^2\right)\\
&= \frac{25\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\operatorname{Subst}\left(\int \left(-136+40x+\frac{2(279+76x)}{3+2x+x^2}\right)dx,x,x^2\right)\\
&= -\frac{17x^2}{2}+\frac{5x^4}{4}+\frac{25\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{8}\operatorname{Subst}\left(\int \frac{279+76x}{3+2x+x^2}\,dx,x,x^2\right)\\
&= -\frac{17x^2}{2}+\frac{5x^4}{4}+\frac{25\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{19}{4}\operatorname{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx,x,x^2\right)+\frac{203}{8}\operatorname{Subs}\\
&= -\frac{17x^2}{2}+\frac{5x^4}{4}+\frac{25\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{19}{4}\log\left(3+2x^2+x^4\right)-\frac{203}{4}\operatorname{Subst}\left(\int \frac{1}{-8-x^2}\right.\\
&= -\frac{17x^2}{2}+\frac{5x^4}{4}+\frac{25\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{203\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{19}{4}\log\left(3+2x^2+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.0299438, size = 66, normalized size = 0.89

$$\frac{1}{16}\left(20x^4-136x^2-\frac{50\left(x^2-3\right)}{x^4+2x^2+3}+76\log\left(x^4+2x^2+3\right)+203\sqrt{2}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (-136*x^2 + 20*x^4 - (50*(-3 + x^2))/(3 + 2*x^2 + x^4) + 203*Sqrt[2]*ArcTan
[(1 + x^2)/Sqrt[2]] + 76*Log[3 + 2*x^2 + x^4])/16
```

Maple [A] time = 0.008, size = 64, normalized size = 0.9

$$\frac{5x^4}{4} - \frac{17x^2}{2} + \frac{1}{2x^4+4x^2+6}\left(-\frac{25x^2}{4} + \frac{75}{4}\right) + \frac{19\ln\left(x^4+2x^2+3\right)}{4} + \frac{203\sqrt{2}}{16}\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5/4*x^4-17/2*x^2+1/2*(-25/4*x^2+75/4)/(x^4+2*x^2+3)+19/4*ln(x^4+2*x^2+3)+20
3/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Maxima [A] time = 1.46919, size = 80, normalized size = 1.08

$$\frac{5}{4}x^4 - \frac{17}{2}x^2 + \frac{203}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right) - \frac{25\left(x^2-3\right)}{8\left(x^4+2x^2+3\right)} + \frac{19}{4}\log\left(x^4+2x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/4*x^4 - 17/2*x^2 + 203/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x
^2 - 3)/(x^4 + 2*x^2 + 3) + 19/4*log(x^4 + 2*x^2 + 3)
```

Fricas [A] time = 1.55717, size = 235, normalized size = 3.18

$$\frac{20x^8 - 96x^6 - 212x^4 + 203\sqrt{2}\left(x^4+2x^2+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right) - 458x^2 + 76\left(x^4+2x^2+3\right)\log\left(x^4+2x^2+3\right) +}{16\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(20*x^8 - 96*x^6 - 212*x^4 + 203*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*
sqrt(2)*(x^2 + 1)) - 458*x^2 + 76*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) +
150)/(x^4 + 2*x^2 + 3)
```

Sympy [A] time = 0.163762, size = 73, normalized size = 0.99

$$\frac{5x^4}{4} - \frac{17x^2}{2} - \frac{25x^2-75}{8x^4+16x^2+24} + \frac{19\log\left(x^4+2x^2+3\right)}{4} + \frac{203\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**4/4 - 17*x**2/2 - (25*x**2 - 75)/(8*x**4 + 16*x**2 + 24) + 19*log(x**4
 + 2*x**2 + 3)/4 + 203*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

Giac [A] time = 1.0805, size = 89, normalized size = 1.2

$$\frac{5}{4}x^4 - \frac{17}{2}x^2 + \frac{203}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right) - \frac{38x^4 + 101x^2 + 39}{8\left(x^4+2x^2+3\right)} + \frac{19}{4}\log\left(x^4+2x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/4*x^4 - 17/2*x^2 + 203/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/8*(38
*x^4 + 101*x^2 + 39)/(x^4 + 2*x^2 + 3) + 19/4*log(x^4 + 2*x^2 + 3)
```

3.103 $\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$

Optimal. Leaf size=65

$$\frac{5x^2}{2} - \frac{25(x^2+3)}{8(x^4+2x^2+3)} - \frac{17}{4}\log(x^4+2x^2+3) - \frac{17\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] (5*x^2)/2 - (25*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) - (17*ArcTan[(1 + x^2)/Sqr
t[2]])/(8*Sqrt[2]) - (17*Log[3 + 2*x^2 + x^4])/4
```

Rubi [A] time = 0.105306, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1663, 1660, 1657, 634, 618, 204, 628}

$$\frac{5x^2}{2} - \frac{25(x^2+3)}{8(x^4+2x^2+3)} - \frac{17}{4}\log(x^4+2x^2+3) - \frac{17\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5*x^2)/2 - (25*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) - (17*ArcTan[(1 + x^2)/Sqr
t[2]])/(8*Sqrt[2]) - (17*Log[3 + 2*x^2 + x^4])/4
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x\left(4+x+3x^2+5x^3\right)}{\left(3+2x+x^2\right)^2}\,dx, x, x^2\right)\\
&= -\frac{25\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)} + \frac{1}{16}\operatorname{Subst}\left(\int \frac{-50-56x+40x^2}{3+2x+x^2}\,dx, x, x^2\right)\\
&= -\frac{25\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)} + \frac{1}{16}\operatorname{Subst}\left(\int \left(40-\frac{34(5+4x)}{3+2x+x^2}\right)dx, x, x^2\right)\\
&= \frac{5x^2}{2} - \frac{25\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)} - \frac{17}{8}\operatorname{Subst}\left(\int \frac{5+4x}{3+2x+x^2}\,dx, x, x^2\right)\\
&= \frac{5x^2}{2} - \frac{25\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)} - \frac{17}{8}\operatorname{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx, x, x^2\right) - \frac{17}{4}\operatorname{Subst}\left(\int \frac{2+}{3+2}\right.\\
&= \frac{5x^2}{2} - \frac{25\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)} - \frac{17}{4}\log\left(3+2x^2+x^4\right) + \frac{17}{4}\operatorname{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2\left(1+\right.\right.\\
&= \frac{5x^2}{2} - \frac{25\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)} - \frac{17\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} - \frac{17}{4}\log\left(3+2x^2+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.0271538, size = 61, normalized size = 0.94

$$\frac{1}{16}\left(40x^2-\frac{50\left(x^2+3\right)}{x^4+2x^2+3}-68\log\left(x^4+2x^2+3\right)-17\sqrt{2}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (40*x^2 - (50*(3 + x^2))/(3 + 2*x^2 + x^4) - 17*Sqrt[2]*ArcTan[(1 + x^2)/Sq
rt[2]] - 68*Log[3 + 2*x^2 + x^4])/16
```

Maple [A] time = 0.008, size = 59, normalized size = 0.9

$$\frac{5x^2}{2} - \frac{1}{2x^4+4x^2+6}\left(\frac{25x^2}{4}+\frac{75}{4}\right) - \frac{17\ln\left(x^4+2x^2+3\right)}{4} - \frac{17\sqrt{2}}{16}\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5/2*x^2-1/2*(25/4*x^2+75/4)/(x^4+2*x^2+3)-17/4*ln(x^4+2*x^2+3)-17/16*2^(1/2
)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Maxima [A] time = 1.47108, size = 73, normalized size = 1.12

$$\frac{5}{2}x^2 - \frac{17}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right) - \frac{25\left(x^2+3\right)}{8\left(x^4+2x^2+3\right)} - \frac{17}{4}\log\left(x^4+2x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/2*x^2 - 17/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x^2 + 3)/(x^4
 + 2*x^2 + 3) - 17/4*log(x^4 + 2*x^2 + 3)
```

Fricas [A] time = 1.50264, size = 219, normalized size = 3.37

$$\frac{40x^6+80x^4-17\sqrt{2}\left(x^4+2x^2+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+70x^2-68\left(x^4+2x^2+3\right)\log\left(x^4+2x^2+3\right)-150}{16\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(40*x^6 + 80*x^4 - 17*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^
2 + 1)) + 70*x^2 - 68*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) - 150)/(x^4 +
2*x^2 + 3)
```

Sympy [A] time = 0.167923, size = 66, normalized size = 1.02

$$\frac{5x^2}{2} - \frac{25x^2+75}{8x^4+16x^2+24} - \frac{17\log\left(x^4+2x^2+3\right)}{4} - \frac{17\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**2/2 - (25*x**2 + 75)/(8*x**4 + 16*x**2 + 24) - 17*log(x**4 + 2*x**2 +
3)/4 - 17*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

Giac [A] time = 1.1123, size = 73, normalized size = 1.12

$$\frac{5}{2}x^2 - \frac{17}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right) - \frac{25\left(x^2+3\right)}{8\left(x^4+2x^2+3\right)} - \frac{17}{4}\log\left(x^4+2x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/2*x^2 - 17/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x^2 + 3)/(x^4
 + 2*x^2 + 3) - 17/4*log(x^4 + 2*x^2 + 3)
```

3.104 $\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$

Optimal. Leaf size=58

$$\frac{25(x^2+1)}{8(x^4+2x^2+3)}+\frac{5}{4}\log\left(x^4+2x^2+3\right)-\frac{23\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] (25*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) - (23*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sq
rt[2]) + (5*Log[3 + 2*x^2 + x^4])/4
```

Rubi [A] time = 0.0669888, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1663, 1660, 634, 618, 204, 628}

$$\frac{25(x^2+1)}{8(x^4+2x^2+3)}+\frac{5}{4}\log\left(x^4+2x^2+3\right)-\frac{23\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (25*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) - (23*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sq
rt[2]) + (5*Log[3 + 2*x^2 + x^4])/4
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{4+x+3x^2+5x^3}{\left(3+2x+x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{25\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\operatorname{Subst}\left(\int \frac{-6+40x}{3+2x+x^2}\,dx,x,x^2\right)\\
&= \frac{25\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{5}{4}\operatorname{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx,x,x^2\right)-\frac{23}{8}\operatorname{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx\right.\\
&= \frac{25\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{5}{4}\log\left(3+2x^2+x^4\right)+\frac{23}{4}\operatorname{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2\left(1+x^2\right)\right)\\
&= \frac{25\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{23\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{5}{4}\log\left(3+2x^2+x^4\right)
\end{aligned}
$$

Mathematica [A] time = 0.0223489, size = 58, normalized size = 1.

$$
\frac{25\left(x^2+1\right)}{8\left(x^4+2x^2+3\right)}+\frac{5}{4}\log\left(x^4+2x^2+3\right)-\frac{23\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (25*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) - (23*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sq
rt[2]) + (5*Log[3 + 2*x^2 + x^4])/4
```

Maple [A] time = 0.01, size = 54, normalized size = 0.9

$$
\frac{1}{2x^4+4x^2+6}\left(\frac{25x^2}{4}+\frac{25}{4}\right)+\frac{5\ln\left(x^4+2x^2+3\right)}{4}-\frac{23\sqrt{2}}{16}\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 1/2*(25/4*x^2+25/4)/(x^4+2*x^2+3)+5/4*ln(x^4+2*x^2+3)-23/16*2^(1/2)*arctan(
1/4*(2*x^2+2)*2^(1/2))
```

Maxima [A] time = 1.47022, size = 66, normalized size = 1.14

$$-\frac{23}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{25\left(x^2+1\right)}{8\left(x^4+2\,x^2+3\right)}+\frac{5}{4}\log\left(x^4+2\,x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -23/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 25/8*(x^2 + 1)/(x^4 + 2*x^2
+ 3) + 5/4*log(x^4 + 2*x^2 + 3)
```

Fricas [A] time = 1.52721, size = 194, normalized size = 3.34

$$-\frac{23\sqrt{2}\left(x^4+2\,x^2+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-50\,x^2-20\left(x^4+2\,x^2+3\right)\log\left(x^4+2\,x^2+3\right)-50}{16\left(x^4+2\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(23*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 50*x^2
- 20*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) - 50)/(x^4 + 2*x^2 + 3)
```

Sympy [A] time = 0.161755, size = 60, normalized size = 1.03

$$\frac{25x^2+25}{8x^4+16x^2+24}+\frac{5\log\left(x^4+2x^2+3\right)}{4}-\frac{23\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] (25*x**2 + 25)/(8*x**4 + 16*x**2 + 24) + 5*log(x**4 + 2*x**2 + 3)/4 - 23*sq
rt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

Giac [A] time = 1.10919, size = 66, normalized size = 1.14

$$-\frac{23}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{25\left(x^2+1\right)}{8\left(x^4+2\,x^2+3\right)}+\frac{5}{4}\log\left(x^4+2\,x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] -23/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 25/8*(x^2 + 1)/(x^4 + 2*x^2
+ 3) + 5/4*log(x^4 + 2*x^2 + 3)
```

3.105 $\int \frac{4+x^2+3x^4+5x^6}{x(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=66

$$\frac{25(1-x^2)}{24(x^4+2x^2+3)}-\frac{1}{9}\log\left(x^4+2x^2+3\right)+\frac{89\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{72\sqrt{2}}+\frac{4\log(x)}{9}$$

```
[Out] (25*(1 - x^2))/(24*(3 + 2*x^2 + x^4)) + (89*ArcTan[(1 + x^2)/Sqrt[2]])/(72*
Sqrt[2]) + (4*Log[x])/9 - Log[3 + 2*x^2 + x^4]/9
```

Rubi [A] time = 0.108079, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1663, 1646, 800, 634, 618, 204, 628}

$$\frac{25(1-x^2)}{24(x^4+2x^2+3)}-\frac{1}{9}\log\left(x^4+2x^2+3\right)+\frac{89\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{72\sqrt{2}}+\frac{4\log(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] (25*(1 - x^2))/(24*(3 + 2*x^2 + x^4)) + (89*ArcTan[(1 + x^2)/Sqrt[2]])/(72*
Sqrt[2]) + (4*Log[x])/9 - Log[3 + 2*x^2 + x^4]/9
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{x\left(3+2x^2+x^4\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x\left(3+2x+x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{25\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{1}{16}\operatorname{Subst}\left(\int \frac{\frac{32}{3}+\frac{70x}{3}}{x\left(3+2x+x^2\right)}\,dx,x,x^2\right)\\
&= \frac{25\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{1}{16}\operatorname{Subst}\left(\int \left(\frac{32}{9x}-\frac{2(-73+16x)}{9\left(3+2x+x^2\right)}\right)dx,x,x^2\right)\\
&= \frac{25\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{4\log(x)}{9}-\frac{1}{72}\operatorname{Subst}\left(\int \frac{-73+16x}{3+2x+x^2}\,dx,x,x^2\right)\\
&= \frac{25\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{4\log(x)}{9}-\frac{1}{9}\operatorname{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx,x,x^2\right)+\frac{89}{72}\operatorname{Subst}\left(\int \frac{1}{3+2\ldots}\right.\\
&= \frac{25\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{4\log(x)}{9}-\frac{1}{9}\log\left(3+2x^2+x^4\right)-\frac{89}{36}\operatorname{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2\left(1+\ldots\right.\right.\\
&= \frac{25\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{89\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{72\sqrt{2}}+\frac{4\log(x)}{9}-\frac{1}{9}\log\left(3+2x^2+x^4\right)
\end{aligned}$$

Mathematica [C] time = 0.0602343, size = 93, normalized size = 1.41

$$\frac{1}{288}\left(-\frac{300\left(x^2-1\right)}{x^4+2x^2+3}-\sqrt{2}\left(16\sqrt{2}+89i\right)\log\left(x^2-i\sqrt{2}+1\right)+\sqrt{2}\left(-16\sqrt{2}+89i\right)\log\left(x^2+i\sqrt{2}+1\right)+128\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] ((-300*(-1 + x^2))/(3 + 2*x^2 + x^4) + 128*Log[x] - Sqrt[2]*(89*I + 16*Sqrt
[2])*Log[1 - I*Sqrt[2] + x^2] + Sqrt[2]*(89*I - 16*Sqrt[2])*Log[1 + I*Sqrt[
2] + x^2])/288
```

Maple [A] time = 0.01, size = 58, normalized size = 0.9

$$-\frac{1}{18x^4+36x^2+54}\left(\frac{75x^2}{4}-\frac{75}{4}\right)-\frac{\ln\left(x^4+2x^2+3\right)}{9}+\frac{89\sqrt{2}}{144}\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)+\frac{4\ln(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x)
```

```
[Out] -1/18*(75/4*x^2-75/4)/(x^4+2*x^2+3)-1/9*ln(x^4+2*x^2+3)+89/144*2^(1/2)*arct
an(1/4*(2*x^2+2)*2^(1/2))+4/9*ln(x)
```

Maxima [A] time = 1.47864, size = 74, normalized size = 1.12

$$\frac{89}{144}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-\frac{25\left(x^2-1\right)}{24\left(x^4+2x^2+3\right)}-\frac{1}{9}\log\left(x^4+2x^2+3\right)+\frac{2}{9}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 89/144*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/24*(x^2 - 1)/(x^4 + 2*x^2
 + 3) - 1/9*log(x^4 + 2*x^2 + 3) + 2/9*log(x^2)
```

Fricas [A] time = 1.48087, size = 238, normalized size = 3.61

$$\frac{89\sqrt{2}\left(x^4+2x^2+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-150x^2-16\left(x^4+2x^2+3\right)\log\left(x^4+2x^2+3\right)+64\left(x^4+2x^2+3\right)\log(x)}{144\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(89*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 150*x^2
 - 16*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) + 64*(x^4 + 2*x^2 + 3)*log(x)
+ 150)/(x^4 + 2*x^2 + 3)
```

Sympy [A] time = 0.173623, size = 65, normalized size = 0.98

$$-\frac{25x^2-25}{24x^4+48x^2+72}+\frac{4\log(x)}{9}-\frac{\log\left(x^4+2x^2+3\right)}{9}+\frac{89\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x/(x**4+2*x**2+3)**2,x)
```

```
[Out] -(25*x**2 - 25)/(24*x**4 + 48*x**2 + 72) + 4*log(x)/9 - log(x**4 + 2*x**2 +
 3)/9 + 89*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/144
```

Giac [A] time = 1.07984, size = 84, normalized size = 1.27

$$\frac{89}{144}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)+\frac{8\,x^4-59\,x^2+99}{72\left(x^4+2\,x^2+3\right)}-\frac{1}{9}\,\log\left(x^4+2\,x^2+3\right)+\frac{2}{9}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 89/144*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/72*(8*x^4 - 59*x^2 + 99)/(
x^4 + 2*x^2 + 3) - 1/9*log(x^4 + 2*x^2 + 3) + 2/9*log(x^2)
```

3.106 $\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2} dx$

Optimal. Leaf size=71

$$-\frac{25\left(x^2+5\right)}{72\left(x^4+2x^2+3\right)}-\frac{2}{9x^2}+\frac{13}{108}\log\left(x^4+2x^2+3\right)-\frac{71\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}}-\frac{13\log(x)}{27}$$

```
[Out] -2/(9*x^2) - (25*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) - (71*ArcTan[(1 + x^2)/S
qrt[2]])/(216*Sqrt[2]) - (13*Log[x])/27 + (13*Log[3 + 2*x^2 + x^4])/108
```

Rubi [A] time = 0.133811, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1663, 1646, 1628, 634, 618, 204, 628}

$$-\frac{25\left(x^2+5\right)}{72\left(x^4+2x^2+3\right)}-\frac{2}{9x^2}+\frac{13}{108}\log\left(x^4+2x^2+3\right)-\frac{71\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}}-\frac{13\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -2/(9*x^2) - (25*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) - (71*ArcTan[(1 + x^2)/S
qrt[2]])/(216*Sqrt[2]) - (13*Log[x])/27 + (13*Log[3 + 2*x^2 + x^4])/108
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^2\left(3+2x+x^2\right)^2}\,dx,x,x^2\right)\\
&= -\frac{25\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}+\frac{1}{16}\operatorname{Subst}\left(\int \frac{\frac{32}{3}-\frac{40x}{9}-\frac{50x^2}{9}}{x^2\left(3+2x+x^2\right)}\,dx,x,x^2\right)\\
&= -\frac{25\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}+\frac{1}{16}\operatorname{Subst}\left(\int\left(\frac{32}{9x^2}-\frac{104}{27x}+\frac{2(-19+52x)}{27\left(3+2x+x^2\right)}\right)dx,x,x^2\right)\\
&= -\frac{2}{9x^2}-\frac{25\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{13\log(x)}{27}+\frac{1}{216}\operatorname{Subst}\left(\int \frac{-19+52x}{3+2x+x^2}\,dx,x,x^2\right)\\
&= -\frac{2}{9x^2}-\frac{25\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{13\log(x)}{27}+\frac{13}{108}\operatorname{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx,x,x^2\right)-\frac{71}{216}\operatorname{Sub}\\
&= -\frac{2}{9x^2}-\frac{25\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{13\log(x)}{27}+\frac{13}{108}\log\left(3+2x^2+x^4\right)+\frac{71}{108}\operatorname{Subst}\left(\int \frac{1}{-8-x^2}\right.\\
&= -\frac{2}{9x^2}-\frac{25\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{71\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{216\sqrt{2}}-\frac{13\log(x)}{27}+\frac{13}{108}\log\left(3+2x^2+x^4\right)
\end{aligned}
$$

Mathematica [C] time = 0.0512072, size = 97, normalized size = 1.37

$$
\frac{1}{864}\left(-\frac{300\left(x^2+5\right)}{x^4+2x^2+3}-\frac{192}{x^2}+\sqrt{2}\left(52\sqrt{2}+71i\right)\log\left(x^2-i\sqrt{2}+1\right)+\sqrt{2}\left(52\sqrt{2}-71i\right)\log\left(x^2+i\sqrt{2}+1\right)-416\log(x)\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] (-192/x^2 - (300*(5 + x^2))/(3 + 2*x^2 + x^4) - 416*Log[x] + Sqrt[2]*(71*I
+ 52*Sqrt[2])*Log[1 - I*Sqrt[2] + x^2] + Sqrt[2]*(-71*I + 52*Sqrt[2])*Log[1
```

```
+ I*Sqrt[2] + x^2])/864
```

Maple [A] time = 0.014, size = 63, normalized size = 0.9

$$\frac{1}{54\,x^4+108\,x^2+162}\left(-\frac{75\,x^2}{4}-\frac{375}{4}\right)+\frac{13\,\ln\left(x^4+2\,x^2+3\right)}{108}-\frac{71\,\sqrt{2}}{432}\arctan\left(\frac{\left(2\,x^2+2\right)\sqrt{2}}{4}\right)-\frac{2}{9\,x^2}-\frac{13\,\ln(x)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x)
```

```
[Out] 1/54*(-75/4*x^2-375/4)/(x^4+2*x^2+3)+13/108*ln(x^4+2*x^2+3)-71/432*2^(1/2)*
arctan(1/4*(2*x^2+2)*2^(1/2))-2/9/x^2-13/27*ln(x)
```

Maxima [A] time = 1.45779, size = 89, normalized size = 1.25

$$-\frac{71}{432}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)-\frac{41\,x^4+157\,x^2+48}{72\left(x^6+2\,x^4+3\,x^2\right)}+\frac{13}{108}\log\left(x^4+2\,x^2+3\right)-\frac{13}{54}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -71/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/72*(41*x^4 + 157*x^2 + 48
)/(x^6 + 2*x^4 + 3*x^2) + 13/108*log(x^4 + 2*x^2 + 3) - 13/54*log(x^2)
```

Fricas [A] time = 1.61482, size = 275, normalized size = 3.87

$$-\frac{246\,x^4+71\,\sqrt{2}\left(x^6+2\,x^4+3\,x^2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)+942\,x^2-52\left(x^6+2\,x^4+3\,x^2\right)\log\left(x^4+2\,x^2+3\right)+208\left(x^6 -\right.}{432\left(x^6+2\,x^4+3\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/432*(246*x^4 + 71*sqrt(2)*(x^6 + 2*x^4 + 3*x^2)*arctan(1/2*sqrt(2)*(x^2
+ 1)) + 942*x^2 - 52*(x^6 + 2*x^4 + 3*x^2)*log(x^4 + 2*x^2 + 3) + 208*(x^6
+ 2*x^4 + 3*x^2)*log(x) + 288)/(x^6 + 2*x^4 + 3*x^2)
```

Sympy [A] time = 0.200721, size = 75, normalized size = 1.06

$$-\frac{41x^4+157x^2+48}{72x^6+144x^4+216x^2}-\frac{13\log(x)}{27}+\frac{13\log\left(x^4+2x^2+3\right)}{108}-\frac{71\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**3/(x**4+2*x**2+3)**2,x)
```

```
[Out] -(41*x**4 + 157*x**2 + 48)/(72*x**6 + 144*x**4 + 216*x**2) - 13*log(x)/27 +
 13*log(x**4 + 2*x**2 + 3)/108 - 71*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2
)/432
```

Giac [A] time = 1.09812, size = 89, normalized size = 1.25

$$-\frac{71}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-\frac{41x^4+157x^2+48}{72\left(x^6+2x^4+3x^2\right)}+\frac{13}{108}\log\left(x^4+2x^2+3\right)-\frac{13}{54}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] -71/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/72*(41*x^4 + 157*x^2 + 48
)/(x^6 + 2*x^4 + 3*x^2) + 13/108*log(x^4 + 2*x^2 + 3) - 13/54*log(x^2)
```

3.107 $\int \frac{4+x^2+3x^4+5x^6}{x^5(3+2x^2+x^4)^2} dx$

Optimal. Leaf size=80

$$\frac{25(5x^2+7)}{216(x^4+2x^2+3)}+\frac{13}{54x^2}-\frac{1}{9x^4}-\frac{13}{108}\log\left(x^4+2x^2+3\right)+\frac{125\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}}+\frac{13\log(x)}{27}$$

```
[Out] -1/(9*x^4) + 13/(54*x^2) + (25*(7 + 5*x^2))/(216*(3 + 2*x^2 + x^4)) + (125*
ArcTan[(1 + x^2)/Sqrt[2]])/(216*Sqrt[2]) + (13*Log[x])/27 - (13*Log[3 + 2*x
^2 + x^4])/108
```

Rubi [A] time = 0.136748, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1663, 1646, 1628, 634, 618, 204, 628}

$$\frac{25(5x^2+7)}{216(x^4+2x^2+3)}+\frac{13}{54x^2}-\frac{1}{9x^4}-\frac{13}{108}\log\left(x^4+2x^2+3\right)+\frac{125\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}}+\frac{13\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -1/(9*x^4) + 13/(54*x^2) + (25*(7 + 5*x^2))/(216*(3 + 2*x^2 + x^4)) + (125*
ArcTan[(1 + x^2)/Sqrt[2]])/(216*Sqrt[2]) + (13*Log[x])/27 - (13*Log[3 + 2*x
^2 + x^4])/108
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^3\left(3+2x+x^2\right)^2}\,dx, x, x^2\right)\\
&= \frac{25\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{1}{16}\operatorname{Subst}\left(\int \frac{\frac{32}{3}-\frac{40x}{9}+\frac{200x^2}{27}+\frac{250x^3}{27}}{x^3\left(3+2x+x^2\right)}\,dx, x, x^2\right)\\
&= \frac{25\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{1}{16}\operatorname{Subst}\left(\int\left(\frac{32}{9x^3}-\frac{104}{27x^2}+\frac{104}{27x}-\frac{2(-73+52x)}{27\left(3+2x+x^2\right)}\right)dx, x, x^2\right)\\
&= -\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{13\log(x)}{27}-\frac{1}{216}\operatorname{Subst}\left(\int \frac{-73+52x}{3+2x+x^2}\,dx, x, x^2\right)\\
&= -\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{13\log(x)}{27}-\frac{13}{108}\operatorname{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx, x, x^2\right)+\\
&= -\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{13\log(x)}{27}-\frac{13}{108}\log\left(3+2x^2+x^4\right)-\frac{125}{108}\operatorname{Subst}\Big(\\
&= -\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{125\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{216\sqrt{2}}+\frac{13\log(x)}{27}-\frac{13}{108}\log\left(3+2x^2+\right.
\end{aligned}$$

Mathematica [C] time = 0.0613122, size = 105, normalized size = 1.31

$$\frac{1}{864}\left(\frac{100\left(5x^2+7\right)}{x^4+2x^2+3}+\frac{208}{x^2}-\frac{96}{x^4}-\sqrt{2}\left(52\sqrt{2}+125i\right)\log\left(x^2-i\sqrt{2}+1\right)+\sqrt{2}\left(-52\sqrt{2}+125i\right)\log\left(x^2+i\sqrt{2}+1\right)+4\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] (-96/x^4 + 208/x^2 + (100*(7 + 5*x^2))/(3 + 2*x^2 + x^4) + 416*Log[x] - Sqr
t[2]*(125*I + 52*Sqrt[2])*Log[1 - I*Sqrt[2] + x^2] + Sqrt[2]*(125*I - 52*Sq
rt[2])*Log[1 + I*Sqrt[2] + x^2])/864
```

Maple [A] time = 0.012, size = 68, normalized size = 0.9

$$-\frac{1}{54\,x^4+108\,x^2+162}\left(-\frac{125\,x^2}{4}-\frac{175}{4}\right)-\frac{13\,\ln\left(x^4+2\,x^2+3\right)}{108}+\frac{125\,\sqrt{2}}{432}\arctan\left(\frac{\left(2\,x^2+2\right)\sqrt{2}}{4}\right)-\frac{1}{9\,x^4}+\frac{13}{54\,x^2}+13$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x)
```

```
[Out] -1/54*(-125/4*x^2-175/4)/(x^4+2*x^2+3)-13/108*ln(x^4+2*x^2+3)+125/432*2^(1/
2)*arctan(1/4*(2*x^2+2)*2^(1/2))-1/9/x^4+13/54/x^2+13/27*ln(x)
```

Maxima [A] time = 1.4745, size = 96, normalized size = 1.2

$$\frac{125}{432}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)+\frac{59\,x^6+85\,x^4+36\,x^2-24}{72\left(x^8+2\,x^6+3\,x^4\right)}-\frac{13}{108}\,\log\left(x^4+2\,x^2+3\right)+\frac{13}{54}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 125/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/72*(59*x^6 + 85*x^4 + 36*
x^2 - 24)/(x^8 + 2*x^6 + 3*x^4) - 13/108*log(x^4 + 2*x^2 + 3) + 13/54*log(x
^2)
```

Fricas [A] time = 1.49019, size = 289, normalized size = 3.61

$$\frac{354\,x^6+510\,x^4+125\,\sqrt{2}\left(x^8+2\,x^6+3\,x^4\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)+216\,x^2-52\left(x^8+2\,x^6+3\,x^4\right)\log\left(x^4+2\,x^2+3\right)+}{432\left(x^8+2\,x^6+3\,x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/432*(354*x^6 + 510*x^4 + 125*sqrt(2)*(x^8 + 2*x^6 + 3*x^4)*arctan(1/2*sqr
t(2)*(x^2 + 1)) + 216*x^2 - 52*(x^8 + 2*x^6 + 3*x^4)*log(x^4 + 2*x^2 + 3) +
 208*(x^8 + 2*x^6 + 3*x^4)*log(x) - 144)/(x^8 + 2*x^6 + 3*x^4)
```

Sympy [A] time = 0.214274, size = 80, normalized size = 1.

$$\frac{13\log(x)}{27}-\frac{13\log\left(x^4+2x^2+3\right)}{108}+\frac{125\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}+\frac{59x^6+85x^4+36x^2-24}{72x^8+144x^6+216x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**5/(x**4+2*x**2+3)**2,x)
```

```
[Out] 13*log(x)/27 - 13*log(x**4 + 2*x**2 + 3)/108 + 125*sqrt(2)*atan(sqrt(2)*x**
2/2 + sqrt(2)/2)/432 + (59*x**6 + 85*x**4 + 36*x**2 - 24)/(72*x**8 + 144*x*
*6 + 216*x**4)
```

Giac [A] time = 1.08324, size = 107, normalized size = 1.34

$$\frac{125}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)+\frac{26x^4+177x^2+253}{216\left(x^4+2x^2+3\right)}-\frac{39x^4-26x^2+12}{108x^4}-\frac{13}{108}\log\left(x^4+2x^2+3\right)+\frac{13}{54}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 125/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/216*(26*x^4 + 177*x^2 + 2
53)/(x^4 + 2*x^2 + 3) - 1/108*(39*x^4 - 26*x^2 + 12)/x^4 - 13/108*log(x^4 +
 2*x^2 + 3) + 13/54*log(x^2)
```

3.108 $\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=87

$$\frac{25\left(1-7x^2\right)}{648\left(x^4+2x^2+3\right)}-\frac{13}{54x^2}+\frac{13}{108x^4}-\frac{2}{27x^6}-\frac{61}{972}\log\left(x^4+2x^2+3\right)-\frac{1237\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{1944\sqrt{2}}+\frac{61\log(x)}{243}$$

```
[Out] -2/(27*x^6) + 13/(108*x^4) - 13/(54*x^2) + (25*(1 - 7*x^2))/(648*(3 + 2*x^2
 + x^4)) - (1237*ArcTan[(1 + x^2)/Sqrt[2]])/(1944*Sqrt[2]) + (61*Log[x])/24
3 - (61*Log[3 + 2*x^2 + x^4])/972
```

Rubi [A] time = 0.148986, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1663, 1646, 1628, 634, 618, 204, 628}

$$\frac{25\left(1-7x^2\right)}{648\left(x^4+2x^2+3\right)}-\frac{13}{54x^2}+\frac{13}{108x^4}-\frac{2}{27x^6}-\frac{61}{972}\log\left(x^4+2x^2+3\right)-\frac{1237\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{1944\sqrt{2}}+\frac{61\log(x)}{243}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^7*(3 + 2*x^2 + x^4)^2),x]
```


```
[Out] -2/(27*x^6) + 13/(108*x^4) - 13/(54*x^2) + (25*(1 - 7*x^2))/(648*(3 + 2*x^2
 + x^4)) - (1237*ArcTan[(1 + x^2)/Sqrt[2]])/(1944*Sqrt[2]) + (61*Log[x])/24
3 - (61*Log[3 + 2*x^2 + x^4])/972
```


Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^4\left(3+2x+x^2\right)^2}\,dx,x,x^2\right)$$

$$= \frac{25\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)} + \frac{1}{16}\,\text{Subst}\left(\int \frac{\frac{32}{3}-\frac{40x}{9}+\frac{200x^2}{27}+\frac{800x^3}{81}-\frac{350x^4}{81}}{x^4\left(3+2x+x^2\right)}\,dx,x,x^2\right)$$

$$= \frac{25\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)} + \frac{1}{16}\,\text{Subst}\left(\int \left(\frac{32}{9x^4}-\frac{104}{27x^3}+\frac{104}{27x^2}+\frac{488}{243x}-\frac{2(1481+244x)}{243\left(3+2x+x^2\right)}\right)d$$

$$= -\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}+\frac{61\log(x)}{243}-\frac{\text{Subst}\left(\int \frac{1481+244x}{3+2x+x^2}\,dx,x,x^2\right)}{1944}$$

$$= -\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}+\frac{61\log(x)}{243}-\frac{61}{972}\,\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,d\right.$$

$$= -\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}+\frac{61\log(x)}{243}-\frac{61}{972}\log\left(3+2x^2+x^4\right)+\frac{12}{97}$$

$$= -\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}-\frac{1237\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{1944\sqrt{2}}+\frac{61\log(x)}{243}-\frac{61}{972}\log$$

Mathematica [C] time = 0.0689521, size = 110, normalized size = 1.26

$$\frac{-\frac{300\left(7x^2-1\right)}{x^4+2x^2+3}-\frac{1872}{x^2}+\frac{936}{x^4}-\frac{576}{x^6}+\sqrt{2}\left(-244\sqrt{2}+1237i\right)\log\left(x^2-i\sqrt{2}+1\right)-\sqrt{2}\left(244\sqrt{2}+1237i\right)\log\left(x^2+i\sqrt{2}+1\right)+}{7776}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^7*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] (-576/x^6 + 936/x^4 - 1872/x^2 - (300*(-1 + 7*x^2))/(3 + 2*x^2 + x^4) + 195
2*Log[x] + Sqrt[2]*(1237*I - 244*Sqrt[2])*Log[1 - I*Sqrt[2] + x^2] - Sqrt[2
]*(1237*I + 244*Sqrt[2])*Log[1 + I*Sqrt[2] + x^2])/7776
```

Maple [A] time = 0.013, size = 73, normalized size = 0.8

$$-\frac{1}{486\,x^4+972\,x^2+1458}\left(\frac{525\,x^2}{4}-\frac{75}{4}\right)-\frac{61\,\ln\left(x^4+2\,x^2+3\right)}{972}-\frac{1237\,\sqrt{2}}{3888}\arctan\left(\frac{\left(2\,x^2+2\right)\sqrt{2}}{4}\right)-\frac{2}{27\,x^6}+\frac{13}{108\,x^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x)
```

```
[Out] -1/486*(525/4*x^2-75/4)/(x^4+2*x^2+3)-61/972*ln(x^4+2*x^2+3)-1237/3888*2^(1
/2)*arctan(1/4*(2*x^2+2)*2^(1/2))-2/27/x^6+13/108/x^4-13/54/x^2+61/243*ln(x
)
```

Maxima [A] time = 1.49926, size = 103, normalized size = 1.18

$$-\frac{1237}{3888}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-\frac{331\,x^8+209\,x^6+360\,x^4-138\,x^2+144}{648\left(x^{10}+2\,x^8+3\,x^6\right)}-\frac{61}{972}\log\left(x^4+2\,x^2+3\right)+\frac{61}{486}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -1237/3888*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/648*(331*x^8 + 209*x^6
 + 360*x^4 - 138*x^2 + 144)/(x^10 + 2*x^8 + 3*x^6) - 61/972*log(x^4 + 2*x^2
 + 3) + 61/486*log(x^2)
```

Fricas [A] time = 1.52743, size = 317, normalized size = 3.64

$$-\frac{1986\,x^8+1254\,x^6+2160\,x^4+1237\,\sqrt{2}\left(x^{10}+2\,x^8+3\,x^6\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-828\,x^2+244\left(x^{10}+2\,x^8+3\,x^6\right)\log}{3888\left(x^{10}+2\,x^8+3\,x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/3888*(1986*x^8 + 1254*x^6 + 2160*x^4 + 1237*sqrt(2)*(x^10 + 2*x^8 + 3*x^
6)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 828*x^2 + 244*(x^10 + 2*x^8 + 3*x^6)*log
(x^4 + 2*x^2 + 3) - 976*(x^10 + 2*x^8 + 3*x^6)*log(x) + 864)/(x^10 + 2*x^8
+ 3*x^6)
```

Sympy [A] time = 0.236714, size = 85, normalized size = 0.98

$$\frac{61\log(x)}{243}-\frac{61\log\left(x^4+2x^2+3\right)}{972}-\frac{1237\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{3888}-\frac{331x^8+209x^6+360x^4-138x^2+144}{648x^{10}+1296x^8+1944x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**7/(x**4+2*x**2+3)**2,x)
```

```
[Out] 61*log(x)/243 - 61*log(x**4 + 2*x**2 + 3)/972 - 1237*sqrt(2)*atan(sqrt(2)*x
**2/2 + sqrt(2)/2)/3888 - (331*x**8 + 209*x**6 + 360*x**4 - 138*x**2 + 144)
/(648*x**10 + 1296*x**8 + 1944*x**6)
```

Giac [A] time = 1.09403, size = 113, normalized size = 1.3

$$-\frac{1237}{3888}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{122x^4-281x^2+441}{1944\left(x^4+2x^2+3\right)}-\frac{671x^6+702x^4-351x^2+216}{2916x^6}-\frac{61}{972}\log\left(x^4+2x^2+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] -1237/3888*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/1944*(122*x^4 - 281*x^
2 + 441)/(x^4 + 2*x^2 + 3) - 1/2916*(671*x^6 + 702*x^4 - 351*x^2 + 216)/x^6
 - 61/972*log(x^4 + 2*x^2 + 3) + 61/486*log(x^2)
```

3.109 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$

Optimal. Leaf size=248

$$\frac{5x^7}{7} - \frac{17x^5}{5} + \frac{19x^3}{3} + \frac{25(5x^2+3)x}{8(x^4+2x^2+3)} - \frac{1}{32}\sqrt{\frac{1}{2}(618291\sqrt{3}-262771)}\log\left(x^2-\sqrt{2(\sqrt{3}-1)}x+\sqrt{3}\right) + \frac{1}{32}\sqrt{\frac{1}{2}(61829$$

```
[Out] 38*x + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7 + (25*x*(3 + 5*x^2))/(8*(3 + 2*x
^2 + x^4)) + (Sqrt[(262771 + 618291*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3
])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(262771 + 618291*Sqrt[3])/2]*
ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(-
262771 + 618291*Sqrt[3])/2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
32 + (Sqrt[(-262771 + 618291*Sqrt[3])/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3]
)]*x + x^2])/32
```

Rubi [A] time = 0.344904, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1668, 1676, 1169, 634, 618, 204, 628}

$$\frac{5x^7}{7} - \frac{17x^5}{5} + \frac{19x^3}{3} + \frac{25(5x^2+3)x}{8(x^4+2x^2+3)} - \frac{1}{32}\sqrt{\frac{1}{2}(618291\sqrt{3}-262771)}\log\left(x^2-\sqrt{2(\sqrt{3}-1)}x+\sqrt{3}\right) + \frac{1}{32}\sqrt{\frac{1}{2}(61829$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 38*x + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7 + (25*x*(3 + 5*x^2))/(8*(3 + 2*x
^2 + x^4)) + (Sqrt[(262771 + 618291*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3
])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(262771 + 618291*Sqrt[3])/2]*
ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(-
262771 + 618291*Sqrt[3])/2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
32 + (Sqrt[(-262771 + 618291*Sqrt[3])/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3]
)]*x + x^2])/32
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^8\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx = \frac{25x\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \frac{-450-1650x^2+1200x^4-336x^8+240x^{10}}{3+2x^2+x^4}\,dx$$

$$= \frac{25x\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \left(1824+912x^2-816x^4+240x^6-\frac{6\left(987+1339x^2\right)}{3+2x^2+x^4}\right)dx$$

$$= 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{8}\int \frac{987+1339x^2}{3+2x^2+x^4}\,dx$$

$$= 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{\int \frac{987\sqrt{2\left(-1+\sqrt{3}\right)}-\left(987-1339\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{16\sqrt{6\left(-1+\sqrt{3}\right)}}-\frac{\int$$

$$= 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{32}\left(1339+329\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1\right.}}$$

$$= 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{32}\sqrt{\frac{1}{2}\left(-262771+618291\sqrt{3}\right)}\log\left(\right.$$

$$= 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\tan^{-1}$$

Mathematica [C] time = 0.177918, size = 145, normalized size = 0.58

$$\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}+\frac{25\left(5x^2+3\right)x}{8\left(x^4+2x^2+3\right)}+38x-\frac{\left(1339\sqrt{2}+352i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}}-\frac{\left(1339\sqrt{2}-352i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 38*x + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7 + (25*x*(3 + 5*x^2))/(8*(3 + 2*x
^2 + x^4)) - ((352*I + 1339*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqr
t[2 - (2*I)*Sqrt[2]]) - ((-352*I + 1339*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2
]]])/(16*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [B] time = 0.105, size = 427, normalized size = 1.7

$$\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}+38x-\frac{1}{x^4+2x^2+3}\left(-\frac{125x^3}{8}-\frac{75x}{8}\right)-\frac{505\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{64}-\frac{11}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5/7*x^7-17/5*x^5+19/3*x^3+38*x-(-125/8*x^3-75/8*x)/(x^4+2*x^2+3)-505/64*ln(
x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)-11/4*ln(x^
2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-505/32/(2+2*3^(1/2))
^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2)
)*3^(1/2)-11/2/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3
^(1/2))^(1/2))*(-2+2*3^(1/2))-329/8/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3
^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)+505/64*ln(x^2+3^(1/2)+x*(-2+2*3
^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+11/4*ln(x^2+3^(1/2)+x*(-2+2*3^(
1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-505/32/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-
2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)-11/2/(2+2*3
^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*
3^(1/2))-329/8/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3
^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{5}{7}x^7-\frac{17}{5}x^5+\frac{19}{3}x^3+38x+\frac{25\left(5x^3+3x\right)}{8\left(x^4+2x^2+3\right)}-\frac{1}{8}\int\frac{1339x^2+987}{x^4+2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/7*x^7 - 17/5*x^5 + 19/3*x^3 + 38*x + 25/8*(5*x^3 + 3*x)/(x^4 + 2*x^2 + 3)
 - 1/8*integrate((1339*x^2 + 987)/(x^4 + 2*x^2 + 3), x)
```

Fricas [B] time = 1.7466, size = 2484, normalized size = 10.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/338902147590720*(242072962564800*x^11 - 668121376678848*x^9 + 56806455215
2064*x^7 + 13714240239171136*x^5 - 102773860*14158657803^(1/4)*sqrt(68699)*
sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(262771*sqrt(3) + 1854873)*arctan(1/3
14508955473231302631193738 2*sqrt(50431867201)*14158657803^(3/4)*sqrt(68699)
*sqrt(3*14158657803^(1/4)*sqrt(68699)*(1339*sqrt(3)*x - 987*x)*sqrt(262771*
sqrt(3) + 1854873) + 453886804809*x^2 + 453886804809*sqrt(3))*(329*sqrt(3)*
sqrt(2) - 1339*sqrt(2))*sqrt(262771*sqrt(3) + 1854873) - 1/2078771306904899
4*14158657803^(3/4)*sqrt(68699)*(329*sqrt(3)*sqrt(2)*x - 1339*sqrt(2)*x)*sq
rt(262771*sqrt(3) + 1854873) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 1027738
60*14158657803^(1/4)*sqrt(68699)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(262
771*sqrt(3) + 1854873)*arctan(1/314508955473231302631193738 2*sqrt(504318672
01)*14158657803^(3/4)*sqrt(68699)*sqrt(-3*14158657803^(1/4)*sqrt(68699)*(13
39*sqrt(3)*x - 987*x)*sqrt(262771*sqrt(3) + 1854873) + 453886804809*x^2 + 4
53886804809*sqrt(3))*(329*sqrt(3)*sqrt(2) - 1339*sqrt(2))*sqrt(262771*sqrt(
3) + 1854873) - 1/20787713069048994*14158657803^(3/4)*sqrt(68699)*(329*sqrt
(3)*sqrt(2)*x - 1339*sqrt(2)*x)*sqrt(262771*sqrt(3) + 1854873) - 1/2*sqrt(3
)*sqrt(2) + 1/2*sqrt(2)) + 35*14158657803^(1/4)*sqrt(68699)*(1854873*x^4 +
3709746*x^2 - 262771*sqrt(3)*(x^4 + 2*x^2 + 3) + 5564619)*sqrt(262771*sqrt(
3) + 1854873)*log(3*14158657803^(1/4)*sqrt(68699)*(1339*sqrt(3)*x - 987*x)*
sqrt(262771*sqrt(3) + 1854873) + 453886804809*x^2 + 453886804809*sqrt(3)) -
 35*14158657803^(1/4)*sqrt(68699)*(1854873*x^4 + 3709746*x^2 - 262771*sqrt(
3)*(x^4 + 2*x^2 + 3) + 5564619)*sqrt(262771*sqrt(3) + 1854873)*log(-3*14158
657803^(1/4)*sqrt(68699)*(1339*sqrt(3)*x - 987*x)*sqrt(262771*sqrt(3) + 185
4873) + 453886804809*x^2 + 453886804809*sqrt(3)) + 37491050077223400*x^3 +
41812052459005080*x)/(x^4 + 2*x^2 + 3)
```

Sympy [A] time = 0.537435, size = 71, normalized size = 0.29

$$\frac{5x^7}{7} - \frac{17x^5}{5} + \frac{19x^3}{3} + 38x + \frac{125x^3 + 75x}{8x^4 + 16x^2 + 24} + \operatorname{RootSum}\left(1048576t^4 + 538155008t^2 + 1146851282043, \left(t \mapsto t\log\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**7/7 - 17*x**5/5 + 19*x**3/3 + 38*x + (125*x**3 + 75*x)/(8*x**4 + 16*x*
*2 + 24) + RootSum(1048576*_t**4 + 538155008*_t**2 + 1146851282043, Lambda(
_t, _t*log(-16547840*_t**3/453886804809 - 11974973632*_t/453886804809 + x))
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^6 + 3x^4 + x^2 + 4\right)x^8}{\left(x^4 + 2x^2 + 3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^8/(x^4 + 2*x^2 + 3)^2, x)
```

3.110 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$

Optimal. Leaf size=237

$$x^5 - \frac{17x^3}{3} + \frac{25(3-x^2)x}{8(x^4+2x^2+3)} + \frac{3}{32}\sqrt{\frac{3}{2}(8669+5011\sqrt{3})}\log\left(x^2-\sqrt{2(\sqrt{3}-1)}x+\sqrt{3}\right) - \frac{3}{32}\sqrt{\frac{3}{2}(8669+5011\sqrt{3})}\,\mathrm{l}$$

```
[Out] 19*x - (17*x^3)/3 + x^5 + (25*x*(3 - x^2))/(8*(3 + 2*x^2 + x^4)) + (3*Sqrt[
(3*(-8669 + 5011*Sqrt[3]))/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*
(1 + Sqrt[3])]])/16 - (3*Sqrt[(3*(-8669 + 5011*Sqrt[3]))/2]*ArcTan[(Sqrt[2*
(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 + (3*Sqrt[(3*(8669 + 5011
*Sqrt[3]))/2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32 - (3*Sqrt[(
3*(8669 + 5011*Sqrt[3]))/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
32
```

Rubi [A] time = 0.293025, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1668, 1676, 1169, 634, 618, 204, 628}

$$x^5 - \frac{17x^3}{3} + \frac{25(3-x^2)x}{8(x^4+2x^2+3)} + \frac{3}{32}\sqrt{\frac{3}{2}(8669+5011\sqrt{3})}\log\left(x^2-\sqrt{2(\sqrt{3}-1)}x+\sqrt{3}\right) - \frac{3}{32}\sqrt{\frac{3}{2}(8669+5011\sqrt{3})}\,\mathrm{l}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```



```
[Out] 19*x - (17*x^3)/3 + x^5 + (25*x*(3 - x^2))/(8*(3 + 2*x^2 + x^4)) + (3*Sqrt[
(3*(-8669 + 5011*Sqrt[3]))/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*
(1 + Sqrt[3])]])/16 - (3*Sqrt[(3*(-8669 + 5011*Sqrt[3]))/2]*ArcTan[(Sqrt[2*
(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 + (3*Sqrt[(3*(8669 + 5011
*Sqrt[3]))/2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32 - (3*Sqrt[(
3*(8669 + 5011*Sqrt[3]))/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
32
```


Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^6\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx = \frac{25x\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)} + \frac{1}{48}\int \frac{-450+1050x^2-336x^6+240x^8}{3+2x^2+x^4}\,dx$$

$$= \frac{25x\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)} + \frac{1}{48}\int \left(912-816x^2+240x^4-\frac{54\left(59-31x^2\right)}{3+2x^2+x^4}\right)dx$$

$$= 19x-\frac{17x^3}{3}+x^5+\frac{25x\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{9}{8}\int \frac{59-31x^2}{3+2x^2+x^4}\,dx$$

$$= 19x-\frac{17x^3}{3}+x^5+\frac{25x\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{32}\left(3\sqrt{3\left(1+\sqrt{3}\right)}\right)\int \frac{59\sqrt{2\left(-1+\sqrt{3}\right)}-\left(59+\cdots\right.}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x\cdots}$$

$$= 19x-\frac{17x^3}{3}+x^5+\frac{25x\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{16}\left(3\sqrt{\frac{3}{2}\left(3182-1829\sqrt{3}\right)}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\cdots\right.}}$$

$$= 19x-\frac{17x^3}{3}+x^5+\frac{25x\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\cdots\right.}\right.$$

$$= 19x-\frac{17x^3}{3}+x^5+\frac{25x\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{3}{16}\sqrt{\frac{3}{2}\left(-8669+5011\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}\cdots}{\sqrt{2\left(1+\sqrt{3}\cdots\right.}}\right.$$

Mathematica [C] time = 0.164887, size = 132, normalized size = 0.56

$$x^5-\frac{17x^3}{3}-\frac{25\left(x^2-3\right)x}{8\left(x^4+2x^2+3\right)}+19x+\frac{9\left(31\sqrt{2}+90i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}}+\frac{9\left(31\sqrt{2}-90i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 19*x - (17*x^3)/3 + x^5 - (25*x*(-3 + x^2))/(8*(3 + 2*x^2 + x^4)) + (9*(90*
I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]])
 + (9*(-90*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I
)*Sqrt[2]])
```

Maple [B] time = 0.026, size = 419, normalized size = 1.8

$$x^5-\frac{17\,x^3}{3}+19\,x+\frac{1}{x^4+2\,x^2+3}\left(-\frac{25\,x^3}{8}+\frac{75\,x}{8}\right)+\frac{57\,\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\,\sqrt{3}}\right)\sqrt{-2+2\,\sqrt{3}}\sqrt{3}}{16}+\frac{405\,\ln\left(x^2 -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] x^5-17/3*x^3+19*x+(-25/8*x^3+75/8*x)/(x^4+2*x^2+3)+57/16*ln(x^2+3^(1/2)-x*(
-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+405/64*ln(x^2+3^(1/2)-x*(
-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+57/8/(2+2*3^(1/2))^(1/2)*arctan((
2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+405/3
2/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)
)*(-2+2*3^(1/2))-177/8/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2)
)/(2+2*3^(1/2))^(1/2))*3^(1/2)-57/16*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))
*(-2+2*3^(1/2))^(1/2)*3^(1/2)-405/64*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))
*(-2+2*3^(1/2))^(1/2)+57/8/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(
1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+405/32/(2+2*3^(1/2))^(1/2
)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))-177
/8/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2
))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x^5-\frac{17}{3}\,x^3+19\,x-\frac{25\left(x^3-3\,x\right)}{8\left(x^4+2\,x^2+3\right)}+\frac{9}{8}\int\frac{31\,x^2-59}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] x^5 - 17/3*x^3 + 19*x - 25/8*(x^3 - 3*x)/(x^4 + 2*x^2 + 3) + 9/8*integrate(
(31*x^2 - 59)/(x^4 + 2*x^2 + 3), x)
```

Fricas [B] time = 1.66491, size = 2055, normalized size = 8.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/287671488*(287671488*x^9 - 1054795456*x^7 + 3068495872*x^5 + 3588*6779732
67^(1/4)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(-43440359*sqrt(3) + 7533036
3)*arctan(1/1822344999502852422*677973267^(3/4)*sqrt(4494867)*sqrt(4494867*
x^2 + 677973267^(1/4)*(31*sqrt(3)*x + 59*x)*sqrt(-43440359*sqrt(3) + 753303
63) + 4494867*sqrt(3))*(59*sqrt(3)*sqrt(2) + 93*sqrt(2))*sqrt(-43440359*sqr
t(3) + 75330363) - 1/405428013666*677973267^(3/4)*(59*sqrt(3)*sqrt(2)*x + 9
3*sqrt(2)*x)*sqrt(-43440359*sqrt(3) + 75330363) - 1/2*sqrt(3)*sqrt(2) + 1/2
*sqrt(2)) + 3588*677973267^(1/4)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(-43
440359*sqrt(3) + 75330363)*arctan(1/1822344999502852422*677973267^(3/4)*sqr
t(4494867)*sqrt(4494867*x^2 - 677973267^(1/4)*(31*sqrt(3)*x + 59*x)*sqrt(-4
3440359*sqrt(3) + 75330363) + 4494867*sqrt(3))*(59*sqrt(3)*sqrt(2) + 93*sqr
t(2))*sqrt(-43440359*sqrt(3) + 75330363) - 1/405428013666*677973267^(3/4)*(
59*sqrt(3)*sqrt(2)*x + 93*sqrt(2)*x)*sqrt(-43440359*sqrt(3) + 75330363) + 1
/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 5142127848*x^3 - 3*677973267^(1/4)*(150
33*x^4 + 30066*x^2 + 8669*sqrt(3)*(x^4 + 2*x^2 + 3) + 45099)*sqrt(-43440359
*sqrt(3) + 75330363)*log(4494867*x^2 + 677973267^(1/4)*(31*sqrt(3)*x + 59*x
)*sqrt(-43440359*sqrt(3) + 75330363) + 4494867*sqrt(3)) + 3*677973267^(1/4)
*(15033*x^4 + 30066*x^2 + 8669*sqrt(3)*(x^4 + 2*x^2 + 3) + 45099)*sqrt(-434
40359*sqrt(3) + 75330363)*log(4494867*x^2 - 677973267^(1/4)*(31*sqrt(3)*x +
 59*x)*sqrt(-43440359*sqrt(3) + 75330363) + 4494867*sqrt(3)) + 19094195016*
x)/(x^4 + 2*x^2 + 3)
```

Sympy [A] time = 0.541024, size = 63, normalized size = 0.27

$$x^5 - \frac{17x^3}{3} + 19x - \frac{25x^3 - 75x}{8x^4 + 16x^2 + 24} + 3\,\mathrm{RootSum}\left(1048576t^4 - 53262336t^2 + 677973267, \left(t \mapsto t\log\left(-\frac{2490368t^3}{13484601} + \right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] x**5 - 17*x**3/3 + 19*x - (25*x**3 - 75*x)/(8*x**4 + 16*x**2 + 24) + 3*Root
Sum(1048576*_t**4 - 53262336*_t**2 + 677973267, Lambda(_t, _t*log(-2490368*
_t**3/13484601 + 20518496*_t/4494867 + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^6 + 3\,x^4 + x^2 + 4\right)x^6}{\left(x^4 + 2\,x^2 + 3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^6/(x^4 + 2*x^2 + 3)^2, x)
```

3.111 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=232

$$\frac{5x^3}{3}-\frac{25\left(x^2+3\right)x}{8\left(x^4+2x^2+3\right)}-\frac{1}{32}\sqrt{\frac{1}{2}\left(26499\sqrt{3}-14395\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{2}\left(26499\sqrt{3}-14395\right)}\text{lo}$$

```
[Out] -17*x + (5*x^3)/3 - (25*x*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) - (Sqrt[(14395 +
 26499*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3
])]])/16 + (Sqrt[(14395 + 26499*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]
+ 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(-14395 + 26499*Sqrt[3])/2]*Log[S
qrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32 + (Sqrt[(-14395 + 26499*Sqrt[3
])/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32
```

Rubi [A] time = 0.292392, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1668, 1676, 1169, 634, 618, 204, 628}

$$\frac{5x^3}{3}-\frac{25\left(x^2+3\right)x}{8\left(x^4+2x^2+3\right)}-\frac{1}{32}\sqrt{\frac{1}{2}\left(26499\sqrt{3}-14395\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{2}\left(26499\sqrt{3}-14395\right)}\text{lo}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```


```
[Out] -17*x + (5*x^3)/3 - (25*x*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) - (Sqrt[(14395 +
 26499*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3
])]])/16 + (Sqrt[(14395 + 26499*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]
+ 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(-14395 + 26499*Sqrt[3])/2]*Log[S
qrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32 + (Sqrt[(-14395 + 26499*Sqrt[3
])/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32
```


Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx = -\frac{25x\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \frac{450-150x^2-336x^4+240x^6}{3+2x^2+x^4}\,dx$$

$$= -\frac{25x\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \left(-816+240x^2+\frac{6\left(483+127x^2\right)}{3+2x^2+x^4}\right)dx$$

$$= -17x+\frac{5x^3}{3}-\frac{25x\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{8}\int \frac{483+127x^2}{3+2x^2+x^4}\,dx$$

$$= -17x+\frac{5x^3}{3}-\frac{25x\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{\int \frac{483\sqrt{2\left(-1+\sqrt{3}\right)}-\left(483-127\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{16\sqrt{6\left(-1+\sqrt{3}\right)}}+\frac{\int \frac{483\sqrt{2\left(-1+\sqrt{3}\right)}+\left(483-\right.}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x}}{16\sqrt{6\left(-1+\right.}}$$

$$= -17x+\frac{5x^3}{3}-\frac{25x\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{32}\left(127+161\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx+$$

$$= -17x+\frac{5x^3}{3}-\frac{25x\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{32}\sqrt{\frac{1}{2}\left(-14395+26499\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right.}\right.$$

$$= -17x+\frac{5x^3}{3}-\frac{25x\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2}{\sqrt{2\left(1+\sqrt{3}\right)}}\right.$$

Mathematica [C] time = 0.161821, size = 129, normalized size = 0.56

$$\frac{5x^3}{3} - \frac{25\left(x^2+3\right)x}{8\left(x^4+2x^2+3\right)} - 17x + \frac{\left(127\sqrt{2}-356i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}} + \frac{\left(127\sqrt{2}+356i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] -17*x + (5*x^3)/3 - (25*x*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) + ((-356*I + 127
*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]]) + ((3
56*I + 127*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I)*Sqrt[
2]])
```

Maple [B] time = 0.022, size = 416, normalized size = 1.8

$$\frac{5\,x^3}{3} - 17\,x + \frac{1}{x^4+2\,x^2+3}\left(-\frac{25\,x^3}{8} - \frac{75\,x}{8}\right) - \frac{17\,\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\,\sqrt{3}}\right)\sqrt{-2+2\,\sqrt{3}}\sqrt{3}}{64} - \frac{89\,\ln\left(x^2+\sqrt{3}-x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5/3*x^3-17*x+(-25/8*x^3-75/8*x)/(x^4+2*x^2+3)-17/64*ln(x^2+3^(1/2)-x*(-2+2*
3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)-89/32*ln(x^2+3^(1/2)-x*(-2+2*3
^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-17/32/(2+2*3^(1/2))^(1/2)*arctan((2*x-(
-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)-89/16/(2+2
*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+
2*3^(1/2))+161/8/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2
*3^(1/2))^(1/2))*3^(1/2)+17/64*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2
*3^(1/2))^(1/2)*3^(1/2)+89/32*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*
3^(1/2))^(1/2)-17/32/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/
(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)-89/16/(2+2*3^(1/2))^(1/2)*arcta
n((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))+161/8/(2+2
*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{5}{3}\,x^3 - 17\,x - \frac{25\left(x^3+3\,x\right)}{8\left(x^4+2\,x^2+3\right)} + \frac{1}{8}\int\frac{127\,x^2+483}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 - 17*x - 25/8*(x^3 + 3*x)/(x^4 + 2*x^2 + 3) + 1/8*integrate((127*x^
2 + 483)/(x^4 + 2*x^2 + 3), x)
```

Fricas [B] time = 1.77465, size = 2102, normalized size = 9.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/1295793216*(2159655360*x^7 - 17709173952*x^5 - 123268*143883^(1/4)*sqrt(2
19)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(14395*sqrt(3) + 79497)*arctan(1/
6583502378326137 66*sqrt(24746051)*143883^(3/4)*sqrt(219)*sqrt(11*143883^(1/
4)*sqrt(219)*(127*sqrt(3)*x - 483*x)*sqrt(14395*sqrt(3) + 79497) + 22271445
9*x^2 + 222714459*sqrt(3))*(161*sqrt(3)*sqrt(2) - 127*sqrt(2))*sqrt(14395*s
qrt(3) + 79497) - 1/8868084822*143883^(3/4)*sqrt(219)*(161*sqrt(3)*sqrt(2)*
x - 127*sqrt(2)*x)*sqrt(14395*sqrt(3) + 79497) + 1/2*sqrt(3)*sqrt(2) - 1/2*
sqrt(2)) - 123268*143883^(1/4)*sqrt(219)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*
sqrt(14395*sqrt(3) + 79497)*arctan(1/658350237832613766*sqrt(24746051)*1438
83^(3/4)*sqrt(219)*sqrt(-11*143883^(1/4)*sqrt(219)*(127*sqrt(3)*x - 483*x)*
sqrt(14395*sqrt(3) + 79497) + 222714459*x^2 + 222714459*sqrt(3))*(161*sqrt(
3)*sqrt(2) - 127*sqrt(2))*sqrt(14395*sqrt(3) + 79497) - 1/8868084822*143883
^(3/4)*sqrt(219)*(161*sqrt(3)*sqrt(2)*x - 127*sqrt(2)*x)*sqrt(14395*sqrt(3)
 + 79497) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) - 143883^(1/4)*sqrt(219)*(79
497*x^4 + 158994*x^2 - 14395*sqrt(3)*(x^4 + 2*x^2 + 3) + 238491)*sqrt(14395
*sqrt(3) + 79497)*log(11*143883^(1/4)*sqrt(219)*(127*sqrt(3)*x - 483*x)*sqr
t(14395*sqrt(3) + 79497) + 222714459*x^2 + 222714459*sqrt(3)) + 143883^(1/4
)*sqrt(219)*(79497*x^4 + 158994*x^2 - 14395*sqrt(3)*(x^4 + 2*x^2 + 3) + 238
491)*sqrt(14395*sqrt(3) + 79497)*log(-11*143883^(1/4)*sqrt(219)*(127*sqrt(3
)*x - 483*x)*sqrt(14395*sqrt(3) + 79497) + 222714459*x^2 + 222714459*sqrt(3
)) - 41627357064*x^3 - 78233515416*x)/(x^4 + 2*x^2 + 3)
```

Sympy [A] time = 0.532538, size = 58, normalized size = 0.25

$$\frac{5x^3}{3} - 17x - \frac{25x^3 + 75x}{8x^4 + 16x^2 + 24} + \text{RootSum}\left(1048576t^4 + 29480960t^2 + 2106591003, \left(t \mapsto t\log\left(\frac{557056t^3}{816619683} + \frac{1666000}{8166196}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**3/3 - 17*x - (25*x**3 + 75*x)/(8*x**4 + 16*x**2 + 24) + RootSum(104857
6*_t**4 + 29480960*_t**2 + 2106591003, Lambda(_t, _t*log(557056*_t**3/81661
9683 + 166600064*_t/816619683 + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^6 + 3\,x^4 + x^2 + 4\right)x^4}{\left(x^4 + 2\,x^2 + 3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^4/(x^4 + 2*x^2 + 3)^2, x)
```

3.112 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=225

$$\frac{25\left(x^2+1\right)x}{8\left(x^4+2x^2+3\right)}-\frac{1}{32}\sqrt{\frac{1}{6}\left(12899\sqrt{3}-19291\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{6}\left(12899\sqrt{3}-19291\right)}\log\left(x^2+\right.$$

```
[Out] 5*x + (25*x*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) + (Sqrt[(19291 + 12899*Sqrt[3]
)/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sq
rt[(19291 + 12899*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*
(1 + Sqrt[3])]])/16 - (Sqrt[(-19291 + 12899*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[
2*(-1 + Sqrt[3])]*x + x^2])/32 + (Sqrt[(-19291 + 12899*Sqrt[3])/6]*Log[Sqrt
[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32
```

Rubi [A] time = 0.29714, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1668, 1676, 1169, 634, 618, 204, 628}

$$\frac{25\left(x^2+1\right)x}{8\left(x^4+2x^2+3\right)}-\frac{1}{32}\sqrt{\frac{1}{6}\left(12899\sqrt{3}-19291\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{6}\left(12899\sqrt{3}-19291\right)}\log\left(x^2+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (25*x*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) + (Sqrt[(19291 + 12899*Sqrt[3]
)/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sq
rt[(19291 + 12899*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*
(1 + Sqrt[3])]])/16 - (Sqrt[(-19291 + 12899*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[
2*(-1 + Sqrt[3])]*x + x^2])/32 + (Sqrt[(-19291 + 12899*Sqrt[3])/6]*Log[Sqrt
[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx = \frac{25x\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \frac{-150-186x^2+240x^4}{3+2x^2+x^4}\,dx$$

$$= \frac{25x\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \left(240-\frac{6\left(145+111x^2\right)}{3+2x^2+x^4}\right)dx$$

$$= 5x+\frac{25x\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{8}\int \frac{145+111x^2}{3+2x^2+x^4}\,dx$$

$$= 5x+\frac{25x\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{\int \frac{145\sqrt{2\left(-1+\sqrt{3}\right)}-\left(145-111\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{16\sqrt{6\left(-1+\sqrt{3}\right)}}-\frac{\int \frac{145\sqrt{2\left(-1+\sqrt{3}\right)}+\left(145-111\sqrt{3}\right)x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{16\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= 5x+\frac{25x\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{96}\left(333+145\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx-\frac{1}{96}\left(333\right.$$

$$= 5x+\frac{25x\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{32}\sqrt{\frac{1}{6}\left(-19291+12899\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)+$$

$$= 5x+\frac{25x\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{16}\sqrt{}$$

Mathematica [C] time = 0.170382, size = 121, normalized size = 0.54

$$\frac{25\left(x^3+x\right)}{8\left(x^4+2x^2+3\right)}+5x-\frac{\left(111\sqrt{2}-34i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}}-\frac{\left(111\sqrt{2}+34i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (25*(x + x^3))/(8*(3 + 2*x^2 + x^4)) - ((-34*I + 111*Sqrt[2])*ArcTan[
x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]]) - ((34*I + 111*Sqrt[2]
)*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [B] time = 0.018, size = 412, normalized size = 1.8

$$5x-\frac{1}{x^4+2x^2+3}\left(-\frac{25x^3}{8}-\frac{25x}{8}\right)-\frac{47\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{96}+\frac{17\ln\left(x^2+\sqrt{3}-x\sqrt{-2+\right.}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5*x-(-25/8*x^3-25/8*x)/(x^4+2*x^2+3)-47/96*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^
(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+17/64*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(
1/2))*(-2+2*3^(1/2))^(1/2)-47/48/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1
/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+17/32/(2+2*3^(1/2))
^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2)
)-145/24/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2)
)^(1/2))*3^(1/2)+47/96*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2)
)^(1/2)*3^(1/2)-17/64*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))
^(1/2)-47/48/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(
1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+17/32/(2+2*3^(1/2))^(1/2)*arctan((2*x+(
-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))-145/24/(2+2*3^(1/2
))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$5x+\frac{25\left(x^3+x\right)}{8\left(x^4+2x^2+3\right)}-\frac{1}{8}\int\frac{111x^2+145}{x^4+2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5*x + 25/8*(x^3 + x)/(x^4 + 2*x^2 + 3) - 1/8*integrate((111*x^2 + 145)/(x^4
 + 2*x^2 + 3), x)
```

Fricas [B] time = 1.75631, size = 2072, normalized size = 9.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/19736089152*(98680445760*x^5 + 31876*499152603^(1/4)*sqrt(2)*(x^4 + 2*x^2
 + 3)*sqrt(248834609*sqrt(3) + 499152603)*arctan(1/245328660180049420330 2*4
99152603^(3/4)*sqrt(308376393)*sqrt(308376393*x^2 + 499152603^(1/4)*(145*sq
rt(3)*x - 333*x)*sqrt(248834609*sqrt(3) + 499152603) + 308376393*sqrt(3))*(
111*sqrt(3)*sqrt(2) - 145*sqrt(2))*sqrt(248834609*sqrt(3) + 499152603) - 1/
7955494186614*499152603^(3/4)*(111*sqrt(3)*sqrt(2)*x - 145*sqrt(2)*x)*sqrt(
248834609*sqrt(3) + 499152603) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 31876
*499152603^(1/4)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(248834609*sqrt(3) + 4991526
03)*arctan(1/245328660180049420330 2*499152603^(3/4)*sqrt(308376393)*sqrt(30
8376393*x^2 - 499152603^(1/4)*(145*sqrt(3)*x - 333*x)*sqrt(248834609*sqrt(3
) + 499152603) + 308376393*sqrt(3))*(111*sqrt(3)*sqrt(2) - 145*sqrt(2))*sqr
t(248834609*sqrt(3) + 499152603) - 1/7955494186614*499152603^(3/4)*(111*sqr
t(3)*sqrt(2)*x - 145*sqrt(2)*x)*sqrt(248834609*sqrt(3) + 499152603) - 1/2*s
qrt(3)*sqrt(2) + 1/2*sqrt(2)) + 259036170120*x^3 + 499152603^(1/4)*(19291*x
^4 + 38582*x^2 - 12899*sqrt(3)*(x^4 + 2*x^2 + 3) + 57873)*sqrt(248834609*sq
rt(3) + 499152603)*log(308376393*x^2 + 499152603^(1/4)*(145*sqrt(3)*x - 333
*x)*sqrt(248834609*sqrt(3) + 499152603) + 308376393*sqrt(3)) - 499152603^(1
/4)*(19291*x^4 + 38582*x^2 - 12899*sqrt(3)*(x^4 + 2*x^2 + 3) + 57873)*sqrt(
248834609*sqrt(3) + 499152603)*log(308376393*x^2 - 499152603^(1/4)*(145*sqr
t(3)*x - 333*x)*sqrt(248834609*sqrt(3) + 499152603) + 308376393*sqrt(3)) +
357716615880*x)/(x^4 + 2*x^2 + 3)
```

Sympy [A] time = 0.535457, size = 51, normalized size = 0.23

$$5x + \frac{25x^3 + 25x}{8x^4 + 16x^2 + 24} + \operatorname{RootSum}\left(3145728t^4 + 39507968t^2 + 166384201, \left(t \mapsto t \log\left(-\frac{9240576t^3}{102792131} - \frac{95003488t}{102792131} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x + (25*x**3 + 25*x)/(8*x**4 + 16*x**2 + 24) + RootSum(3145728*_t**4 + 39
507968*_t**2 + 166384201, Lambda(_t, _t*log(-9240576*_t**3/102792131 - 9500
3488*_t/102792131 + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^6 + 3\,x^4 + x^2 + 4\right)x^2}{\left(x^4 + 2\,x^2 + 3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^2/(x^4 + 2*x^2 + 3)^2, x)
```

3.113 $\int \frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=224

$$\frac{25x\left(1-x^2\right)}{24\left(x^4+2x^2+3\right)}+\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(x^2\right.$$

```
[Out] (25*x*(1 - x^2))/(24*(3 + 2*x^2 + x^4)) - (Sqrt[(-11567 + 12897*Sqrt[3])/6]
*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/48 + (Sqrt[(
-11567 + 12897*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1
+ Sqrt[3])]])/48 + (Sqrt[(11567 + 12897*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-
1 + Sqrt[3])]*x + x^2])/96 - (Sqrt[(11567 + 12897*Sqrt[3])/6]*Log[Sqrt[3] +
 Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/96
```

Rubi [A] time = 0.215101, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1678, 1169, 634, 618, 204, 628}

$$\frac{25x\left(1-x^2\right)}{24\left(x^4+2x^2+3\right)}+\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(x^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^2,x]
```


```
[Out] (25*x*(1 - x^2))/(24*(3 + 2*x^2 + x^4)) - (Sqrt[(-11567 + 12897*Sqrt[3])/6]
*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/48 + (Sqrt[(
-11567 + 12897*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1
+ Sqrt[3])]])/48 + (Sqrt[(11567 + 12897*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-
1 + Sqrt[3])]*x + x^2])/96 - (Sqrt[(11567 + 12897*Sqrt[3])/6]*Log[Sqrt[3] +
 Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/96
```


Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^2}\,dx &= \frac{25x\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \frac{14+190x^2}{3+2x^2+x^4}\,dx\\
&= \frac{25x\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{\int \frac{14\sqrt{2\left(-1+\sqrt{3}\right)}-\left(14-190\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{96\sqrt{6\left(-1+\sqrt{3}\right)}}+\frac{\int \frac{14\sqrt{2\left(-1+\sqrt{3}\right)}+\left(14-190\sqrt{3}\right)x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{96\sqrt{6\left(-1+\sqrt{3}\right)}}\\
&= \frac{25x\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{\left(7-95\sqrt{3}\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{96\sqrt{6\left(-1+\sqrt{3}\right)}}+\frac{1}{288}\left(285+7\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1\right.}}\\
&= \frac{25x\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{1}{96}\sqrt{\frac{11567}{6}+\frac{4299\sqrt{3}}{2}}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)-\frac{1}{96}\sqrt{\frac{11567}{6}-}\\
&= \frac{25x\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}-\frac{1}{48}\sqrt{\frac{1}{6}\left(-11567+12897\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{48}\sqrt{\frac{1}{6}\left(-1\right.}
\end{aligned}
$$

Mathematica [C] time = 0.273742, size = 115, normalized size = 0.51

$$
\frac{1}{48}\left(-\frac{50x\left(x^2-1\right)}{x^4+2x^2+3}+\frac{\left(95+44i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{\left(95-44i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] ((-50*x*(-1 + x^2))/(3 + 2*x^2 + x^4) + ((95 + (44*I)*Sqrt[2])*ArcTan[x/Sqr
t[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((95 - (44*I)*Sqrt[2])*ArcTan[x/Sq
rt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/48
```

Maple [B] time = 0.02, size = 408, normalized size = 1.8

$$\frac{1}{x^4+2x^2+3}\left(-\frac{25x^3}{24}+\frac{25x}{24}\right)+\frac{139\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{576}+\frac{11\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right.}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] (-25/24*x^3+25/24*x)/(x^4+2*x^2+3)+139/576*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^
(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+11/48*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(
1/2))*(-2+2*3^(1/2))^(1/2)+139/288/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^
(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+11/24/(2+2*3^(1/2
))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/
2))+7/72/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2)
)^(1/2))*3^(1/2)-139/576*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/
2))^(1/2)*3^(1/2)-11/48*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2
))^(1/2)+139/288/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2
*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+11/24/(2+2*3^(1/2))^(1/2)*arctan((2
*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))+7/72/(2+2*3^(1
/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{25\left(x^3-x\right)}{24\left(x^4+2x^2+3\right)}+\frac{1}{24}\int\frac{95x^2+7}{x^4+2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -25/24*(x^3 - x)/(x^4 + 2*x^2 + 3) + 1/24*integrate((95*x^2 + 7)/(x^4 + 2*x
^2 + 3), x)
```

Fricas [B] time = 1.72396, size = 1993, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/33461214912*(54052*6160467^(1/4)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(-1491795
99*sqrt(3) + 498997827)*arctan(1/29015889224422097862*sqrt(19364129)*616046
7^(3/4)*sqrt(174277161*x^2 + 6160467^(1/4)*(7*sqrt(3)*x - 285*x)*sqrt(-1491
79599*sqrt(3) + 498997827) + 174277161*sqrt(3))*(95*sqrt(3)*sqrt(2) - 7*sqr
```

```
t(2))*sqrt(-149179599*sqrt(3) + 498997827) - 1/499478343426*6160467^(3/4)*(
95*sqrt(3)*sqrt(2)*x - 7*sqrt(2)*x)*sqrt(-149179599*sqrt(3) + 498997827) +
1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 54052*6160467^(1/4)*sqrt(2)*(x^4 + 2*x
^2 + 3)*sqrt(-149179599*sqrt(3) + 498997827)*arctan(1/29015889224422097862*
sqrt(19364129)*6160467^(3/4)*sqrt(174277161*x^2 - 6160467^(1/4)*(7*sqrt(3)*
x - 285*x)*sqrt(-149179599*sqrt(3) + 498997827) + 174277161*sqrt(3))*(95*sq
rt(3)*sqrt(2) - 7*sqrt(2))*sqrt(-149179599*sqrt(3) + 498997827) - 1/4994783
43426*6160467^(3/4)*(95*sqrt(3)*sqrt(2)*x - 7*sqrt(2)*x)*sqrt(-149179599*sq
rt(3) + 498997827) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 34855432200*x^3 -
 6160467^(1/4)*(11567*x^4 + 23134*x^2 + 12897*sqrt(3)*(x^4 + 2*x^2 + 3) + 3
4701)*sqrt(-149179599*sqrt(3) + 498997827)*log(174277161*x^2 + 6160467^(1/4
)*(7*sqrt(3)*x - 285*x)*sqrt(-149179599*sqrt(3) + 498997827) + 174277161*sq
rt(3)) + 6160467^(1/4)*(11567*x^4 + 23134*x^2 + 12897*sqrt(3)*(x^4 + 2*x^2
+ 3) + 34701)*sqrt(-149179599*sqrt(3) + 498997827)*log(174277161*x^2 - 6160
467^(1/4)*(7*sqrt(3)*x - 285*x)*sqrt(-149179599*sqrt(3) + 498997827) + 1742
77161*sqrt(3)) - 34855432200*x)/(x^4 + 2*x^2 + 3)
```

Sympy [A] time = 0.522246, size = 48, normalized size = 0.21

$$-\frac{25x^3-25x}{24x^4+48x^2+72}+\operatorname{RootSum}\left(28311552t^4-23689216t^2+18481401,\left(t\mapsto t\log\left(\frac{40992768t^3}{19364129}-\frac{48423104t}{58092387}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] -(25*x**3 - 25*x)/(24*x**4 + 48*x**2 + 72) + RootSum(28311552*_t**4 - 23689
216*_t**2 + 18481401, Lambda(_t, _t*log(40992768*_t**3/19364129 - 48423104*
_t/58092387 + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^6+3\,x^4+x^2+4}{\left(x^4+2\,x^2+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)/(x^4 + 2*x^2 + 3)^2, x)
```

3.114 $\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=229

$$-\frac{25x\left(x^2+5\right)}{72\left(x^4+2x^2+3\right)}-\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(x^2+\sqrt{2(\right.$$

```
[Out] -4/(9*x) - (25*x*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) + (Sqrt[(-965 + 699*Sqrt
[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/48 -
(Sqrt[(-965 + 699*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*
(1 + Sqrt[3])]])/48 - (Sqrt[(965 + 699*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1
 + Sqrt[3])]*x + x^2])/96 + (Sqrt[(965 + 699*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt
[2*(-1 + Sqrt[3])]*x + x^2])/96
```

Rubi [A] time = 0.310033, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1669, 1664, 1169, 634, 618, 204, 628}

$$-\frac{25x\left(x^2+5\right)}{72\left(x^4+2x^2+3\right)}-\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(x^2+\sqrt{2(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^2),x]
```


```
[Out] -4/(9*x) - (25*x*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) + (Sqrt[(-965 + 699*Sqrt
[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/48 -
(Sqrt[(-965 + 699*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*
(1 + Sqrt[3])]])/48 - (Sqrt[(965 + 699*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1
 + Sqrt[3])]*x + x^2])/96 + (Sqrt[(965 + 699*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt
[2*(-1 + Sqrt[3])]*x + x^2])/96
```


Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^2}\,dx &= -\frac{25x\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \frac{64+\frac{170x^2}{3}-\frac{50x^4}{3}}{x^2\left(3+2x^2+x^4\right)}\,dx\\
&= -\frac{25x\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \left(\frac{64}{3x^2}-\frac{2\left(-7+19x^2\right)}{3+2x^2+x^4}\right)dx\\
&= -\frac{4}{9x}-\frac{25x\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{1}{24}\int \frac{-7+19x^2}{3+2x^2+x^4}\,dx\\
&= -\frac{4}{9x}-\frac{25x\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{\int \frac{-7\sqrt{2\left(-1+\sqrt{3}\right)}-\left(-7-19\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{48\sqrt{6\left(-1+\sqrt{3}\right)}}-\frac{\int \frac{-7\sqrt{2\left(-1+\sqrt{3}\right)}+\left(-7-19\sqrt{3}\right)x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{48\sqrt{6\left(-1+\sqrt{3}\right)}}\\
&= -\frac{4}{9x}-\frac{25x\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{1}{48}\sqrt{\frac{1}{6}\left(566-133\sqrt{3}\right)}\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx-\frac{1}{48}\sqrt{\cdots}\\
&= -\frac{4}{9x}-\frac{25x\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)+\frac{1}{96}\sqrt{\cdots}\\
&= -\frac{4}{9x}-\frac{25x\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}+\frac{1}{48}\sqrt{\frac{1}{6}\left(-965+699\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{48}\sqrt{\frac{1}{6}\left(\cdots\right.}
\end{aligned}$$

Mathematica [C] time = 0.184795, size = 126, normalized size = 0.55

$$-\frac{25x\left(x^2+5\right)}{72\left(x^4+2x^2+3\right)}-\frac{4}{9x}-\frac{\left(19\sqrt{2}+26i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{48\sqrt{2-2i\sqrt{2}}}-\frac{\left(19\sqrt{2}-26i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{48\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -4/(9*x) - (25*x*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) - ((26*I + 19*Sqrt[2])*A
rcTan[x/Sqrt[1 - I*Sqrt[2]]])/(48*Sqrt[2 - (2*I)*Sqrt[2]]) - ((-26*I + 19*S
qrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(48*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [B] time = 0.023, size = 414, normalized size = 1.8

$$-\frac{1}{9x^4+18x^2+27}\left(\frac{25x^3}{8}+\frac{125x}{8}\right)-\frac{\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{18}-\frac{13\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{}}\right.}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x)
```

```
[Out] -1/9*(25/8*x^3+125/8*x)/(x^4+2*x^2+3)-1/18*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^
(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)-13/192*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^
(1/2))*(-2+2*3^(1/2))^(1/2)-1/9/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/
2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)-13/96/(2+2*3^(1/2))^
(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))
+7/72/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(
1/2))*3^(1/2)+1/18*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1
/2)*3^(1/2)+13/192*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1
/2)-1/9/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))
^(1/2))*(-2+2*3^(1/2))*3^(1/2)-13/96/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*
3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))+7/72/(2+2*3^(1/2))^(1/2
)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)-4/9/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{19x^4+63x^2+32}{24\left(x^5+2x^3+3x\right)}-\frac{1}{24}\int\frac{19x^2-7}{x^4+2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(19*x^4 + 63*x^2 + 32)/(x^5 + 2*x^3 + 3*x) - 1/24*integrate((19*x^2 -
 7)/(x^4 + 2*x^2 + 3), x)
```

Fricas [B] time = 1.70574, size = 1871, normalized size = 8.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/208156608*(164790648*x^4 - 2068*1465803^(1/4)*sqrt(2)*(x^5 + 2*x^3 + 3*x
)*sqrt(-674535*sqrt(3) + 1465803)*arctan(1/547726639257666*1465803^(3/4)*sq
rt(120461)*sqrt(1084149*x^2 + 1465803^(1/4)*(7*sqrt(3)*x + 57*x)*sqrt(-6745
35*sqrt(3) + 1465803) + 1084149*sqrt(3))*(19*sqrt(3)*sqrt(2) + 7*sqrt(2))*s
qrt(-674535*sqrt(3) + 1465803) - 1/1515640302*1465803^(3/4)*(19*sqrt(3)*sqr
t(2)*x + 7*sqrt(2)*x)*sqrt(-674535*sqrt(3) + 1465803) - 1/2*sqrt(3)*sqrt(2)
 + 1/2*sqrt(2)) - 2068*1465803^(1/4)*sqrt(2)*(x^5 + 2*x^3 + 3*x)*sqrt(-6745
35*sqrt(3) + 1465803)*arctan(1/547726639257666*1465803^(3/4)*sqrt(120461)*s
qrt(1084149*x^2 - 1465803^(1/4)*(7*sqrt(3)*x + 57*x)*sqrt(-674535*sqrt(3) +
 1465803) + 1084149*sqrt(3))*(19*sqrt(3)*sqrt(2) + 7*sqrt(2))*sqrt(-674535*
sqrt(3) + 1465803) - 1/1515640302*1465803^(3/4)*(19*sqrt(3)*sqrt(2)*x + 7*s
qrt(2)*x)*sqrt(-674535*sqrt(3) + 1465803) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(
2)) - 1465803^(1/4)*(965*x^5 + 1930*x^3 + 699*sqrt(3)*(x^5 + 2*x^3 + 3*x) +
 2895*x)*sqrt(-674535*sqrt(3) + 1465803)*log(1084149*x^2 + 1465803^(1/4)*(7
*sqrt(3)*x + 57*x)*sqrt(-674535*sqrt(3) + 1465803) + 1084149*sqrt(3)) + 146
5803^(1/4)*(965*x^5 + 1930*x^3 + 699*sqrt(3)*(x^5 + 2*x^3 + 3*x) + 2895*x)*
sqrt(-674535*sqrt(3) + 1465803)*log(1084149*x^2 - 1465803^(1/4)*(7*sqrt(3)*
x + 57*x)*sqrt(-674535*sqrt(3) + 1465803) + 1084149*sqrt(3)) + 546411096*x^
2 + 277542144)/(x^5 + 2*x^3 + 3*x)
```

Sympy [A] time = 0.550947, size = 53, normalized size = 0.23

$$-\frac{19x^4+63x^2+32}{24x^5+48x^3+72x}+\operatorname{RootSum}\left(28311552t^4-1976320t^2+54289,\left(t\mapsto t\log\left(-\frac{28311552t^3}{120461}+\frac{1103968t}{120461}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+2*x**2+3)**2,x)
```

```
[Out] -(19*x**4 + 63*x**2 + 32)/(24*x**5 + 48*x**3 + 72*x) + RootSum(28311552*_t*
*4 - 1976320*_t**2 + 54289, Lambda(_t, _t*log(-28311552*_t**3/120461 + 1103
968*_t/120461 + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^6+3\,x^4+x^2+4}{\left(x^4+2\,x^2+3\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^2), x)
```

3.115 $\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^2} \, dx$

Optimal. Leaf size=238

$$\frac{25x\left(5x^2+7\right)}{216\left(x^4+2x^2+3\right)}-\frac{4}{27x^3}+\frac{1}{864}\sqrt{\frac{1}{6}\left(56673\sqrt{3}-6073\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{864}\sqrt{\frac{1}{6}\left(56673\sqrt{3}-6073\right.}$$

```
[Out] -4/(27*x^3) + 13/(27*x) + (25*x*(7 + 5*x^2))/(216*(3 + 2*x^2 + x^4)) - (Sqr
t[(6073 + 56673*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1
 + Sqrt[3])]])/432 + (Sqrt[(6073 + 56673*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + S
qrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/432 + (Sqrt[(-6073 + 56673*Sqrt[3])
/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/864 - (Sqrt[(-6073 + 566
73*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/864
```

Rubi [A] time = 0.335361, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1669, 1664, 1169, 634, 618, 204, 628}

$$\frac{25x\left(5x^2+7\right)}{216\left(x^4+2x^2+3\right)}-\frac{4}{27x^3}+\frac{1}{864}\sqrt{\frac{1}{6}\left(56673\sqrt{3}-6073\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{864}\sqrt{\frac{1}{6}\left(56673\sqrt{3}-6073\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -4/(27*x^3) + 13/(27*x) + (25*x*(7 + 5*x^2))/(216*(3 + 2*x^2 + x^4)) - (Sqr
t[(6073 + 56673*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1
 + Sqrt[3])]])/432 + (Sqrt[(6073 + 56673*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + S
qrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/432 + (Sqrt[(-6073 + 56673*Sqrt[3])
/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/864 - (Sqrt[(-6073 + 566
73*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/864
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^2}\,dx = \frac{25x\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \frac{64-\frac{80x^2}{3}+\frac{50x^4}{9}+\frac{250x^6}{9}}{x^4\left(3+2x^2+x^4\right)}\,dx$$

$$= \frac{25x\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \left(\frac{64}{3x^4}-\frac{208}{9x^2}+\frac{2\left(137+229x^2\right)}{9\left(3+2x^2+x^4\right)}\right)dx$$

$$= -\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{1}{216}\int \frac{137+229x^2}{3+2x^2+x^4}\,dx$$

$$= -\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{\int \frac{137\sqrt{2\left(-1+\sqrt{3}\right)}-\left(137-229\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{432\sqrt{6\left(-1+\sqrt{3}\right)}}+\frac{\int \frac{137\sqrt{2\left(-1+\sqrt{3}\right)}+\left(137-\right.}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x}}{432\sqrt{6\left(-1+\right.}}$$

$$= -\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{1}{432}\sqrt{\frac{1}{6}\left(88046+31373\sqrt{3}\right)}\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x}$$

$$= -\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{1}{864}\sqrt{\frac{1}{6}\left(-6073+56673\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right.}\right.$$

$$= -\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}-\frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2}{\sqrt{2\left(1+\sqrt{3}\right)}}\right.$$

Mathematica [C] time = 0.310279, size = 131, normalized size = 0.55

$$\frac{1}{864}\left(\frac{4\left(229x^6+351x^4+248x^2-96\right)}{x^3\left(x^4+2x^2+3\right)}+\frac{2\left(229+46i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{2\left(229-46i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] ((4*(-96 + 248*x^2 + 351*x^4 + 229*x^6))/(x^3*(3 + 2*x^2 + x^4)) + (2*(229
+ (46*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (2*(
229 - (46*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/8
64
```

Maple [B] time = 0.023, size = 419, normalized size = 1.8

$$\frac{1}{27\,x^4+54\,x^2+81}\left(\frac{125\,x^3}{8}+\frac{175\,x}{8}\right)+\frac{275\,\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\,\sqrt{3}}\right)\sqrt{-2+2\,\sqrt{3}}\sqrt{3}}{5184}+\frac{23\,\ln\left(x^2+\sqrt{3}-x\sqrt{-2}\right.}{864}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x)
```

```
[Out] 1/27*(125/8*x^3+175/8*x)/(x^4+2*x^2+3)+275/5184*ln(x^2+3^(1/2)-x*(-2+2*3^(1
/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+23/864*ln(x^2+3^(1/2)-x*(-2+2*3^(1
/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+275/2592/(2+2*3^(1/2))^(1/2)*arctan((2*x-(
-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+23/432/(2+
2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2
+2*3^(1/2))+137/648/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(
2+2*3^(1/2))^(1/2))*3^(1/2)-275/5184*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))
*(-2+2*3^(1/2))^(1/2)*3^(1/2)-23/864*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))
*(-2+2*3^(1/2))^(1/2)+275/2592/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2
))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+23/432/(2+2*3^(1/2))^
(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))
+137/648/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2)
)^(1/2))*3^(1/2)-4/27/x^3+13/27/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{229\,x^6+351\,x^4+248\,x^2-96}{216\left(x^7+2\,x^5+3\,x^3\right)}+\frac{1}{216}\int\frac{229\,x^2+137}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 1/216*(229*x^6 + 351*x^4 + 248*x^2 - 96)/(x^7 + 2*x^5 + 3*x^3) + 1/216*inte
grate((229*x^2 + 137)/(x^4 + 2*x^2 + 3), x)
```

Fricas [B] time = 1.72255, size = 2205, normalized size = 9.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/2261454002496*(2397560030424*x^6 + 3674862754056*x^4 - 277108*118956627^(
1/4)*sqrt(6297)*sqrt(2)*(x^7 + 2*x^5 + 3*x^3)*sqrt(6073*sqrt(3) + 170019)*a
rctan(1/295480530439458889122*118956627^(3/4)*sqrt(81861)*sqrt(6297)*sqrt(3
*118956627^(1/4)*sqrt(6297)*(137*sqrt(3)*x - 687*x)*sqrt(6073*sqrt(3) + 170
019) + 3926135421*x^2 + 3926135421*sqrt(3))*(229*sqrt(3)*sqrt(2) - 137*sqrt
(2))*sqrt(6073*sqrt(3) + 170019) - 1/16481916497358*118956627^(3/4)*sqrt(62
97)*(229*sqrt(3)*sqrt(2)*x - 137*sqrt(2)*x)*sqrt(6073*sqrt(3) + 170019) + 1
/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 277108*118956627^(1/4)*sqrt(6297)*sqrt(
2)*(x^7 + 2*x^5 + 3*x^3)*sqrt(6073*sqrt(3) + 170019)*arctan(1/2954805304394
58889122*118956627^(3/4)*sqrt(81861)*sqrt(6297)*sqrt(-3*118956627^(1/4)*sqr
t(6297)*(137*sqrt(3)*x - 687*x)*sqrt(6073*sqrt(3) + 170019) + 3926135421*x^
2 + 3926135421*sqrt(3))*(229*sqrt(3)*sqrt(2) - 137*sqrt(2))*sqrt(6073*sqrt(
3) + 170019) - 1/16481916497358*118956627^(3/4)*sqrt(6297)*(229*sqrt(3)*sqr
t(2)*x - 137*sqrt(2)*x)*sqrt(6073*sqrt(3) + 170019) - 1/2*sqrt(3)*sqrt(2) +
 1/2*sqrt(2)) - 118956627^(1/4)*sqrt(6297)*(6073*x^7 + 12146*x^5 + 18219*x^
3 - 56673*sqrt(3)*(x^7 + 2*x^5 + 3*x^3))*sqrt(6073*sqrt(3) + 170019)*log(3*
118956627^(1/4)*sqrt(6297)*(137*sqrt(3)*x - 687*x)*sqrt(6073*sqrt(3) + 1700
19) + 3926135421*x^2 + 3926135421*sqrt(3)) + 118956627^(1/4)*sqrt(6297)*(60
73*x^7 + 12146*x^5 + 18219*x^3 - 56673*sqrt(3)*(x^7 + 2*x^5 + 3*x^3))*sqrt(
6073*sqrt(3) + 170019)*log(-3*118956627^(1/4)*sqrt(6297)*(137*sqrt(3)*x - 6
87*x)*sqrt(6073*sqrt(3) + 170019) + 3926135421*x^2 + 3926135421*sqrt(3)) +
2596484225088*x^2 - 1005090667776)/(x^7 + 2*x^5 + 3*x^3)
```

Sympy [A] time = 0.575375, size = 60, normalized size = 0.25

$$\mathrm{RootSum}\left(2293235712t^4 + 12437504t^2 + 4405801, \left(t \mapsto t\log\left(\frac{19707494400t^3}{145412423} + \frac{357152768t}{145412423} + x\right)\right)\right) + \frac{229x^6 + 351x^4}{216x^7 + 432}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+2*x**2+3)**2,x)
```

```
[Out] RootSum(2293235712*_t**4 + 12437504*_t**2 + 4405801, Lambda(_t, _t*log(1970
7494400*_t**3/145412423 + 357152768*_t/145412423 + x))) + (229*x**6 + 351*x
**4 + 248*x**2 - 96)/(216*x**7 + 432*x**5 + 648*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^6 + 3\,x^4 + x^2 + 4}{\left(x^4 + 2\,x^2 + 3\right)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^4), x)
```

3.116 $\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=245

$$\frac{25x\left(1-7x^2\right)}{648\left(x^4+2x^2+3\right)}+\frac{13}{81x^3}-\frac{4}{45x^5}-\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{2592}+\frac{\sqrt{\frac{1}{6}\left(1139381+6884\right.}}{}$$

```
[Out] -4/(45*x^5) + 13/(81*x^3) - 13/(27*x) + (25*x*(1 - 7*x^2))/(648*(3 + 2*x^2
+ x^4)) + (Sqrt[(-1139381 + 688419*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]
)] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/1296 - (Sqrt[(-1139381 + 688419*Sqrt[3])/
6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/1296 - (Sq
rt[(1139381 + 688419*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x
^2])/2592 + (Sqrt[(1139381 + 688419*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 +
Sqrt[3])]*x + x^2])/2592
```

Rubi [A] time = 0.328981, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1669, 1664, 1169, 634, 618, 204, 628}

$$\frac{25x\left(1-7x^2\right)}{648\left(x^4+2x^2+3\right)}+\frac{13}{81x^3}-\frac{4}{45x^5}-\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{2592}+\frac{\sqrt{\frac{1}{6}\left(1139381+6884\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(3 + 2*x^2 + x^4)^2),x]
```


```
[Out] -4/(45*x^5) + 13/(81*x^3) - 13/(27*x) + (25*x*(1 - 7*x^2))/(648*(3 + 2*x^2
+ x^4)) + (Sqrt[(-1139381 + 688419*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]
)] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/1296 - (Sqrt[(-1139381 + 688419*Sqrt[3])/
6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/1296 - (Sq
rt[(1139381 + 688419*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x
^2])/2592 + (Sqrt[(1139381 + 688419*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 +
Sqrt[3])]*x + x^2])/2592
```


Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
```

```
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2}dx = \frac{25x\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)} + \frac{1}{48}\int \frac{64-\frac{80x^2}{3}+\frac{400x^4}{9}+\frac{1550x^6}{27}-\frac{350x^8}{27}}{x^6\left(3+2x^2+x^4\right)}dx$$

$$= \frac{25x\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)} + \frac{1}{48}\int \left(\frac{64}{3x^6}-\frac{208}{9x^4}+\frac{208}{9x^2}-\frac{2\left(-463+487x^2\right)}{27\left(3+2x^2+x^4\right)}\right)dx$$

$$= -\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}-\frac{1}{648}\int \frac{-463+487x^2}{3+2x^2+x^4}dx$$

$$= -\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}-\frac{\int \frac{-463\sqrt{2\left(-1+\sqrt{3}\right)}-\left(-463-487\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}dx}{1296\sqrt{6\left(-1+\sqrt{3}\right)}}-\frac{\int \frac{-463\sqrt{2\left(-\right.}}{\sqrt{3}+}}{129}$$

$$= -\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}-\frac{\left(1461-463\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}dx}{7776}+\frac{\left(-1\right.}{}$$

$$= -\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}-\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1\right.}\right.}{2592}$$

$$= -\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}+\frac{\sqrt{\frac{1}{6}\left(-1139381+688419\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right.}}{\sqrt{2\left(1+\sqrt{\right.}}}\right.}{1296}$$

Mathematica [C] time = 0.303282, size = 140, normalized size = 0.57

$$\frac{-\frac{4\left(2435x^8+2475x^6+3928x^4-984x^2+864\right)}{x^5\left(x^4+2x^2+3\right)}-\frac{10i\left(475\sqrt{2}-487i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{10i\left(475\sqrt{2}+487i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{12960}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] ((-4*(864 - 984*x^2 + 3928*x^4 + 2475*x^6 + 2435*x^8))/(x^5*(3 + 2*x^2 + x^
4)) - ((10*I)*(-487*I + 475*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1
- I*Sqrt[2]] + ((10*I)*(487*I + 475*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])
/Sqrt[1 + I*Sqrt[2]])/12960
```

Maple [B] time = 0.023, size = 424, normalized size = 1.7

$$-\frac{1}{27x^4+54x^2+81}\left(\frac{175x^3}{24}-\frac{25x}{24}\right)-\frac{481\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{7776}-\frac{475\ln\left(x^2+\sqrt{3}-x\sqrt{-2+\right.}{5184}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x)
```

[Out] -1/27*(175/24*x^3-25/24*x)/(x^4+2*x^2+3)-481/7776*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)-475/5184*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-481/3888/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)-475/2592/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))+463/1944/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)+481/7776*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+475/5184*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-481/3888/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)-475/2592/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))+463/1944/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)-4/45/x^5+13/81/x^3-13/27/x

---

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2435\,x^8+2475\,x^6+3928\,x^4-984\,x^2+864}{3240\left(x^9+2\,x^7+3\,x^5\right)}-\frac{1}{648}\int\frac{487\,x^2-463}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="maxima")

[Out] -1/3240*(2435*x^8 + 2475*x^6 + 3928*x^4 - 984*x^2 + 864)/(x^9 + 2*x^7 + 3*x^5) - 1/648*integrate((487*x^2 - 463)/(x^4 + 2*x^2 + 3), x)

Fricas [B] time = 1.74124, size = 2392, normalized size = 9.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="fricas")

[Out] -1/1478473537631040*(1111136748188760*x^8 + 1129389507912600*x^6 + 1792421004881088*x^4 - 4971380*216699003^(1/4)*sqrt(2)*(x^9 + 2*x^7 + 3*x^5)*sqrt(-784371528639*sqrt(3) + 1421762158683)*arctan(1/614486622356872175645371 8*sqrt(704195977)*216699003^(3/4)*sqrt(57039874137*x^2 + 216699003^(1/4)*(463*sqrt(3)*x + 1461*x)*sqrt(-784371528639*sqrt(3) + 1421762158683) + 57039874137*sqrt(3))*(487*sqrt(3)*sqrt(2) + 463*sqrt(2))*sqrt(-784371528639*sqrt(3) + 1421762158683) - 1/969563780580726*216699003^(3/4)*(487*sqrt(3)*sqrt(2)*x + 463*sqrt(2)*x)*sqrt(-784371528639*sqrt(3) + 1421762158683) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) - 4971380*216699003^(1/4)*sqrt(2)*(x^9 + 2*x^7 + 3*x^5)*sqrt(-784371528639*sqrt(3) + 1421762158683)*arctan(1/6144866223568721756453718*sqrt(704195977)*216699003^(3/4)*sqrt(57039874137*x^2 - 216699003^(1/4)*(463*sqrt(3)*x + 1461*x)*sqrt(-784371528639*sqrt(3) + 1421762158683) + 57039874137*sqrt(3))*(487*sqrt(3)*sqrt(2) + 463*sqrt(2))*sqrt(-784371528639*sqrt(3) + 1421762158683) - 1/969563780580726*216699003^(3/4)*(487*sqrt(3)*sqrt(2)*x + 463*sqrt(2)*x)*sqrt(-784371528639*sqrt(3) + 1421762158683) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 5*216699003^(1/4)*(1139381*x^9 + 2278762*x^7 + 3418143*x^5 + 688419*sqrt(3)*(x^9 + 2*x^7 + 3*x^5))*sqrt(-784371528639*sqrt(3) + 1421762158683)*log(57039874137*x^2 + 216699003^(1/4)*(463*sqrt(3)*x + 1461*x)*sqrt(-784371528639*sqrt(3) + 1421762158683) + 57039874137*sqrt(3)) + 5*216699003^(1/4)*(1139381*x^9 + 2278762*x^7 + 3418143*x^5 + 6884

```
19*sqrt(3)*(x^9 + 2*x^7 + 3*x^5))*sqrt(-784371528639*sqrt(3) + 142176215868
3)*log(57039874137*x^2 - 216699003^(1/4)*(463*sqrt(3)*x + 1461*x)*sqrt(-784
371528639*sqrt(3) + 1421762158683) + 57039874137*sqrt(3)) - 449017889206464
*x^2 + 394259610034944)/(x^9 + 2*x^7 + 3*x^5)
```

Sympy [A] time = 0.590268, size = 65, normalized size = 0.27

$$\operatorname{RootSum}\left(20639121408t^4 - 2333452288t^2 + 72233001, \left(t \mapsto t\log\left(-\frac{206821195776t^3}{704195977} + \frac{38757503008t}{2112587931} + x\right)\right)\right) - \frac{2435}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+2*x**2+3)**2,x)
```

```
[Out] RootSum(20639121408*_t**4 - 2333452288*_t**2 + 72233001, Lambda(_t, _t*log(
-206821195776*_t**3/704195977 + 38757503008*_t/2112587931 + x))) - (2435*x*
*8 + 2475*x**6 + 3928*x**4 - 984*x**2 + 864)/(3240*x**9 + 6480*x**7 + 9720*
x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^6 + 3\,x^4 + x^2 + 4}{\left(x^4 + 2\,x^2 + 3\right)^2 x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^2*x^6), x)
```

3.117 $\int \frac{x^{10}\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=243

$$x^5-9x^3+\frac{\left(252x^2+3305\right)x}{64\left(x^4+2x^2+3\right)}-\frac{25\left(7x^2+15\right)x}{16\left(x^4+2x^2+3\right)^2}+\frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{}{5}$$

```
[Out] 58*x - 9*x^3 + x^5 - (25*x*(15 + 7*x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(330
5 + 252*x^2))/(64*(3 + 2*x^2 + x^4)) + (3*Sqrt[-8595619 + 7678611*Sqrt[3]]*
ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (3*Sqrt
[-8595619 + 7678611*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(
1 + Sqrt[3])]])/256 + (3*Sqrt[8595619 + 7678611*Sqrt[3]]*Log[Sqrt[3] - Sqrt
[2*(-1 + Sqrt[3])]*x + x^2])/512 - (3*Sqrt[8595619 + 7678611*Sqrt[3]]*Log[S
qrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512
```

Rubi [A] time = 0.359897, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {1668, 1678, 1676, 1169, 634, 618, 204, 628}

$$x^5-9x^3+\frac{\left(252x^2+3305\right)x}{64\left(x^4+2x^2+3\right)}-\frac{25\left(7x^2+15\right)x}{16\left(x^4+2x^2+3\right)^2}+\frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```


```
[Out] 58*x - 9*x^3 + x^5 - (25*x*(15 + 7*x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(330
5 + 252*x^2))/(64*(3 + 2*x^2 + x^4)) + (3*Sqrt[-8595619 + 7678611*Sqrt[3]]*
ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (3*Sqrt
[-8595619 + 7678611*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(
1 + Sqrt[3])]])/256 + (3*Sqrt[8595619 + 7678611*Sqrt[3]]*Log[Sqrt[3] - Sqrt
[2*(-1 + Sqrt[3])]*x + x^2])/512 - (3*Sqrt[8595619 + 7678611*Sqrt[3]]*Log[S
qrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512
```


Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
```

```
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{10}\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}\,dx = -\frac{25x\left(15+7x^2\right)}{16\left(3+2x^2+x^4\right)^2} + \frac{1}{96}\int \frac{2250-2850x^2-4800x^4+2400x^6-672x^{10}+480x^{12}}{\left(3+2x^2+x^4\right)^2}$$

$$= -\frac{25x\left(15+7x^2\right)}{16\left(3+2x^2+x^4\right)^2} + \frac{x\left(3305+252x^2\right)}{64\left(3+2x^2+x^4\right)} + \frac{\int \frac{-201960+193248x^2+87552x^4-78336x^6+23040x^8}{3+2x^2+x^4}}{4608}$$

$$= -\frac{25x\left(15+7x^2\right)}{16\left(3+2x^2+x^4\right)^2} + \frac{x\left(3305+252x^2\right)}{64\left(3+2x^2+x^4\right)} + \frac{\int \left(267264-124416x^2+23040x^4-\frac{216}{}\right.}{4608}$$

$$= 58x-9x^3+x^5-\frac{25x\left(15+7x^2\right)}{16\left(3+2x^2+x^4\right)^2} + \frac{x\left(3305+252x^2\right)}{64\left(3+2x^2+x^4\right)} - \frac{3}{64}\int \frac{4647-148x^2}{3+2x^2+x^4}\,dx$$

$$= 58x-9x^3+x^5-\frac{25x\left(15+7x^2\right)}{16\left(3+2x^2+x^4\right)^2} + \frac{x\left(3305+252x^2\right)}{64\left(3+2x^2+x^4\right)} - \frac{1}{256}\sqrt{3\left(1+\sqrt{3}\right)}\int \frac{464}{}$$

$$= 58x-9x^3+x^5-\frac{25x\left(15+7x^2\right)}{16\left(3+2x^2+x^4\right)^2} + \frac{x\left(3305+252x^2\right)}{64\left(3+2x^2+x^4\right)} - \frac{1}{256}\left(3\sqrt{7220107-458}\right.$$

$$= 58x-9x^3+x^5-\frac{25x\left(15+7x^2\right)}{16\left(3+2x^2+x^4\right)^2} + \frac{x\left(3305+252x^2\right)}{64\left(3+2x^2+x^4\right)} + \frac{3}{512}\sqrt{8595619+76786}$$

$$= 58x-9x^3+x^5-\frac{25x\left(15+7x^2\right)}{16\left(3+2x^2+x^4\right)^2} + \frac{x\left(3305+252x^2\right)}{64\left(3+2x^2+x^4\right)} + \frac{3}{256}\sqrt{-8595619+7678}$$

Mathematica [C] time = 0.222089, size = 156, normalized size = 0.64

$$x^5-9x^3+\frac{\left(252x^2+3305\right)x}{64\left(x^4+2x^2+3\right)}-\frac{25\left(7x^2+15\right)x}{16\left(x^4+2x^2+3\right)^2}+58x+\frac{3\left(148\sqrt{2}+4795i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{128\sqrt{2-2i\sqrt{2}}}+\frac{3\left(148\sqrt{2}-4795i\right)}{128\sqrt{2+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] 58*x - 9*x^3 + x^5 - (25*x*(15 + 7*x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(330
5 + 252*x^2))/(64*(3 + 2*x^2 + x^4)) + (3*(4795*I + 148*Sqrt[2])*ArcTan[x/S
qrt[1 - I*Sqrt[2]]])/(128*Sqrt[2 - (2*I)*Sqrt[2]]) + (3*(-4795*I + 148*Sqrt
[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(128*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [B] time = 0.02, size = 429, normalized size = 1.8

$$x^5-9\,x^3+58\,x+\frac{1}{\left(x^4+2\,x^2+3\right)^2}\left(\frac{63\,x^7}{16}+\frac{3809\,x^5}{64}+\frac{3333\,x^3}{32}+\frac{8415\,x}{64}\right)+\frac{5091\,\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\,\sqrt{3}}\right)\sqrt{-}}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

```
[Out] x^5-9*x^3+58*x+(63/16*x^7+3809/64*x^5+3333/32*x^3+8415/64*x)/(x^4+2*x^2+3)^
2+5091/1024*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(
1/2)+14385/1024*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)
+5091/512/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2
))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+14385/512/(2+2*3^(1/2))^(1/2)*arctan((2*x-
(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))-4647/64/(2+2*3^(1
/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)-5
091/1024*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2
)-14385/1024*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+50
91/512/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^
(1/2))*(-2+2*3^(1/2))*3^(1/2)+14385/512/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2
+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))-4647/64/(2+2*3^(1/2)
)^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x^5 - 9\,x^3 + 58\,x + \frac{252\,x^7 + 3809\,x^5 + 6666\,x^3 + 8415\,x}{64\left(x^8 + 4\,x^6 + 10\,x^4 + 12\,x^2 + 9\right)} + \frac{3}{64}\int \frac{148\,x^2 - 4647}{x^4 + 2\,x^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] x^5 - 9*x^3 + 58*x + 1/64*(252*x^7 + 3809*x^5 + 6666*x^3 + 8415*x)/(x^8 + 4
*x^6 + 10*x^4 + 12*x^2 + 9) + 3/64*integrate((148*x^2 - 4647)/(x^4 + 2*x^2
+ 3), x)
```

Fricas [B] time = 1.7942, size = 2853, normalized size = 11.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/18808834881088512*(18808834881088512*x^13 - 94044174405442560*x^11 + 6018
82716194832384*x^9 + 2970620359031916864*x^7 + 10166469141273357744*x^5 + 5
7410392*2183743218123^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqr
t(-66002414605209*sqrt(3) + 176883200667963)*arctan(1/863545621466021963404
537403089353*sqrt(6122667604521)*2183743218123^(3/4)*sqrt(55104008440689*x^
2 + 2183743218123^(1/4)*(148*sqrt(3)*sqrt(2)*x + 4647*sqrt(2)*x)*sqrt(-6600
2414605209*sqrt(3) + 176883200667963) + 55104008440689*sqrt(3))*(1549*sqrt(
3) + 148)*sqrt(-66002414605209*sqrt(3) + 176883200667963) - 1/4701358281741
8600331*2183743218123^(3/4)*(1549*sqrt(3)*x + 148*x)*sqrt(-66002414605209*s
qrt(3) + 176883200667963) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 57410392*2
183743218123^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-660024
14605209*sqrt(3) + 176883200667963)*arctan(1/863545621466021963404537403089
353*sqrt(6122667604521)*2183743218123^(3/4)*sqrt(55104008440689*x^2 - 21837
43218123^(1/4)*(148*sqrt(3)*sqrt(2)*x + 4647*sqrt(2)*x)*sqrt(-6600241460520
9*sqrt(3) + 176883200667963) + 55104008440689*sqrt(3))*(1549*sqrt(3) + 148)
*sqrt(-66002414605209*sqrt(3) + 176883200667963) - 1/47013582817418600331*2
183743218123^(3/4)*(1549*sqrt(3)*x + 148*x)*sqrt(-66002414605209*sqrt(3) +
176883200667963) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 1352649115995281020
8*x^3 - 2183743218123^(1/4)*(8595619*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4
```

```
+ 12*x^2 + 9) + 23035833*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(
-66002414605209*sqrt(3) + 176883200667963)*log(55104008440689*x^2 + 2183743
218123^(1/4)*(148*sqrt(3)*sqrt(2)*x + 4647*sqrt(2)*x)*sqrt(-66002414605209*
sqrt(3) + 176883200667963) + 55104008440689*sqrt(3)) + 2183743218123^(1/4)*
(8595619*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 23035833*sqr
t(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(-66002414605209*sqrt(3) + 17
6883200667963)*log(55104008440689*x^2 - 2183743218123^(1/4)*(148*sqrt(3)*sq
rt(2)*x + 4647*sqrt(2)*x)*sqrt(-66002414605209*sqrt(3) + 176883200667963) +
 55104008440689*sqrt(3)) + 12291279706746325584*x)/(x^8 + 4*x^6 + 10*x^4 +
12*x^2 + 9)
```

Sympy [A] time = 0.591164, size = 82, normalized size = 0.34

$$x^5 - 9x^3 + 58x + \frac{252x^7 + 3809x^5 + 6666x^3 + 8415x}{64x^8 + 256x^6 + 640x^4 + 768x^2 + 576} + 3\,\mathrm{RootSum}\Big(17179869184t^4 - 2253289947136t^2 + 17688$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] x**5 - 9*x**3 + 58*x + (252*x**7 + 3809*x**5 + 6666*x**3 + 8415*x)/(64*x**8
 + 256*x**6 + 640*x**4 + 768*x**2 + 576) + 3*RootSum(17179869184*_t**4 - 22
53289947136*_t**2 + 176883200667963, Lambda(_t, _t*log(-56941871104*_t**3/5
5104008440689 - 1957224667904*_t/55104008440689 + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^6 + 3\,x^4 + x^2 + 4\right)x^{10}}{\left(x^4 + 2\,x^2 + 3\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^10/(x^4 + 2*x^2 + 3)^3, x)
```

3.118 $\int \frac{x^8\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=242

$$\frac{5x^3}{3}-\frac{\left(835x^2+1468\right)x}{64\left(x^4+2x^2+3\right)}+\frac{25\left(5x^2+3\right)x}{16\left(x^4+2x^2+3\right)^2}-\frac{21}{512}\sqrt{22721\sqrt{3}-34271}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{21}{512}\sqrt{22721\sqrt{}}$$

```
[Out] -27*x + (5*x^3)/3 + (25*x*(3 + 5*x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(1468
+ 835*x^2))/(64*(3 + 2*x^2 + x^4)) - (21*Sqrt[34271 + 22721*Sqrt[3]]*ArcTan
[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (21*Sqrt[3427
1 + 22721*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3
])]])/256 - (21*Sqrt[-34271 + 22721*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqr
t[3])]*x + x^2])/512 + (21*Sqrt[-34271 + 22721*Sqrt[3]]*Log[Sqrt[3] + Sqrt[
2*(-1 + Sqrt[3])]*x + x^2])/512
```

Rubi [A] time = 0.309967, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {1668, 1678, 1676, 1169, 634, 618, 204, 628}

$$\frac{5x^3}{3}-\frac{\left(835x^2+1468\right)x}{64\left(x^4+2x^2+3\right)}+\frac{25\left(5x^2+3\right)x}{16\left(x^4+2x^2+3\right)^2}-\frac{21}{512}\sqrt{22721\sqrt{3}-34271}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{21}{512}\sqrt{22721\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] -27*x + (5*x^3)/3 + (25*x*(3 + 5*x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(1468
+ 835*x^2))/(64*(3 + 2*x^2 + x^4)) - (21*Sqrt[34271 + 22721*Sqrt[3]]*ArcTan
[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (21*Sqrt[3427
1 + 22721*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3
])]])/256 - (21*Sqrt[-34271 + 22721*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqr
t[3])]*x + x^2])/512 + (21*Sqrt[-34271 + 22721*Sqrt[3]]*Log[Sqrt[3] + Sqrt[
2*(-1 + Sqrt[3])]*x + x^2])/512
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
```

```
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^8 \left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3} dx = \frac{25x\left(3+5x^2\right)}{16\left(3+2x^2+x^4\right)^2} + \frac{1}{96}\int \frac{-450-1050x^2+2400x^4-672x^8+480x^{10}}{\left(3+2x^2+x^4\right)^2} dx$$

$$= \frac{25x\left(3+5x^2\right)}{16\left(3+2x^2+x^4\right)^2} - \frac{x\left(1468+835x^2\right)}{64\left(3+2x^2+x^4\right)} + \frac{\int \frac{98496+27432x^2-78336x^4+23040x^6}{3+2x^2+x^4} dx}{4608}$$

$$= \frac{25x\left(3+5x^2\right)}{16\left(3+2x^2+x^4\right)^2} - \frac{x\left(1468+835x^2\right)}{64\left(3+2x^2+x^4\right)} + \frac{\int \left(-124416+23040x^2+\frac{1512\left(312+137x^2\right)}{3+2x^2+x^4}\right) dx}{4608}$$

$$= -27x + \frac{5x^3}{3} + \frac{25x\left(3+5x^2\right)}{16\left(3+2x^2+x^4\right)^2} - \frac{x\left(1468+835x^2\right)}{64\left(3+2x^2+x^4\right)} + \frac{21}{64}\int \frac{312+137x^2}{3+2x^2+x^4} dx$$

$$= -27x + \frac{5x^3}{3} + \frac{25x\left(3+5x^2\right)}{16\left(3+2x^2+x^4\right)^2} - \frac{x\left(1468+835x^2\right)}{64\left(3+2x^2+x^4\right)} + \frac{1}{256}\left(7\sqrt{3}\left(1+\sqrt{3}\right)\right)\int \frac{312\sqrt{}}{}$$

$$= -27x + \frac{5x^3}{3} + \frac{25x\left(3+5x^2\right)}{16\left(3+2x^2+x^4\right)^2} - \frac{x\left(1468+835x^2\right)}{64\left(3+2x^2+x^4\right)} - \frac{1}{512}\left(21\sqrt{-34271+22721\sqrt{3}}\right)$$

$$= -27x + \frac{5x^3}{3} + \frac{25x\left(3+5x^2\right)}{16\left(3+2x^2+x^4\right)^2} - \frac{x\left(1468+835x^2\right)}{64\left(3+2x^2+x^4\right)} - \frac{21}{512}\sqrt{-34271+22721\sqrt{3}}\log\left(\right.$$

$$= -27x + \frac{5x^3}{3} + \frac{25x\left(3+5x^2\right)}{16\left(3+2x^2+x^4\right)^2} - \frac{x\left(1468+835x^2\right)}{64\left(3+2x^2+x^4\right)} - \frac{21}{256}\sqrt{34271+22721\sqrt{3}}\tan^{-1}$$

Mathematica [C] time = 0.210776, size = 155, normalized size = 0.64

$$\frac{5x^3}{3} - \frac{\left(835x^2+1468\right)x}{64\left(x^4+2x^2+3\right)} + \frac{25\left(5x^2+3\right)x}{16\left(x^4+2x^2+3\right)^2} - 27x + \frac{21\left(137\sqrt{2}-175i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{128\sqrt{2-2i\sqrt{2}}} + \frac{21\left(137\sqrt{2}+175i\right)\tan^{-1}\left(\right.}{128\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] -27*x + (5*x^3)/3 + (25*x*(3 + 5*x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(1468
+ 835*x^2))/(64*(3 + 2*x^2 + x^4)) + (21*(-175*I + 137*Sqrt[2])*ArcTan[x/Sq
rt[1 - I*Sqrt[2]]])/(128*Sqrt[2 - (2*I)*Sqrt[2]]) + (21*(175*I + 137*Sqrt[2
])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(128*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [B] time = 0.022, size = 426, normalized size = 1.8

$$\frac{5x^3}{3} - 27x + \frac{1}{\left(x^4+2x^2+3\right)^2}\left(-\frac{835x^7}{64} - \frac{1569x^5}{32} - \frac{4941x^3}{64} - \frac{513x}{8}\right) + \frac{693\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

```
[Out] 5/3*x^3-27*x+(-835/64*x^7-1569/32*x^5-4941/64*x^3-513/8*x)/(x^4+2*x^2+3)^2+
693/1024*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2
)-3675/1024*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+693
/512/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1
/2))*(-2+2*3^(1/2))*3^(1/2)-3675/512/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*
3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))+273/8/(2+2*3^(1/2))^(1/
2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)-693/1024*
ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+3675/10
24*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+693/512/(2+2
*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+
2*3^(1/2))*3^(1/2)-3675/512/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^
(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))+273/8/(2+2*3^(1/2))^(1/2)*arctan
((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{5}{3}x^3 - 27x - \frac{835x^7 + 3138x^5 + 4941x^3 + 4104x}{64\left(x^8 + 4x^6 + 10x^4 + 12x^2 + 9\right)} + \frac{21}{64}\int \frac{137x^2 + 312}{x^4 + 2x^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 - 27*x - 1/64*(835*x^7 + 3138*x^5 + 4941*x^3 + 4104*x)/(x^8 + 4*x^6
 + 10*x^4 + 12*x^2 + 9) + 21/64*integrate((137*x^2 + 312)/(x^4 + 2*x^2 + 3)
, x)
```

Fricas [B] time = 1.8097, size = 2421, normalized size = 10.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/954779317248*(1591298862080*x^11 - 19413846117376*x^9 - 99660064046704*x^
7 - 285508852710816*x^5 - 2298072*1548731523^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 1
0*x^4 + 12*x^2 + 9)*sqrt(778671391*sqrt(3) + 1548731523)*arctan(1/197530213
71716480527209*1548731523^(3/4)*sqrt(932401677)*sqrt(932401677*x^2 + 154873
1523^(1/4)*(137*sqrt(3)*sqrt(2)*x - 312*sqrt(2)*x)*sqrt(778671391*sqrt(3) +
 1548731523) + 932401677*sqrt(3))*sqrt(778671391*sqrt(3) + 1548731523)*(104
*sqrt(3) - 137) - 1/21185098503117*1548731523^(3/4)*(104*sqrt(3)*x - 137*x)
*sqrt(778671391*sqrt(3) + 1548731523) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2))
- 2298072*1548731523^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt
(778671391*sqrt(3) + 1548731523)*arctan(1/19753021371716480527209*154873152
3^(3/4)*sqrt(932401677)*sqrt(932401677*x^2 - 1548731523^(1/4)*(137*sqrt(3)*
sqrt(2)*x - 312*sqrt(2)*x)*sqrt(778671391*sqrt(3) + 1548731523) + 932401677
*sqrt(3))*sqrt(778671391*sqrt(3) + 1548731523)*(104*sqrt(3) - 137) - 1/2118
5098503117*1548731523^(3/4)*(104*sqrt(3)*x - 137*x)*sqrt(778671391*sqrt(3)
+ 1548731523) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) - 368738756006544*x^3 +
21*1548731523^(1/4)*(34271*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 +
 9) - 68163*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(778671391*sqr
t(3) + 1548731523)*log(932401677*x^2 + 1548731523^(1/4)*(137*sqrt(3)*sqrt(2
)*x - 312*sqrt(2)*x)*sqrt(778671391*sqrt(3) + 1548731523) + 932401677*sqrt(
```

```
3)) - 21*1548731523^(1/4)*(34271*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12
*x^2 + 9) - 68163*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(7786713
91*sqrt(3) + 1548731523)*log(932401677*x^2 - 1548731523^(1/4)*(137*sqrt(3)*
sqrt(2)*x - 312*sqrt(2)*x)*sqrt(778671391*sqrt(3) + 1548731523) + 932401677
*sqrt(3)) - 293236597809792*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [A] time = 0.606788, size = 80, normalized size = 0.33

$$\frac{5x^3}{3} - 27x - \frac{835x^7 + 3138x^5 + 4941x^3 + 4104x}{64x^8 + 256x^6 + 640x^4 + 768x^2 + 576} + 21\,\mathrm{RootSum}\left(17179869184t^4 + 8983937024t^2 + 1548731523, \left(t \mapsto\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] 5*x**3/3 - 27*x - (835*x**7 + 3138*x**5 + 4941*x**3 + 4104*x)/(64*x**8 + 25
6*x**6 + 640*x**4 + 768*x**2 + 576) + 21*RootSum(17179869184*_t**4 + 898393
7024*_t**2 + 1548731523, Lambda(_t, _t*log(-1107296256*_t**3/310800559 + 43
8857984*_t/310800559 + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^6 + 3\,x^4 + x^2 + 4\right)x^8}{\left(x^4 + 2\,x^2 + 3\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^8/(x^4 + 2*x^2 + 3)^3, x)
```

3.119 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx$

Optimal. Leaf size=235

$$\frac{7(58x^2+11)x}{64(x^4+2x^2+3)}+\frac{25(3-x^2)x}{16(x^4+2x^2+3)^2}-\frac{1}{512}\sqrt{1176531\sqrt{3}-827621}\log\left(x^2-\sqrt{2(\sqrt{3}-1)}x+\sqrt{3}\right)+\frac{1}{512}\sqrt{117653}$$

```
[Out] 5*x + (25*x*(3 - x^2))/(16*(3 + 2*x^2 + x^4)^2) + (7*x*(11 + 58*x^2))/(64*(
3 + 2*x^2 + x^4)) + (Sqrt[827621 + 1176531*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sq
rt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (Sqrt[827621 + 1176531*Sqrt[3]
]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (Sqrt
[-827621 + 1176531*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
512 + (Sqrt[-827621 + 1176531*Sqrt[3]]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]
*x + x^2])/512
```

Rubi [A] time = 0.300497, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {1668, 1678, 1676, 1169, 634, 618, 204, 628}

$$\frac{7(58x^2+11)x}{64(x^4+2x^2+3)}+\frac{25(3-x^2)x}{16(x^4+2x^2+3)^2}-\frac{1}{512}\sqrt{1176531\sqrt{3}-827621}\log\left(x^2-\sqrt{2(\sqrt{3}-1)}x+\sqrt{3}\right)+\frac{1}{512}\sqrt{117653}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```


```
[Out] 5*x + (25*x*(3 - x^2))/(16*(3 + 2*x^2 + x^4)^2) + (7*x*(11 + 58*x^2))/(64*(
3 + 2*x^2 + x^4)) + (Sqrt[827621 + 1176531*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sq
rt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (Sqrt[827621 + 1176531*Sqrt[3]
]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (Sqrt
[-827621 + 1176531*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
512 + (Sqrt[-827621 + 1176531*Sqrt[3]]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]
*x + x^2])/512
```


Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
```

```
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^6\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}\,dx = \frac{25x\left(3-x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{1}{96}\int \frac{-450+1650x^2-672x^6+480x^8}{\left(3+2x^2+x^4\right)^2}\,dx$$

$$= \frac{25x\left(3-x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x\left(11+58x^2\right)}{64\left(3+2x^2+x^4\right)}+\frac{\int \frac{-12744-49104x^2+23040x^4}{3+2x^2+x^4}\,dx}{4608}$$

$$= \frac{25x\left(3-x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x\left(11+58x^2\right)}{64\left(3+2x^2+x^4\right)}+\frac{\int \left(23040-\frac{72\left(1137+1322x^2\right)}{3+2x^2+x^4}\right)dx}{4608}$$

$$= 5x+\frac{25x\left(3-x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x\left(11+58x^2\right)}{64\left(3+2x^2+x^4\right)}-\frac{1}{64}\int \frac{1137+1322x^2}{3+2x^2+x^4}\,dx$$

$$= 5x+\frac{25x\left(3-x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x\left(11+58x^2\right)}{64\left(3+2x^2+x^4\right)}-\frac{\int \frac{1137\sqrt{2\left(-1+\sqrt{3}\right)}-\left(1137-1322\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{128\sqrt{6\left(-1+\sqrt{3}\right)}}-\cdots \int$$

$$= 5x+\frac{25x\left(3-x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x\left(11+58x^2\right)}{64\left(3+2x^2+x^4\right)}-\frac{1}{256}\left(1322+379\sqrt{3}\right)\int \frac{}{\sqrt{3}-\sqrt{2\left(-\cdots\right.}}$$

$$= 5x+\frac{25x\left(3-x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x\left(11+58x^2\right)}{64\left(3+2x^2+x^4\right)}-\frac{1}{512}\sqrt{-827621+1176531\sqrt{3}}\log\left(\sqrt{3}\cdots\right.$$

$$= 5x+\frac{25x\left(3-x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x\left(11+58x^2\right)}{64\left(3+2x^2+x^4\right)}+\frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\tan^{-1}\left(\cdots\right.$$

Mathematica [C] time = 0.330087, size = 138, normalized size = 0.59

$$\frac{1}{256}\left(\frac{4x\left(320x^8+1686x^6+4089x^4+5112x^2+3411\right)}{\left(x^4+2x^2+3\right)^2}-\frac{i\left(185\sqrt{2}-2644i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{i\left(185\sqrt{2}+2644i\right)\tan^{-1}\cdots}{\sqrt{1+i\sqrt{2}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] ((4*x*(3411 + 5112*x^2 + 4089*x^4 + 1686*x^6 + 320*x^8))/(3 + 2*x^2 + x^4)^
2 - (I*(-2644*I + 185*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sq
rt[2]] + (I*(2644*I + 185*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 +
I*Sqrt[2]])/256
```

Maple [B] time = 0.022, size = 422, normalized size = 1.8

$$5\,x-\frac{1}{\left(x^4+2\,x^2+3\right)^2}\left(-\frac{203\,x^7}{32}-\frac{889\,x^5}{64}-\frac{159\,x^3}{8}-\frac{531\,x}{64}\right)-\frac{943\,\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\,\sqrt{3}}\right)\sqrt{-2+2\,\sqrt{3}}\sqrt{3}}{1024}-\cdots 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

```
[Out] 5*x-(-203/32*x^7-889/64*x^5-159/8*x^3-531/64*x)/(x^4+2*x^2+3)^2-943/1024*ln
(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)-185/1024*
ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-943/512/(2+2*3^
(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3
^(1/2))*3^(1/2)-185/512/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2
))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))-379/64/(2+2*3^(1/2))^(1/2)*arctan((2
*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)+943/1024*ln(x^2+3^(1/
2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+185/1024*ln(x^2+3^(
1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-943/512/(2+2*3^(1/2))^(1/
2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^
(1/2)-185/512/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^
(1/2))^(1/2))*(-2+2*3^(1/2))-379/64/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3
^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$5\,x + \frac{406\,x^7 + 889\,x^5 + 1272\,x^3 + 531\,x}{64\left(x^8 + 4\,x^6 + 10\,x^4 + 12\,x^2 + 9\right)} - \frac{1}{64}\int \frac{1322\,x^2 + 1137}{x^4 + 2\,x^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 5*x + 1/64*(406*x^7 + 889*x^5 + 1272*x^3 + 531*x)/(x^8 + 4*x^6 + 10*x^4 + 1
2*x^2 + 9) - 1/64*integrate((1322*x^2 + 1137)/(x^4 + 2*x^2 + 3), x)
```

Fricas [B] time = 1.77273, size = 2670, normalized size = 11.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/4759173538071552*(23795867690357760*x^9 + 125374477893572448*x^7 + 304066
571830852752*x^5 - 10534088*4152675581883^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x
^4 + 12*x^2 + 9)*sqrt(973721762751*sqrt(3) + 4152675581883)*arctan(1/847120
6900375217227324302495633*4152675581883^(3/4)*sqrt(516403378697)*sqrt(46476
30408273*x^2 + 4152675581883^(1/4)*(1322*sqrt(3)*sqrt(2)*x - 1137*sqrt(2)*x
)*sqrt(973721762751*sqrt(3) + 4152675581883) + 4647630408273*sqrt(3))*sqrt(
973721762751*sqrt(3) + 4152675581883)*(379*sqrt(3) - 1322) - 1/546808125187
5840963*4152675581883^(3/4)*(379*sqrt(3)*x - 1322*x)*sqrt(973721762751*sqrt
(3) + 4152675581883) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 10534088*415267
5581883^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(973721762751
*sqrt(3) + 4152675581883)*arctan(1/8471206900375217227324302495633*41526755
81883^(3/4)*sqrt(516403378697)*sqrt(4647630408273*x^2 - 4152675581883^(1/4)
*(1322*sqrt(3)*sqrt(2)*x - 1137*sqrt(2)*x)*sqrt(973721762751*sqrt(3) + 4152
675581883) + 4647630408273*sqrt(3))*sqrt(973721762751*sqrt(3) + 41526755818
83)*(379*sqrt(3) - 1322) - 1/5468081251875840963*4152675581883^(3/4)*(379*s
qrt(3)*x - 1322*x)*sqrt(973721762751*sqrt(3) + 4152675581883) - 1/2*sqrt(3)
*sqrt(2) + 1/2*sqrt(2)) + 380138986353465216*x^3 - 4152675581883^(1/4)*(827
621*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) - 3529593*sqrt(2)*(
```

```
x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(973721762751*sqrt(3) + 41526755818
83)*log(4647630408273*x^2 + 4152675581883^(1/4)*(1322*sqrt(3)*sqrt(2)*x - 1
137*sqrt(2)*x)*sqrt(973721762751*sqrt(3) + 4152675581883) + 4647630408273*s
qrt(3)) + 4152675581883^(1/4)*(827621*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4
 + 12*x^2 + 9) - 3529593*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(
973721762751*sqrt(3) + 4152675581883)*log(4647630408273*x^2 - 4152675581883
^(1/4)*(1322*sqrt(3)*sqrt(2)*x - 1137*sqrt(2)*x)*sqrt(973721762751*sqrt(3)
+ 4152675581883) + 4647630408273*sqrt(3)) + 25364907716190724 8*x)/(x^8 + 4*
x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [A] time = 0.586753, size = 71, normalized size = 0.3

$$5x + \frac{406x^7 + 889x^5 + 1272x^3 + 531x}{64x^8 + 256x^6 + 640x^4 + 768x^2 + 576} + \mathrm{RootSum}\left(17179869184t^4 + 216955879424t^2 + 4152675581883, \left(t \mapsto\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] 5*x + (406*x**7 + 889*x**5 + 1272*x**3 + 531*x)/(64*x**8 + 256*x**6 + 640*x
**4 + 768*x**2 + 576) + RootSum(17179869184*_t**4 + 216955879424*_t**2 + 41
52675581883, Lambda(_t, _t*log(-31641829376*_t**3/1549210136091 - 455309168
896*_t/1549210136091 + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^6 + 3\,x^4 + x^2 + 4\right)x^6}{\left(x^4 + 2\,x^2 + 3\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^6/(x^4 + 2*x^2 + 3)^3, x)
```

3.120 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

Optimal. Leaf size=238

$$\frac{x(238-59x^2)}{64(x^4+2x^2+3)}-\frac{25x(x^2+3)}{16(x^4+2x^2+3)^2}+\frac{1}{512}\sqrt{3\left(48835+32827\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{512}\sqrt{3\left(48835+\right.}$$

```
[Out] (-25*x*(3 + x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(238 - 59*x^2))/(64*(3 + 2*
x^2 + x^4)) - (Sqrt[3*(-48835 + 32827*Sqrt[3])]*ArcTan[(Sqrt[2*(-1 + Sqrt[3
])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[3*(-48835 + 32827*Sqrt[3])]*
ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[3
*(48835 + 32827*Sqrt[3])]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/51
2 - (Sqrt[3*(48835 + 32827*Sqrt[3])]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x
 + x^2])/512
```

Rubi [A] time = 0.290093, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1668, 1678, 1169, 634, 618, 204, 628}

$$\frac{x(238-59x^2)}{64(x^4+2x^2+3)}-\frac{25x(x^2+3)}{16(x^4+2x^2+3)^2}+\frac{1}{512}\sqrt{3\left(48835+32827\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{512}\sqrt{3\left(48835+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] (-25*x*(3 + x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(238 - 59*x^2))/(64*(3 + 2*
x^2 + x^4)) - (Sqrt[3*(-48835 + 32827*Sqrt[3])]*ArcTan[(Sqrt[2*(-1 + Sqrt[3
])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[3*(-48835 + 32827*Sqrt[3])]*
ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[3
*(48835 + 32827*Sqrt[3])]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/51
2 - (Sqrt[3*(48835 + 32827*Sqrt[3])]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x
 + x^2])/512
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
```

```
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4 (4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^3} dx = -\frac{25x (3 + x^2)}{16 (3 + 2x^2 + x^4)^2} + \frac{1}{96} \int \frac{450 - 750x^2 - 672x^4 + 480x^6}{(3 + 2x^2 + x^4)^2} dx$$

$$= -\frac{25x (3 + x^2)}{16 (3 + 2x^2 + x^4)^2} + \frac{x (238 - 59x^2)}{64 (3 + 2x^2 + x^4)} + \frac{\int \frac{-9936+18792x^2}{3+2x^2+x^4} dx}{4608}$$

$$= -\frac{25x (3 + x^2)}{16 (3 + 2x^2 + x^4)^2} + \frac{x (238 - 59x^2)}{64 (3 + 2x^2 + x^4)} + \frac{\int \frac{-9936\sqrt{2(-1+\sqrt{3})}-(-9936-18792\sqrt{3})x}{\sqrt{3}-\sqrt{2(-1+\sqrt{3})}x+x^2} dx}{9216\sqrt{6 (-1 + \sqrt{3})}} + \frac{\int \frac{-9}{}}{}$$

$$= -\frac{25x (3 + x^2)}{16 (3 + 2x^2 + x^4)^2} + \frac{x (238 - 59x^2)}{64 (3 + 2x^2 + x^4)} + \frac{1}{256} (261 - 46\sqrt{3}) \int \frac{1}{\sqrt{3} - \sqrt{2 (-1 + \sqrt{3})}x}$$

$$= -\frac{25x (3 + x^2)}{16 (3 + 2x^2 + x^4)^2} + \frac{x (238 - 59x^2)}{64 (3 + 2x^2 + x^4)} + \frac{1}{512}\sqrt{146505 + 98481\sqrt{3}} \log \left(\sqrt{3} - \sqrt{2 (-1}\right.$$

$$= -\frac{25x (3 + x^2)}{16 (3 + 2x^2 + x^4)^2} + \frac{x (238 - 59x^2)}{64 (3 + 2x^2 + x^4)} - \frac{1}{256}\sqrt{3 (-48835 + 32827\sqrt{3})} \tan^{-1} \left(\frac{\sqrt{2 (-}}{\sqrt{2}}\right.$$

Mathematica [C] time = 0.303537, size = 129, normalized size = 0.54

$$\frac{1}{256} \left(\frac{4x (-59x^6 + 120x^4 + 199x^2 + 414)}{(x^4 + 2x^2 + 3)^2} + \frac{3 (174 + 133i\sqrt{2}) \tan^{-1} \left(\frac{x}{\sqrt{1-i\sqrt{2}}} \right)}{\sqrt{1 - i\sqrt{2}}} + \frac{3 (174 - 133i\sqrt{2}) \tan^{-1} \left(\frac{x}{\sqrt{1+i\sqrt{2}}} \right)}{\sqrt{1 + i\sqrt{2}}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] ((4*x*(414 + 199*x^2 + 120*x^4 - 59*x^6))/(3 + 2*x^2 + x^4)^2 + (3*(174 + (
133*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (3*(17
4 - (133*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/25
6
```

Maple [B] time = 0.019, size = 418, normalized size = 1.8

$$\frac{1}{(x^4 + 2x^2 + 3)^2} \left(-\frac{59 x^7}{64} + \frac{15 x^5}{8} + \frac{199 x^3}{64} + \frac{207 x}{32} \right) + \frac{307 \ln \left(x^2 + \sqrt{3} - x\sqrt{-2 + 2\sqrt{3}} \right) \sqrt{-2 + 2\sqrt{3}}\sqrt{3}}{1024} + \frac{399 \ln \left(x^2 \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

```
[Out] (-59/64*x^7+15/8*x^5+199/64*x^3+207/32*x)/(x^4+2*x^2+3)^2+307/1024*ln(x^2+3
^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+399/1024*ln(x^2
+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+307/512/(2+2*3^(1/2))
^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2)
```

```
)*3^(1/2)+399/512/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+
2*3^(1/2))^(1/2))*(-2+2*3^(1/2))-23/32/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+
2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)-307/1024*ln(x^2+3^(1/2)+x*(-
2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)-399/1024*ln(x^2+3^(1/2)+x*
(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+307/512/(2+2*3^(1/2))^(1/2)*arct
an((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+3
99/512/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^
(1/2))*(-2+2*3^(1/2))-23/32/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^
(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{59\,x^7-120\,x^5-199\,x^3-414\,x}{64\left(x^8+4\,x^6+10\,x^4+12\,x^2+9\right)}+\frac{3}{64}\int\frac{87\,x^2-46}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(59*x^7 - 120*x^5 - 199*x^3 - 414*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 +
 9) + 3/64*integrate((87*x^2 - 46)/(x^4 + 2*x^2 + 3), x)
```

Fricas [B] time = 1.8353, size = 2402, normalized size = 10.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/2076490005504*(1914264223824*x^7 - 3893418760320*x^5 + 164728*2909552208
3^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1603106545*sqrt(3
) + 3232835787)*arctan(1/121488027699636551876136 3*29095522083^(3/4)*sqrt(2
027822271)*sqrt(2027822271*x^2 + 29095522083^(1/4)*(87*sqrt(3)*sqrt(2)*x +
46*sqrt(2)*x)*sqrt(-1603106545*sqrt(3) + 3232835787) + 2027822271*sqrt(3))*
(46*sqrt(3) + 261)*sqrt(-1603106545*sqrt(3) + 3232835787) - 1/5991058952110
53*29095522083^(3/4)*(46*sqrt(3)*x + 261*x)*sqrt(-1603106545*sqrt(3) + 3232
835787) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 164728*29095522083^(1/4)*sqr
t(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1603106545*sqrt(3) + 3232835
787)*arctan(1/1214880276996365518761363*29095522083^(3/4)*sqrt(2027822271)*
sqrt(2027822271*x^2 - 29095522083^(1/4)*(87*sqrt(3)*sqrt(2)*x + 46*sqrt(2)*
x)*sqrt(-1603106545*sqrt(3) + 3232835787) + 2027822271*sqrt(3))*(46*sqrt(3)
 + 261)*sqrt(-1603106545*sqrt(3) + 3232835787) - 1/599105895211053*29095522
083^(3/4)*(46*sqrt(3)*x + 261*x)*sqrt(-1603106545*sqrt(3) + 3232835787) + 1
/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 6456586110864*x^3 + 29095522083^(1/4)*(
48835*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 98481*sqrt(2)*(
x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(-1603106545*sqrt(3) + 3232835787)*
log(2027822271*x^2 + 29095522083^(1/4)*(87*sqrt(3)*sqrt(2)*x + 46*sqrt(2)*x
)*sqrt(-1603106545*sqrt(3) + 3232835787) + 2027822271*sqrt(3)) - 2909552208
3^(1/4)*(48835*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 98481*
sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(-1603106545*sqrt(3) + 323
2835787)*log(2027822271*x^2 - 29095522083^(1/4)*(87*sqrt(3)*sqrt(2)*x + 46*
sqrt(2)*x)*sqrt(-1603106545*sqrt(3) + 3232835787) + 2027822271*sqrt(3)) - 1
3432294723104*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [A] time = 0.591851, size = 68, normalized size = 0.29

$$-\frac{59x^7 - 120x^5 - 199x^3 - 414x}{64x^8 + 256x^6 + 640x^4 + 768x^2 + 576} + \text{RootSum}\left(17179869184t^4 - 38405406720t^2 + 29095522083, \left(t \mapsto t\log\left(\frac{103}{60}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] -(59*x**7 - 120*x**5 - 199*x**3 - 414*x)/(64*x**8 + 256*x**6 + 640*x**4 + 7
68*x**2 + 576) + RootSum(17179869184*_t**4 - 38405406720*_t**2 + 2909552208
3, Lambda(_t, _t*log(10301210624*_t**3/6083466813 - 4322999552*_t/202782227
1 + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^6 + 3x^4 + x^2 + 4)x^4}{(x^4 + 2x^2 + 3)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^4/(x^4 + 2*x^2 + 3)^3, x)
```

3.121 $\int \frac{x^2\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=246

$$\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}-\frac{x\left(88x^2+353\right)}{192\left(x^4+2x^2+3\right)}-\frac{11\sqrt{\frac{1}{3}\left(1825+1089\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{1536}+\frac{11\sqrt{\frac{1}{3}\left(1825+1\right.}}{}$$

```
[Out] (25*x*(1 + x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(353 + 88*x^2))/(192*(3 + 2*
x^2 + x^4)) - (11*Sqrt[(-1825 + 1089*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[
3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/768 + (11*Sqrt[(-1825 + 1089*Sqrt[3])/3
]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/768 - (11*S
qrt[(1825 + 1089*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])
/1536 + (11*Sqrt[(1825 + 1089*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3
])]*x + x^2])/1536
```

Rubi [A] time = 0.284363, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1668, 1678, 1169, 634, 618, 204, 628}

$$\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}-\frac{x\left(88x^2+353\right)}{192\left(x^4+2x^2+3\right)}-\frac{11\sqrt{\frac{1}{3}\left(1825+1089\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{1536}+\frac{11\sqrt{\frac{1}{3}\left(1825+1\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] (25*x*(1 + x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(353 + 88*x^2))/(192*(3 + 2*
x^2 + x^4)) - (11*Sqrt[(-1825 + 1089*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[
3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/768 + (11*Sqrt[(-1825 + 1089*Sqrt[3])/3
]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/768 - (11*S
qrt[(1825 + 1089*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])
/1536 + (11*Sqrt[(1825 + 1089*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3
])]*x + x^2])/1536
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
```

```
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}\,dx = \frac{25x\left(1+x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{1}{96}\int \frac{-150+78x^2+480x^4}{\left(3+2x^2+x^4\right)^2}\,dx$$

$$= \frac{25x\left(1+x^2\right)}{16\left(3+2x^2+x^4\right)^2}-\frac{x\left(353+88x^2\right)}{192\left(3+2x^2+x^4\right)}+\frac{\int \frac{6072-2112x^2}{3+2x^2+x^4}\,dx}{4608}$$

$$= \frac{25x\left(1+x^2\right)}{16\left(3+2x^2+x^4\right)^2}-\frac{x\left(353+88x^2\right)}{192\left(3+2x^2+x^4\right)}+\frac{\int \frac{6072\sqrt{2\left(-1+\sqrt{3}\right)}-\left(6072+2112\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{9216\sqrt{6\left(-1+\sqrt{3}\right)}}+\frac{\int \frac{607}{}}{}$$

$$= \frac{25x\left(1+x^2\right)}{16\left(3+2x^2+x^4\right)^2}-\frac{x\left(353+88x^2\right)}{192\left(3+2x^2+x^4\right)}-\frac{\left(11\left(24-23\sqrt{3}\right)\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{2304}$$

$$= \frac{25x\left(1+x^2\right)}{16\left(3+2x^2+x^4\right)^2}-\frac{x\left(353+88x^2\right)}{192\left(3+2x^2+x^4\right)}-\frac{11}{768}\sqrt{\frac{1825}{12}+\frac{363\sqrt{3}}{4}}\log\left(\sqrt{3}-\sqrt{2\left(-1\right.}\right.$$

$$= \frac{25x\left(1+x^2\right)}{16\left(3+2x^2+x^4\right)^2}-\frac{x\left(353+88x^2\right)}{192\left(3+2x^2+x^4\right)}-\frac{11}{768}\sqrt{\frac{1}{3}\left(-1825+1089\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\right.}}{\sqrt{}}\right.$$

Mathematica [C] time = 0.299272, size = 133, normalized size = 0.54

$$\frac{1}{768}\left(-\frac{4x\left(88x^6+529x^4+670x^2+759\right)}{\left(x^4+2x^2+3\right)^2}-\frac{11i\left(31\sqrt{2}-16i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{11i\left(31\sqrt{2}+16i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] ((-4*x*(759 + 670*x^2 + 529*x^4 + 88*x^6))/(3 + 2*x^2 + x^4)^2 - ((11*I)*(-
16*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((1
1*I)*(16*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]]
)/768
```

Maple [B] time = 0.022, size = 418, normalized size = 1.7

$$\frac{1}{\left(x^4+2x^2+3\right)^2}\left(-\frac{11x^7}{24}-\frac{529x^5}{192}-\frac{335x^3}{96}-\frac{253x}{64}\right)-\frac{517\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{9216}-\frac{341\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

```
[Out] (-11/24*x^7-529/192*x^5-335/96*x^3-253/64*x)/(x^4+2*x^2+3)^2-517/9216*ln(x^
2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)-341/3072*ln(
```

```
x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-517/4608/(2+2*3^(1
/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(
1/2))*3^(1/2)-341/1536/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2)
)/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))+253/576/(2+2*3^(1/2))^(1/2)*arctan((2
*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)+517/9216*ln(x^2+3^(1/
2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+341/3072*ln(x^2+3^(
1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-517/4608/(2+2*3^(1/2))^(1
/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3
^(1/2)-341/1536/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*
3^(1/2))^(1/2))*(-2+2*3^(1/2))+253/576/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+
2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{88\,x^7+529\,x^5+670\,x^3+759\,x}{192\left(x^8+4\,x^6+10\,x^4+12\,x^2+9\right)}-\frac{11}{192}\int\frac{8\,x^2-23}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] -1/192*(88*x^7 + 529*x^5 + 670*x^3 + 759*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2
+ 9) - 11/192*integrate((8*x^2 - 23)/(x^4 + 2*x^2 + 3), x)
```

Fricas [B] time = 1.73407, size = 2074, normalized size = 8.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/27952128*(12811392*x^7 + 77013936*x^5 + 1348*sqrt(6)*3^(3/4)*sqrt(2)*(x^
8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1987425*sqrt(3) + 3557763)*arctan(1/
2226179538*sqrt(3707)*sqrt(6)*3^(3/4)*sqrt(sqrt(6)*3^(1/4)*(8*sqrt(3)*x + 2
3*x)*sqrt(-1987425*sqrt(3) + 3557763) + 33363*x^2 + 33363*sqrt(3))*(23*sqrt
(3)*sqrt(2) + 24*sqrt(2))*sqrt(-1987425*sqrt(3) + 3557763) - 1/200178*sqrt(
6)*3^(3/4)*(23*sqrt(3)*sqrt(2)*x + 24*sqrt(2)*x)*sqrt(-1987425*sqrt(3) + 35
57763) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 1348*sqrt(6)*3^(3/4)*sqrt(2)*
(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1987425*sqrt(3) + 3557763)*arctan
(1/2226179538*sqrt(3707)*sqrt(6)*3^(3/4)*sqrt(-sqrt(6)*3^(1/4)*(8*sqrt(3)*x
 + 23*x)*sqrt(-1987425*sqrt(3) + 3557763) + 33363*x^2 + 33363*sqrt(3))*(23*
sqrt(3)*sqrt(2) + 24*sqrt(2))*sqrt(-1987425*sqrt(3) + 3557763) - 1/200178*s
qrt(6)*3^(3/4)*(23*sqrt(3)*sqrt(2)*x + 24*sqrt(2)*x)*sqrt(-1987425*sqrt(3)
+ 3557763) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - sqrt(6)*3^(1/4)*(3267*x^8
 + 13068*x^6 + 32670*x^4 + 39204*x^2 + 1825*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 +
 12*x^2 + 9) + 29403)*sqrt(-1987425*sqrt(3) + 3557763)*log(sqrt(6)*3^(1/4)*
(8*sqrt(3)*x + 23*x)*sqrt(-1987425*sqrt(3) + 3557763) + 33363*x^2 + 33363*s
qrt(3)) + sqrt(6)*3^(1/4)*(3267*x^8 + 13068*x^6 + 32670*x^4 + 39204*x^2 + 1
825*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 29403)*sqrt(-1987425*sqrt
(3) + 3557763)*log(-sqrt(6)*3^(1/4)*(8*sqrt(3)*x + 23*x)*sqrt(-1987425*sqrt
(3) + 3557763) + 33363*x^2 + 33363*sqrt(3)) + 97541280*x^3 + 110498256*x)/(
x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [A] time = 0.592378, size = 68, normalized size = 0.28

$$-\frac{88x^7+529x^5+670x^3+759x}{192x^8+768x^6+1920x^4+2304x^2+1728}+\mathrm{RootSum}\left(463856467968t^4-57887948800t^2+1929229929,\left(t\mapsto t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] -(88*x**7 + 529*x**5 + 670*x**3 + 759*x)/(192*x**8 + 768*x**6 + 1920*x**4 +
 2304*x**2 + 1728) + RootSum(463856467968*_t**4 - 57887948800*_t**2 + 19292
29929, Lambda(_t, _t*log(14193524736*_t**3/54274187 - 17989888*_t/1345641 +
 x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5\,x^6+3\,x^4+x^2+4\right)x^2}{\left(x^4+2\,x^2+3\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)*x^2/(x^4 + 2*x^2 + 3)^3, x)
```

3.122 $\int \frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^3} \, dx$

Optimal. Leaf size=248

$$\frac{25x\left(1-x^2\right)}{48\left(x^4+2x^2+3\right)^2}+\frac{x\left(51x^2+64\right)}{192\left(x^4+2x^2+3\right)}+\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+\right.}$$

```
[Out] (25*x*(1 - x^2))/(48*(3 + 2*x^2 + x^4)^2) + (x*(64 + 51*x^2))/(192*(3 + 2*x
^2 + x^4)) - (Sqrt[(-1291 + 1019*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]
 - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[(-1291 + 1019*Sqrt[3])/3]*ArcTa
n[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[(1291
+ 1019*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512 - (Sq
rt[(1291 + 1019*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
512
```

Rubi [A] time = 0.254342, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1678, 1178, 1169, 634, 618, 204, 628}

$$\frac{25x\left(1-x^2\right)}{48\left(x^4+2x^2+3\right)^2}+\frac{x\left(51x^2+64\right)}{192\left(x^4+2x^2+3\right)}+\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^3,x]
```


```
[Out] (25*x*(1 - x^2))/(48*(3 + 2*x^2 + x^4)^2) + (x*(64 + 51*x^2))/(192*(3 + 2*x
^2 + x^4)) - (Sqrt[(-1291 + 1019*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]
 - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[(-1291 + 1019*Sqrt[3])/3]*ArcTa
n[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[(1291
+ 1019*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512 - (Sq
rt[(1291 + 1019*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
512
```


Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
```

```
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^3}\,dx = \frac{25x\left(1-x^2\right)}{48\left(3+2x^2+x^4\right)^2}+\frac{1}{96}\int \frac{78+230x^2}{\left(3+2x^2+x^4\right)^2}\,dx$$

$$= \frac{25x\left(1-x^2\right)}{48\left(3+2x^2+x^4\right)^2}+\frac{x\left(64+51x^2\right)}{192\left(3+2x^2+x^4\right)}+\frac{\int \frac{-288+1224x^2}{3+2x^2+x^4}\,dx}{4608}$$

$$= \frac{25x\left(1-x^2\right)}{48\left(3+2x^2+x^4\right)^2}+\frac{x\left(64+51x^2\right)}{192\left(3+2x^2+x^4\right)}+\frac{\int \frac{-288\sqrt{2\left(-1+\sqrt{3}\right)}-\left(-288-1224\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{9216\sqrt{6\left(-1+\sqrt{3}\right)}}+\frac{\int \frac{-288\sqrt{2(-1+}}{\sqrt{3}+}}{9216}$$

$$= \frac{25x\left(1-x^2\right)}{48\left(3+2x^2+x^4\right)^2}+\frac{x\left(64+51x^2\right)}{192\left(3+2x^2+x^4\right)}+\frac{1}{768}\left(51-4\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx$$

$$= \frac{25x\left(1-x^2\right)}{48\left(3+2x^2+x^4\right)^2}+\frac{x\left(64+51x^2\right)}{192\left(3+2x^2+x^4\right)}+\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{\right.}\right.}\right.$$

$$= \frac{25x\left(1-x^2\right)}{48\left(3+2x^2+x^4\right)^2}+\frac{x\left(64+51x^2\right)}{192\left(3+2x^2+x^4\right)}-\frac{1}{256}\sqrt{\frac{1}{3}\left(-1291+1019\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right.}}{\sqrt{2\left(1+\sqrt{\right.}}}\right.$$

Mathematica [C] time = 0.302399, size = 129, normalized size = 0.52

$$\frac{1}{768}\left(\frac{4x\left(51x^6+166x^4+181x^2+292\right)}{\left(x^4+2x^2+3\right)^2}+\frac{3\left(34+21i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{3\left(34-21i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] ((4*x*(292 + 181*x^2 + 166*x^4 + 51*x^6))/(3 + 2*x^2 + x^4)^2 + (3*(34 + (2
1*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (3*(34 -
 (21*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/768
```

Maple [B] time = 0.022, size = 418, normalized size = 1.7

$$\frac{1}{\left(x^4+2x^2+3\right)^2}\left(\frac{17x^7}{64}+\frac{83x^5}{96}+\frac{181x^3}{192}+\frac{73x}{48}\right)+\frac{55\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{3072}+\frac{21\ln\left(x^2+\sqrt{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

```
[Out] (17/64*x^7+83/96*x^5+181/192*x^3+73/48*x)/(x^4+2*x^2+3)^2+55/3072*ln(x^2+3^
(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+21/1024*ln(x^2+3
^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+55/1536/(2+2*3^(1/2))^(
1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*
3^(1/2)+21/512/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3
```

```
^(1/2))^(1/2))*(-2+2*3^(1/2))-1/48/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^
(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)-55/3072*ln(x^2+3^(1/2)+x*(-2+2*3
^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)-21/1024*ln(x^2+3^(1/2)+x*(-2+2*
3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+55/1536/(2+2*3^(1/2))^(1/2)*arctan((2*
x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+21/512/
(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*
(-2+2*3^(1/2))-1/48/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(
2+2*3^(1/2))^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{51\,x^7+166\,x^5+181\,x^3+292\,x}{192\left(x^8+4\,x^6+10\,x^4+12\,x^2+9\right)}+\frac{1}{64}\int\frac{17\,x^2-4}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 1/192*(51*x^7 + 166*x^5 + 181*x^3 + 292*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 +
 9) + 1/64*integrate((17*x^2 - 4)/(x^4 + 2*x^2 + 3), x)
```

Fricas [B] time = 1.68028, size = 2261, normalized size = 9.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/7991829504*(2122829712*x^7 + 6909602592*x^5 - 3404*3115083^(1/4)*sqrt(6)*
sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1315529*sqrt(3) +
 3115083)*arctan(1/41378565634793586*3115083^(3/4)*sqrt(2601507)*sqrt(6)*sq
rt(3115083^(1/4)*sqrt(6)*(17*sqrt(3)*x + 4*x)*sqrt(-1315529*sqrt(3) + 31150
83) + 2601507*x^2 + 2601507*sqrt(3))*(4*sqrt(3)*sqrt(2) + 51*sqrt(2))*sqrt(
-1315529*sqrt(3) + 3115083) - 1/15905613798*3115083^(3/4)*sqrt(6)*(4*sqrt(3
)*sqrt(2)*x + 51*sqrt(2)*x)*sqrt(-1315529*sqrt(3) + 3115083) - 1/2*sqrt(3)*
sqrt(2) + 1/2*sqrt(2)) - 3404*3115083^(1/4)*sqrt(6)*sqrt(3)*sqrt(2)*(x^8 +
4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1315529*sqrt(3) + 3115083)*arctan(1/4137
8565634793586*3115083^(3/4)*sqrt(2601507)*sqrt(6)*sqrt(-3115083^(1/4)*sqrt(
6)*(17*sqrt(3)*x + 4*x)*sqrt(-1315529*sqrt(3) + 3115083) + 2601507*x^2 + 26
01507*sqrt(3))*(4*sqrt(3)*sqrt(2) + 51*sqrt(2))*sqrt(-1315529*sqrt(3) + 311
5083) - 1/15905613798*3115083^(3/4)*sqrt(6)*(4*sqrt(3)*sqrt(2)*x + 51*sqrt(
2)*x)*sqrt(-1315529*sqrt(3) + 3115083) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2))
 - 3115083^(1/4)*sqrt(6)*(3057*x^8 + 12228*x^6 + 30570*x^4 + 36684*x^2 + 12
91*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 27513)*sqrt(-1315529*sqrt(
3) + 3115083)*log(3115083^(1/4)*sqrt(6)*(17*sqrt(3)*x + 4*x)*sqrt(-1315529*
sqrt(3) + 3115083) + 2601507*x^2 + 2601507*sqrt(3)) + 3115083^(1/4)*sqrt(6)
*(3057*x^8 + 12228*x^6 + 30570*x^4 + 36684*x^2 + 1291*sqrt(3)*(x^8 + 4*x^6
+ 10*x^4 + 12*x^2 + 9) + 27513)*sqrt(-1315529*sqrt(3) + 3115083)*log(-31150
83^(1/4)*sqrt(6)*(17*sqrt(3)*x + 4*x)*sqrt(-1315529*sqrt(3) + 3115083) + 26
01507*x^2 + 2601507*sqrt(3)) + 7533964272*x^3 + 12154240704*x)/(x^8 + 4*x^6
 + 10*x^4 + 12*x^2 + 9)
```

Sympy [A] time = 0.571005, size = 68, normalized size = 0.27

$$\frac{51x^7+166x^5+181x^3+292x}{192x^8+768x^6+1920x^4+2304x^2+1728}+\operatorname{RootSum}\left(51539607552t^4-338427904t^2+1038361,\left(t\mapsto t\log\left(\frac{553648\ldots}{8671\ldots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] (51*x**7 + 166*x**5 + 181*x**3 + 292*x)/(192*x**8 + 768*x**6 + 1920*x**4 +
2304*x**2 + 1728) + RootSum(51539607552*_t**4 - 338427904*_t**2 + 1038361,
Lambda(_t, _t*log(5536481280*_t**3/867169 - 19920128*_t/867169 + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^6+3\,x^4+x^2+4}{\left(x^4+2\,x^2+3\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)/(x^4 + 2*x^2 + 3)^3, x)
```

3.123 $\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=253

$$-\frac{25x\left(x^2+5\right)}{144\left(x^4+2x^2+3\right)^2}-\frac{x\left(242x^2+325\right)}{1728\left(x^4+2x^2+3\right)}-\frac{\sqrt{\frac{1}{3}\left(55161\sqrt{3}-59711\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{4608}+\frac{\sqrt{\frac{1}{3}\left(55161\sqrt{\right.}\right.}}{}$$

```
[Out] -4/(27*x) - (25*x*(5 + x^2))/(144*(3 + 2*x^2 + x^4)^2) - (x*(325 + 242*x^2)
)/(1728*(3 + 2*x^2 + x^4)) + (Sqrt[(59711 + 55161*Sqrt[3])/3]*ArcTan[(Sqrt[
2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2304 - (Sqrt[(59711 + 5516
1*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])
/2304 - (Sqrt[(-59711 + 55161*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3
])]*x + x^2])/4608 + (Sqrt[(-59711 + 55161*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2
*(-1 + Sqrt[3])]*x + x^2])/4608
```

Rubi [A] time = 0.342795, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1669, 1664, 1169, 634, 618, 204, 628}

$$-\frac{25x\left(x^2+5\right)}{144\left(x^4+2x^2+3\right)^2}-\frac{x\left(242x^2+325\right)}{1728\left(x^4+2x^2+3\right)}-\frac{\sqrt{\frac{1}{3}\left(55161\sqrt{3}-59711\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{4608}+\frac{\sqrt{\frac{1}{3}\left(55161\sqrt{\right.}\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^3),x]
```


```
[Out] -4/(27*x) - (25*x*(5 + x^2))/(144*(3 + 2*x^2 + x^4)^2) - (x*(325 + 242*x^2)
)/(1728*(3 + 2*x^2 + x^4)) + (Sqrt[(59711 + 55161*Sqrt[3])/3]*ArcTan[(Sqrt[
2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2304 - (Sqrt[(59711 + 5516
1*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])
/2304 - (Sqrt[(-59711 + 55161*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3
])]*x + x^2])/4608 + (Sqrt[(-59711 + 55161*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2
*(-1 + Sqrt[3])]*x + x^2])/4608
```


Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
```

```
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx = -\frac{25x\left(5+x^2\right)}{144\left(3+2x^2+x^4\right)^2}+\frac{1}{96}\int \frac{128+30x^2-\frac{250x^4}{3}}{x^2\left(3+2x^2+x^4\right)^2}\,dx$$

$$= -\frac{25x\left(5+x^2\right)}{144\left(3+2x^2+x^4\right)^2}-\frac{x\left(325+242x^2\right)}{1728\left(3+2x^2+x^4\right)}+\frac{\int \frac{2048-\frac{56x^2}{3}-\frac{1936x^4}{3}}{x^2\left(3+2x^2+x^4\right)}\,dx}{4608}$$

$$= -\frac{25x\left(5+x^2\right)}{144\left(3+2x^2+x^4\right)^2}-\frac{x\left(325+242x^2\right)}{1728\left(3+2x^2+x^4\right)}+\frac{\int \left(\frac{2048}{3x^2}-\frac{8\left(173+166x^2\right)}{3+2x^2+x^4}\right)dx}{4608}$$

$$= -\frac{4}{27x}-\frac{25x\left(5+x^2\right)}{144\left(3+2x^2+x^4\right)^2}-\frac{x\left(325+242x^2\right)}{1728\left(3+2x^2+x^4\right)}-\frac{1}{576}\int \frac{173+166x^2}{3+2x^2+x^4}\,dx$$

$$= -\frac{4}{27x}-\frac{25x\left(5+x^2\right)}{144\left(3+2x^2+x^4\right)^2}-\frac{x\left(325+242x^2\right)}{1728\left(3+2x^2+x^4\right)}-\frac{\int \frac{173\sqrt{2\left(-1+\sqrt{3}\right)}-\left(173-166\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{1152\sqrt{6\left(-1+\sqrt{3}\right)}}- \ldots$$

$$= -\frac{4}{27x}-\frac{25x\left(5+x^2\right)}{144\left(3+2x^2+x^4\right)^2}-\frac{x\left(325+242x^2\right)}{1728\left(3+2x^2+x^4\right)}-\frac{\sqrt{\frac{1}{3}\left(-59711+55161\sqrt{3}\right)}\int \frac{-\sqrt{2(-\ldots}}{\sqrt{3}-\sqrt{2(\ldots}}\ldots}{4608}\ldots$$

$$= -\frac{4}{27x}-\frac{25x\left(5+x^2\right)}{144\left(3+2x^2+x^4\right)^2}-\frac{x\left(325+242x^2\right)}{1728\left(3+2x^2+x^4\right)}-\frac{\sqrt{\frac{1}{3}\left(-59711+55161\sqrt{3}\right)}\log\left(\sqrt{3}-\ldots\right.}{4608}\ldots$$

$$= -\frac{4}{27x}-\frac{25x\left(5+x^2\right)}{144\left(3+2x^2+x^4\right)^2}-\frac{x\left(325+242x^2\right)}{1728\left(3+2x^2+x^4\right)}+\frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2(\ldots}}{\ldots}\right.}{2304}\ldots$$

Mathematica [C] time = 0.378739, size = 140, normalized size = 0.55

$$\frac{-\frac{12\left(166x^8+611x^6+1412x^4+1849x^2+768\right)}{x\left(x^4+2x^2+3\right)^2}+\frac{3i\left(7\sqrt{2}+332i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}-\frac{3i\left(7\sqrt{2}-332i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{6912}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^3),x]
```

```
[Out] ((-12*(768 + 1849*x^2 + 1412*x^4 + 611*x^6 + 166*x^8))/(x*(3 + 2*x^2 + x^4)
^2) + ((3*I)*(332*I + 7*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*
Sqrt[2]] - ((3*I)*(-332*I + 7*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[
1 + I*Sqrt[2]])/6912
```

Maple [B] time = 0.023, size = 424, normalized size = 1.7

$$-\frac{1}{27\left(x^4+2x^2+3\right)^2}\left(\frac{121x^7}{32}+\frac{809x^5}{64}+\frac{419x^3}{16}+\frac{2475x}{64}\right)-\frac{325\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{27648}+\frac{7\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x)
```

```
[Out] -1/27*(121/32*x^7+809/64*x^5+419/16*x^3+2475/64*x)/(x^4+2*x^2+3)^2-325/2764
8*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+7/921
6*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-325/13824/(2+
2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2
+2*3^(1/2))*3^(1/2)+7/4608/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(
1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))-173/1728/(2+2*3^(1/2))^(1/2)*arct
an((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)+325/27648*ln(x^2
+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)-7/9216*ln(x^2
+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-325/13824/(2+2*3^(1/2
))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/
2))*3^(1/2)+7/4608/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2
+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))-173/1728/(2+2*3^(1/2))^(1/2)*arctan((2*x+
(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)-4/27/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{166\,x^{8}+611\,x^{6}+1412\,x^{4}+1849\,x^{2}+768}{576\left(x^{9}+4\,x^{7}+10\,x^{5}+12\,x^{3}+9\,x\right)}-\frac{1}{576}\int\frac{166\,x^{2}+173}{x^{4}+2\,x^{2}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] -1/576*(166*x^8 + 611*x^6 + 1412*x^4 + 1849*x^2 + 768)/(x^9 + 4*x^7 + 10*x^
5 + 12*x^3 + 9*x) - 1/576*integrate((166*x^2 + 173)/(x^4 + 2*x^2 + 3), x)
```

Fricas [B] time = 1.73342, size = 2529, normalized size = 10.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/2978955242496*(858518351136*x^8 + 3159968147856*x^6 + 210956*1391283^(1/
4)*sqrt(681)*sqrt(6)*sqrt(3)*sqrt(2)*(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x)*
sqrt(59711*sqrt(3) + 165483)*arctan(1/15811665652336538898*sqrt(11971753)*1
391283^(3/4)*sqrt(681)*sqrt(6)*sqrt(1391283^(1/4)*sqrt(681)*sqrt(6)*(166*sq
rt(3)*x - 173*x)*sqrt(59711*sqrt(3) + 165483) + 107745777*x^2 + 107745777*s
qrt(3))*(173*sqrt(3)*sqrt(2) - 498*sqrt(2))*sqrt(59711*sqrt(3) + 165483) -
1/440249244822*1391283^(3/4)*sqrt(681)*sqrt(6)*(173*sqrt(3)*sqrt(2)*x - 498
*sqrt(2)*x)*sqrt(59711*sqrt(3) + 165483) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2
)) + 210956*1391283^(1/4)*sqrt(681)*sqrt(6)*sqrt(3)*sqrt(2)*(x^9 + 4*x^7 +
10*x^5 + 12*x^3 + 9*x)*sqrt(59711*sqrt(3) + 165483)*arctan(1/47434996957009
616694*sqrt(11971753)*1391283^(3/4)*sqrt(681)*sqrt(6)*sqrt(-9*1391283^(1/4)
*sqrt(681)*sqrt(6)*(166*sqrt(3)*x - 173*x)*sqrt(59711*sqrt(3) + 165483) + 9
69711993*x^2 + 969711993*sqrt(3))*(173*sqrt(3)*sqrt(2) - 498*sqrt(2))*sqrt(
59711*sqrt(3) + 165483) - 1/440249244822*1391283^(3/4)*sqrt(681)*sqrt(6)*(1
73*sqrt(3)*sqrt(2)*x - 498*sqrt(2)*x)*sqrt(59711*sqrt(3) + 165483) - 1/2*sq
rt(3)*sqrt(2) + 1/2*sqrt(2)) + 7302577781952*x^4 - 1391283^(1/4)*sqrt(681)*
```

```
sqrt(6)*(165483*x^9 + 661932*x^7 + 1654830*x^5 + 1985796*x^3 - 59711*sqrt(3
)*(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x) + 1489347*x)*sqrt(59711*sqrt(3) + 1
65483)*log(9*1391283^(1/4)*sqrt(681)*sqrt(6)*(166*sqrt(3)*x - 173*x)*sqrt(5
9711*sqrt(3) + 165483) + 969711993*x^2 + 969711993*sqrt(3)) + 1391283^(1/4)
*sqrt(681)*sqrt(6)*(165483*x^9 + 661932*x^7 + 1654830*x^5 + 1985796*x^3 - 5
9711*sqrt(3)*(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x) + 1489347*x)*sqrt(59711*
sqrt(3) + 165483)*log(-9*1391283^(1/4)*sqrt(681)*sqrt(6)*(166*sqrt(3)*x - 1
73*x)*sqrt(59711*sqrt(3) + 165483) + 969711993*x^2 + 969711993*sqrt(3)) + 9
562653200304*x^2 + 3971940323328)/(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x)
```

Sympy [A] time = 0.607374, size = 73, normalized size = 0.29

$$-\frac{166x^8 + 611x^6 + 1412x^4 + 1849x^2 + 768}{576x^9 + 2304x^7 + 5760x^5 + 6912x^3 + 5184x} + \operatorname{RootSum}\left(4174708211712t^4 + 15652880384t^2 + 37564641, \left(t \mapsto t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+2*x**2+3)**3,x)
```

```
[Out] -(166*x**8 + 611*x**6 + 1412*x**4 + 1849*x**2 + 768)/(576*x**9 + 2304*x**7
+ 5760*x**5 + 6912*x**3 + 5184*x) + RootSum(4174708211712*_t**4 + 156528803
84*_t**2 + 37564641, Lambda(_t, _t*log(-98146713600*_t**3/11971753 - 963936
4864*_t/323237331 + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^6 + 3\,x^4 + x^2 + 4}{\left(x^4 + 2\,x^2 + 3\right)^3 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^3*x^2), x)
```

3.124 $\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=262

$$\frac{25x\left(5x^2+7\right)}{432\left(x^4+2x^2+3\right)^2}+\frac{x\left(1025x^2+1474\right)}{5184\left(x^4+2x^2+3\right)}-\frac{4}{81x^3}+\frac{\sqrt{\frac{1}{3}\left(11240451\sqrt{3}-10004741\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{41472}-\ldots$$

```
[Out] -4/(81*x^3) + 7/(27*x) + (25*x*(7 + 5*x^2))/(432*(3 + 2*x^2 + x^4)^2) + (x*
(1474 + 1025*x^2))/(5184*(3 + 2*x^2 + x^4)) - (Sqrt[(10004741 + 11240451*Sq
rt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/207
36 + (Sqrt[(10004741 + 11240451*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]
+ 2*x)/Sqrt[2*(1 + Sqrt[3])]])/20736 + (Sqrt[(-10004741 + 11240451*Sqrt[3])
/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/41472 - (Sqrt[(-10004741
 + 11240451*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4147
2
```

Rubi [A] time = 0.365888, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1669, 1664, 1169, 634, 618, 204, 628}

$$\frac{25x\left(5x^2+7\right)}{432\left(x^4+2x^2+3\right)^2}+\frac{x\left(1025x^2+1474\right)}{5184\left(x^4+2x^2+3\right)}-\frac{4}{81x^3}+\frac{\sqrt{\frac{1}{3}\left(11240451\sqrt{3}-10004741\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{41472}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^3),x]
```


```
[Out] -4/(81*x^3) + 7/(27*x) + (25*x*(7 + 5*x^2))/(432*(3 + 2*x^2 + x^4)^2) + (x*
(1474 + 1025*x^2))/(5184*(3 + 2*x^2 + x^4)) - (Sqrt[(10004741 + 11240451*Sq
rt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/207
36 + (Sqrt[(10004741 + 11240451*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]
+ 2*x)/Sqrt[2*(1 + Sqrt[3])]])/20736 + (Sqrt[(-10004741 + 11240451*Sqrt[3])
/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/41472 - (Sqrt[(-10004741
 + 11240451*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4147
2
```


Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}dx = \frac{25x\left(7+5x^2\right)}{432\left(3+2x^2+x^4\right)^2}+\frac{1}{96}\int \frac{128-\frac{160x^2}{3}+50x^4+\frac{1250x^6}{9}}{x^4\left(3+2x^2+x^4\right)^2}dx$$

$$= \frac{25x\left(7+5x^2\right)}{432\left(3+2x^2+x^4\right)^2}+\frac{x\left(1474+1025x^2\right)}{5184\left(3+2x^2+x^4\right)}+\frac{\int \frac{2048-\frac{6656x^2}{3}+\frac{2576x^4}{9}+\frac{8200x^6}{9}}{x^4(3+2x^2+x^4)}dx}{4608}$$

$$= \frac{25x\left(7+5x^2\right)}{432\left(3+2x^2+x^4\right)^2}+\frac{x\left(1474+1025x^2\right)}{5184\left(3+2x^2+x^4\right)}+\frac{\int \left(\frac{2048}{3x^4}-\frac{3584}{3x^2}+\frac{8(2242+2369x^2)}{9(3+2x^2+x^4)}\right)dx}{4608}$$

$$= -\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x\left(7+5x^2\right)}{432\left(3+2x^2+x^4\right)^2}+\frac{x\left(1474+1025x^2\right)}{5184\left(3+2x^2+x^4\right)}+\frac{\int \frac{2242+2369x^2}{3+2x^2+x^4}dx}{5184}$$

$$= -\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x\left(7+5x^2\right)}{432\left(3+2x^2+x^4\right)^2}+\frac{x\left(1474+1025x^2\right)}{5184\left(3+2x^2+x^4\right)}+\frac{\int \frac{2242\sqrt{2(-1+\sqrt{3})}-(2242-2369\sqrt{3})x}{\sqrt{3}-\sqrt{2(-1+\sqrt{3})}x+x^2}}{10368\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= -\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x\left(7+5x^2\right)}{432\left(3+2x^2+x^4\right)^2}+\frac{x\left(1474+1025x^2\right)}{5184\left(3+2x^2+x^4\right)}+\frac{\left(2242-2369\sqrt{3}\right)\int \frac{\sqrt{2(-1+}}{\sqrt{3}+\sqrt{2(-1-}}}{20736\sqrt{6\left(-1+\sqrt{3}\right.}}$$

$$= -\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x\left(7+5x^2\right)}{432\left(3+2x^2+x^4\right)^2}+\frac{x\left(1474+1025x^2\right)}{5184\left(3+2x^2+x^4\right)}+\frac{\sqrt{-\frac{10004741}{12}+\frac{3746817\sqrt{3}}{4}}\log\left(\right.}{2073}$$

$$= -\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x\left(7+5x^2\right)}{432\left(3+2x^2+x^4\right)^2}+\frac{x\left(1474+1025x^2\right)}{5184\left(3+2x^2+x^4\right)}-\frac{\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right.}}{20736}$$

Mathematica [C] time = 0.331905, size = 139, normalized size = 0.53

$$\frac{\frac{4\left(2369x^{10}+8644x^8+19939x^6+20090x^4+9024x^2-2304\right)}{x^3\left(x^4+2x^2+3\right)^2}+\frac{\left(4738+127i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{\left(4738-127i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{20736}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^3),x]
```

```
[Out] ((4*(-2304 + 9024*x^2 + 20090*x^4 + 19939*x^6 + 8644*x^8 + 2369*x^10))/(x^3
*(3 + 2*x^2 + x^4)^2) + ((4738 + (127*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[
2]]])/Sqrt[1 - I*Sqrt[2]] + ((4738 - (127*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*S
qrt[2]]])/Sqrt[1 + I*Sqrt[2]])/20736
```

Maple [B] time = 0.024, size = 429, normalized size = 1.6

$$\frac{1}{27\left(x^4+2x^2+3\right)^2}\left(\frac{1025x^7}{192}+\frac{881x^5}{48}+\frac{7523x^3}{192}+\frac{1087x}{32}\right)+\frac{4865\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)\sqrt{-2+2\sqrt{3}}\sqrt{3}}{248832}+\frac{12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x)
```

```
[Out] 1/27*(1025/192*x^7+881/48*x^5+7523/192*x^3+1087/32*x)/(x^4+2*x^2+3)^2+4865/
248832*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+
127/82944*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+4865/
124416/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^
(1/2))*(-2+2*3^(1/2))*3^(1/2)+127/41472/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2
+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))+1121/7776/(2+2*3^(1/
2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)-48
65/248832*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/
2)-127/82944*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)+48
65/124416/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2
))^(1/2))*(-2+2*3^(1/2))*3^(1/2)+127/41472/(2+2*3^(1/2))^(1/2)*arctan((2*x+
(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))+1121/7776/(2+2*3^
(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)
-4/81/x^3+7/27/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2369\,x^{10}+8644\,x^{8}+19939\,x^{6}+20090\,x^{4}+9024\,x^{2}-2304}{5184\left(x^{11}+4\,x^{9}+10\,x^{7}+12\,x^{5}+9\,x^{3}\right)}+\frac{1}{5184}\int\frac{2369\,x^{2}+2242}{x^{4}+2\,x^{2}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 1/5184*(2369*x^10 + 8644*x^8 + 19939*x^6 + 20090*x^4 + 9024*x^2 - 2304)/(x^
11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3) + 1/5184*integrate((2369*x^2 + 2242)/
(x^4 + 2*x^2 + 3), x)
```

Fricas [B] time = 1.74976, size = 2952, normalized size = 11.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/135934787413472256*(62119890312985296*x^10 + 226662866975704896*x^8 + 522
840224968600176*x^6 + 47239676*713236683^(1/4)*sqrt(15419)*sqrt(6)*sqrt(3)*
sqrt(2)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)*sqrt(10004741*sqrt(3) + 33
721353)*arctan(1/276093525919725583675206533346*sqrt(182097141061)*713236683
^(3/4)*sqrt(15419)*sqrt(6)*sqrt(3)*sqrt(713236683^(1/4)*sqrt(15419)*sqrt(6)
*(2369*sqrt(3)*x - 2242*x)*sqrt(10004741*sqrt(3) + 33721353) + 546291423183
*x^2 + 546291423183*sqrt(3))*(2242*sqrt(3)*sqrt(2) - 7107*sqrt(2))*sqrt(100
04741*sqrt(3) + 33721353) - 1/50539604724352062*713236683^(3/4)*sqrt(15419)
*sqrt(6)*(2242*sqrt(3)*sqrt(2)*x - 7107*sqrt(2)*x)*sqrt(10004741*sqrt(3) +
33721353) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 47239676*713236683^(1/4)*s
qrt(15419)*sqrt(6)*sqrt(3)*sqrt(2)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)
*sqrt(10004741*sqrt(3) + 33721353)*arctan(1/828280577759176751025619600038*s
qrt(182097141061)*713236683^(3/4)*sqrt(15419)*sqrt(6)*sqrt(-27*713236683^(1
/4)*sqrt(15419)*sqrt(6)*(2369*sqrt(3)*x - 2242*x)*sqrt(10004741*sqrt(3) + 3
```

```
3721353) + 14749868425941*x^2 + 14749868425941*sqrt(3))*(2242*sqrt(3)*sqrt(
2) - 7107*sqrt(2))*sqrt(10004741*sqrt(3) + 33721353) - 1/50539604724352062*
713236683^(3/4)*sqrt(15419)*sqrt(6)*(2242*sqrt(3)*sqrt(2)*x - 7107*sqrt(2)*
x)*sqrt(10004741*sqrt(3) + 33721353) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) +
 526799745203830560*x^4 - 713236683^(1/4)*sqrt(15419)*sqrt(6)*(33721353*x^1
1 + 134885412*x^9 + 337213530*x^7 + 404656236*x^5 + 303492177*x^3 - 1000474
1*sqrt(3)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3))*sqrt(10004741*sqrt(3) +
 33721353)*log(27*713236683^(1/4)*sqrt(15419)*sqrt(6)*(2369*sqrt(3)*x - 224
2*x)*sqrt(10004741*sqrt(3) + 33721353) + 14749868425941*x^2 + 1474986842594
1*sqrt(3)) + 713236683^(1/4)*sqrt(15419)*sqrt(6)*(33721353*x^11 + 134885412
*x^9 + 337213530*x^7 + 404656236*x^5 + 303492177*x^3 - 10004741*sqrt(3)*(x^
11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3))*sqrt(10004741*sqrt(3) + 33721353)*lo
g(-27*713236683^(1/4)*sqrt(15419)*sqrt(6)*(2369*sqrt(3)*x - 2242*x)*sqrt(10
004741*sqrt(3) + 33721353) + 14749868425941*x^2 + 14749868425941*sqrt(3)) +
 236627222534562816*x^2 - 60415461072654336)/(x^11 + 4*x^9 + 10*x^7 + 12*x^
5 + 9*x^3)
```

Sympy [A] time = 0.6269, size = 80, normalized size = 0.31

$$\operatorname{RootSum}\left(338151365148672t^4 + 2622682824704t^2 + 19257390441, \left(t \mapsto t \log\left(\frac{357010935644160t^3}{182097141061} + \frac{260169578908}{16388742695}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+2*x**2+3)**3,x)
```

```
[Out] RootSum(338151365148672*_t**4 + 2622682824704*_t**2 + 19257390441, Lambda(_
t, _t*log(357010935644160*_t**3/182097141061 + 26016957890816*_t/1638874269
549 + x))) + (2369*x**10 + 8644*x**8 + 19939*x**6 + 20090*x**4 + 9024*x**2
- 2304)/(5184*x**11 + 20736*x**9 + 51840*x**7 + 62208*x**5 + 46656*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^6 + 3\,x^4 + x^2 + 4}{\left(x^4 + 2\,x^2 + 3\right)^3 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] integrate((5*x^6 + 3*x^4 + x^2 + 4)/((x^4 + 2*x^2 + 3)^3*x^4), x)
```

3.125 $\int \frac{x\left(d+ex^2+fx^4+gx^6\right)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=149

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)}{2c^3\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)\left(-c(ag+bf)+b^2g+c^2e\right)}{4c^3}$$

```
[Out] ((c*f - b*g)*x^2)/(2*c^2) + (g*x^4)/(4*c) - ((2*c^3*d - c^2*(b*e + 2*a*f) -
 b^3*g + b*c*(b*f + 3*a*g))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^
3*Sqrt[b^2 - 4*a*c]) + ((c^2*e + b^2*g - c*(b*f + a*g))*Log[a + b*x^2 + c*x
^4])/(4*c^3)
```

Rubi [A] time = 0.294841, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1663, 1657, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)}{2c^3\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)\left(-c(ag+bf)+b^2g+c^2e\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*f - b*g)*x^2)/(2*c^2) + (g*x^4)/(4*c) - ((2*c^3*d - c^2*(b*e + 2*a*f) -
 b^3*g + b*c*(b*f + 3*a*g))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^
3*Sqrt[b^2 - 4*a*c]) + ((c^2*e + b^2*g - c*(b*f + a*g))*Log[a + b*x^2 + c*x
^4])/(4*c^3)
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(d+ex^2+fx^4+gx^6\right)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{d+ex+fx^2+gx^3}{a+bx+cx^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{cf-bg}{c^2}+\frac{gx}{c}+\frac{c^2d-acf+abg+\left(c^2e+b^2g-c(bf+ag)\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx, x, x^2\right) \\
&= \frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}+\frac{\operatorname{Subst}\left(\int \frac{c^2d-acf+abg+\left(c^2e+b^2g-c(bf+ag)\right)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c^2} \\
&= \frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}+\frac{\left(c^2e+b^2g-c(bf+ag)\right)\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^3}+\frac{\left(2c^3d-c^2\right.}{} \\
&= \frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}+\frac{\left(c^2e+b^2g-c(bf+ag)\right)\log\left(a+bx^2+cx^4\right)}{4c^3}-\frac{\left(2c^3d-c^2(be+2af\right.}{} \\
&= \frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}-\frac{\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(c}{}
\end{aligned}
$$

Mathematica [A] time = 0.127054, size = 142, normalized size = 0.95

$$
\frac{\frac{2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)}{\sqrt{4ac-b^2}}+\log\left(a+bx^2+cx^4\right)\left(-c(ag+bf)+b^2g+c^2e\right)+2cx^2(cf-bg)+c^2gx^4}{4c^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(c*f - b*g)*x^2 + c^2*g*x^4 + (2*(2*c^3*d - c^2*(b*e + 2*a*f) - b^3*g
+ b*c*(b*f + 3*a*g))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
4*a*c] + (c^2*e + b^2*g - c*(b*f + a*g))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [B] time = 0.005, size = 357, normalized size = 2.4

$$
\frac{gx^4}{4c}-\frac{bx^2g}{2c^2}+\frac{fx^2}{2c}-\frac{\ln\left(cx^4+bx^2+a\right)ag}{4c^2}+\frac{\ln\left(cx^4+bx^2+a\right)b^2g}{4c^3}-\frac{\ln\left(cx^4+bx^2+a\right)bf}{4c^2}+\frac{\ln\left(cx^4+bx^2+a\right)e}{4c}+\frac{3a}{2c}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*g*x^4/c-1/2/c^2*x^2*b*g+1/2*f*x^2/c-1/4/c^2*ln(c*x^4+b*x^2+a)*a*g+1/4/c
^3*ln(c*x^4+b*x^2+a)*b^2*g-1/4/c^2*ln(c*x^4+b*x^2+a)*b*f+1/4/c*ln(c*x^4+b*x
^2+a)*e+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b
*g-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*f+1/(4*a*c
-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*d-1/2/c^3/(4*a*c-b^2)^(1/
2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*g+1/2/c^2/(4*a*c-b^2)^(1/2)*ar
ctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*f-1/2/c/(4*a*c-b^2)^(1/2)*arctan((2
*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.35085, size = 1021, normalized size = 6.85

$$\left[\frac{(b^2c^2-4\,ac^3)gx^4+2\left((b^2c^2-4\,ac^3)f-(b^3c-4\,abc^2)g\right)x^2+\left(2\,c^3d-bc^2e+(b^2c-2\,ac^2)f-(b^3-3\,abc)g\right)\sqrt{b^2-4\,a}}{4\,(b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2 - 4*a*c^3)*g*x^4 + 2*((b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*b
*c^2)*g)*x^2 + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f - (b^3 - 3*a*b*c)*g
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b
)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c^2 - 4*a*c^3)*e - (b^3*c
 - 4*a*b*c^2)*f + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*g)*log(c*x^4 + b*x^2 + a))/
(b^2*c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*g*x^4 + 2*((b^2*c^2 - 4*a*c^3
)*f - (b^3*c - 4*a*b*c^2)*g)*x^2 - 2*(2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)
*f - (b^3 - 3*a*b*c)*g)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2
+ 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c^2 - 4*a*c^3)*e - (b^3*c - 4*a*b*c^2)*f +
(b^4 - 5*a*b^2*c + 4*a^2*c^2)*g)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4
)]
```

Sympy [B] time = 49.2751, size = 789, normalized size = 5.3

$$\left(-\frac{\sqrt{-4ac+b^2}\left(3abcg-2ac^2f-b^3g+b^2cf-bc^2e+2c^3d\right)}{4c^3\left(4ac-b^2\right)}-\frac{acg-b^2g+bcf-c^2e}{4c^3}\right)\log\left(x^2+\frac{2a^2cg-ab^2g+abcf+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(3*a*b*c*g - 2*a*c**2*f - b**3*g + b**2*c*f - b*c**2*
e + 2*c**3*d)/(4*c**3*(4*a*c - b**2)) - (a*c*g - b**2*g + b*c*f - c**2*e)/(
4*c**3))*log(x**2 + (2*a**2*c*g - a*b**2*g + a*b*c*f + 8*a*c**3*(-sqrt(-4*a
*c + b**2)*(3*a*b*c*g - 2*a*c**2*f - b**3*g + b**2*c*f - b*c**2*e + 2*c**3*
d)/(4*c**3*(4*a*c - b**2)) - (a*c*g - b**2*g + b*c*f - c**2*e)/(4*c**3)) -
2*a*c**2*e - 2*b**2*c**2*(-sqrt(-4*a*c + b**2)*(3*a*b*c*g - 2*a*c**2*f - b*
*3*g + b**2*c*f - b*c**2*e + 2*c**3*d)/(4*c**3*(4*a*c - b**2)) - (a*c*g - b
**2*g + b*c*f - c**2*e)/(4*c**3)) + b*c**2*d)/(3*a*b*c*g - 2*a*c**2*f - b**
3*g + b**2*c*f - b*c**2*e + 2*c**3*d)) + (sqrt(-4*a*c + b**2)*(3*a*b*c*g -
2*a*c**2*f - b**3*g + b**2*c*f - b*c**2*e + 2*c**3*d)/(4*c**3*(4*a*c - b**2
)) - (a*c*g - b**2*g + b*c*f - c**2*e)/(4*c**3))*log(x**2 + (2*a**2*c*g - a
*b**2*g + a*b*c*f + 8*a*c**3*(sqrt(-4*a*c + b**2)*(3*a*b*c*g - 2*a*c**2*f -
 b**3*g + b**2*c*f - b*c**2*e + 2*c**3*d)/(4*c**3*(4*a*c - b**2)) - (a*c*g
- b**2*g + b*c*f - c**2*e)/(4*c**3)) - 2*a*c**2*e - 2*b**2*c**2*(sqrt(-4*a*
c + b**2)*(3*a*b*c*g - 2*a*c**2*f - b**3*g + b**2*c*f - b*c**2*e + 2*c**3*d
)/(4*c**3*(4*a*c - b**2)) - (a*c*g - b**2*g + b*c*f - c**2*e)/(4*c**3)) + b
*c**2*d)/(3*a*b*c*g - 2*a*c**2*f - b**3*g + b**2*c*f - b*c**2*e + 2*c**3*d)
) + g*x**4/(4*c) - x**2*(b*g - c*f)/(2*c**2)
```

Giac [A] time = 1.17534, size = 197, normalized size = 1.32

$$\frac{cgx^4 + 2\,cfx^2 - 2\,bgx^2}{4\,c^2} - \frac{\left(bcf - b^2g + acg - c^2e\right)\log\left(cx^4 + bx^2 + a\right)}{4\,c^3} + \frac{\left(2\,c^3d + b^2cf - 2\,ac^2f - b^3g + 3\,abcg - bc^2e\right)\text{arc}}{2\,\sqrt{-b^2 + 4\,ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(c*g*x^4 + 2*c*f*x^2 - 2*b*g*x^2)/c^2 - 1/4*(b*c*f - b^2*g + a*c*g - c^
2*e)*log(c*x^4 + b*x^2 + a)/c^3 + 1/2*(2*c^3*d + b^2*c*f - 2*a*c^2*f - b^3*
g + 3*a*b*c*g - b*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^
2 + 4*a*c)*c^3)
```

3.126 $\int \frac{x^4\left(d+ex^2+fx^4+gx^6\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=594

$$\frac{x\left(x^2\left(-b^2c(ce-4ag)+bc^2(cd-3af)+2ac^2(ce-ag)+b^3cf+b^4(-g)\right)+a\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] ((c*f - 2*b*g)*x)/c^3 + (g*x^3)/(3*c^2) + (x*(a*(2*c^3*d - c^2*(b*e + 2*a*f
) - b^3*g + b*c*(b*f + 3*a*g)) + (b^3*c*f + b*c^2*(c*d - 3*a*f) - b^4*g - b
^2*c*(c*e - 4*a*g) + 2*a*c^2*(c*e - a*g))*x^2))/(2*c^3*(b^2 - 4*a*c)*(a + b
*x^2 + c*x^4)) - ((3*b^3*c*f - b*c^2*(c*d + 13*a*f) - 5*b^4*g - b^2*c*(c*e
- 24*a*g) + 2*a*c^2*(3*c*e - 7*a*g) - (3*b^4*c*f - 4*a*c^3*(c*d - 5*a*f) -
b^2*c^2*(c*d + 19*a*f) - 5*b^5*g - b^3*c*(c*e - 34*a*g) + 4*a*b*c^2*(2*c*e
- 13*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
 ((3*b^3*c*f - b*c^2*(c*d + 13*a*f) - 5*b^4*g - b^2*c*(c*e - 24*a*g) + 2*a*
c^2*(3*c*e - 7*a*g) + (3*b^4*c*f - 4*a*c^3*(c*d - 5*a*f) - b^2*c^2*(c*d + 1
9*a*f) - 5*b^5*g - b^3*c*(c*e - 34*a*g) + 4*a*b*c^2*(2*c*e - 13*a*g))/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 14.1129, antiderivative size = 594, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1668, 1676, 1166, 205}

$$\frac{x\left(x^2\left(-b^2c(ce-4ag)+bc^2(cd-3af)+2ac^2(ce-ag)+b^3cf+b^4(-g)\right)+a\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((c*f - 2*b*g)*x)/c^3 + (g*x^3)/(3*c^2) + (x*(a*(2*c^3*d - c^2*(b*e + 2*a*f
) - b^3*g + b*c*(b*f + 3*a*g)) + (b^3*c*f + b*c^2*(c*d - 3*a*f) - b^4*g - b
^2*c*(c*e - 4*a*g) + 2*a*c^2*(c*e - a*g))*x^2))/(2*c^3*(b^2 - 4*a*c)*(a + b
*x^2 + c*x^4)) - ((3*b^3*c*f - b*c^2*(c*d + 13*a*f) - 5*b^4*g - b^2*c*(c*e
- 24*a*g) + 2*a*c^2*(3*c*e - 7*a*g) - (3*b^4*c*f - 4*a*c^3*(c*d - 5*a*f) -
b^2*c^2*(c*d + 19*a*f) - 5*b^5*g - b^3*c*(c*e - 34*a*g) + 4*a*b*c^2*(2*c*e
- 13*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
 ((3*b^3*c*f - b*c^2*(c*d + 13*a*f) - 5*b^4*g - b^2*c*(c*e - 24*a*g) + 2*a*
c^2*(3*c*e - 7*a*g) + (3*b^4*c*f - 4*a*c^3*(c*d - 5*a*f) - b^2*c^2*(c*d + 1
9*a*f) - 5*b^5*g - b^3*c*(c*e - 34*a*g) + 4*a*b*c^2*(2*c*e - 13*a*g))/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
```

```
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(d+ex^2+fx^4+gx^6\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(a\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag)\right)+\left(b^3cf+bc^2(cd-3af)-b^4g-b^2c(ce \ldots}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(a\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag)\right)+\left(b^3cf+bc^2(cd-3af)-b^4g-b^2c(ce \ldots}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{(cf-2bg)x}{c^3}+\frac{gx^3}{3c^2}+\frac{x\left(a\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag)\right)+\left(b^3cf+bc^2(cd \ldots}{2c^3\left(b^2-4ac\right)\left(a+bx^2 \ldots}$$

$$= \frac{(cf-2bg)x}{c^3}+\frac{gx^3}{3c^2}+\frac{x\left(a\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag)\right)+\left(b^3cf+bc^2(cd \ldots}{2c^3\left(b^2-4ac\right)\left(a+bx^2 \ldots}$$

$$= \frac{(cf-2bg)x}{c^3}+\frac{gx^3}{3c^2}+\frac{x\left(a\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag)\right)+\left(b^3cf+bc^2(cd \ldots}{2c^3\left(b^2-4ac\right)\left(a+bx^2 \ldots}$$

Mathematica [A] time = 2.84651, size = 721, normalized size = 1.21

$$\frac{\frac{6\sqrt{c}x\left(a^2c\left(3bg-2c\left(f+gx^2\right)\right)+a\left(b^2c\left(f+4gx^2\right)+b^3(-g)-bc^2\left(e+3fx^2\right)+2c^3\left(d+ex^2\right)\right)+bx^2\left(b^2cf+b^3(-g)-bc^2e+c^3d\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-b^2c\left(-ce\sqrt{b^2-4ac}+ \ldots\right.\right.}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (12*Sqrt[c]*(c*f - 2*b*g)*x + 4*c^(3/2)*g*x^3 + (6*Sqrt[c]*x*(b*(c^3*d - b*
c^2*e + b^2*c*f - b^3*g)*x^2 + a^2*c*(3*b*g - 2*c*(f + g*x^2)) + a*(-(b^3*g
) + 2*c^3*(d + e*x^2) - b*c^2*(e + 3*f*x^2) + b^2*c*(f + 4*g*x^2))))/((b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*(-5*b^5*g - b^3*c*(c*e + 3*Sqrt[
b^2 - 4*a*c]*f - 34*a*g) + b^4*(3*c*f + 5*Sqrt[b^2 - 4*a*c]*g) + 2*a*c^2*(-
2*c^2*d - 3*c*Sqrt[b^2 - 4*a*c]*e + 10*a*c*f + 7*a*Sqrt[b^2 - 4*a*c]*g) - b
^2*c*(c^2*d - c*Sqrt[b^2 - 4*a*c]*e + 19*a*c*f + 24*a*Sqrt[b^2 - 4*a*c]*g)
+ b*c^2*(c*(Sqrt[b^2 - 4*a*c]*d + 8*a*e) + 13*a*(Sqrt[b^2 - 4*a*c]*f - 4*a*
g)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(5*b^5*g + b^3*c*(c*e - 3
*Sqrt[b^2 - 4*a*c]*f - 34*a*g) + b^4*(-3*c*f + 5*Sqrt[b^2 - 4*a*c]*g) + b^2
*c*(c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 19*a*c*f - 24*a*Sqrt[b^2 - 4*a*c]*g) +
2*a*c^2*(2*c^2*d - 3*c*Sqrt[b^2 - 4*a*c]*e - 10*a*c*f + 7*a*Sqrt[b^2 - 4*a*
c]*g) + b*c^2*(c*(Sqrt[b^2 - 4*a*c]*d - 8*a*e) + 13*a*(Sqrt[b^2 - 4*a*c]*f
+ 4*a*g)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 -
 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*c^(7/2))
```

Maple [B] time = 0.059, size = 3028, normalized size = 5.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -3/4/c^2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2
^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*f+1/4/c/(4*a*c-b^2)*2^(1/2)/((
(-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*
c)^(1/2))*b^2*e+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/
2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a^2*f-
1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*d+5/(4*a*c-b^2)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*f-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*b^2*d+3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*b*f+1/
2/c/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*b*e+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3
*a^2*g+1/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b^4*g-1/2/c^2/(c*x^4+b*x^2+a
)*a/(4*a*c-b^2)*x*b^2*f+3/4/c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*f-1/4/c/
(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e-13/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*a^2*b*g+17/2/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*a*b^3*g-13/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)
^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a^
2*b*g+17/2/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*b^3*g-1
9/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/
2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*b^2*f-19/4/c/(4*
a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2*f-1/2/c^2/(c*x^4+b*x^2
+a)/(4*a*c-b^2)*x^3*b^3*f+1/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b^2*e+1/c/(
c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*x*f+1/4/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(
```

```
1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d+
3/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*e-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b
*d-3/2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(
1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*e-2/c^3*x*b*g-1/(c*x^4+b*x^2+a)*a/
(4*a*c-b^2)*x*d-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*e-1/2/(c*x^4+b*x^2+a)/(
4*a*c-b^2)*x^3*b*d+f*x/c^2+1/2/c^3/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*b^3*g-2/
c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*b^2*g-3/2/c^2/(c*x^4+b*x^2+a)*a^2/(4*
a*c-b^2)*x*b*g+7/2/c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*a
rctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a^2*g+5/4/c^3/(4*a*c-b
^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+
b^2)^(1/2)-b)*c)^(1/2))*b^4*g-7/2/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*g-5/
4/c^3/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*g+1/3*g*x^3/c^2-5/4/c^3/(4*a*c-b^2
)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2
^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^5*g+6/c^2/(4*a*c-b^2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*a*b^2*g-5/4/c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
b^5*g-6/c^2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*
x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*b^2*g+3/4/c^2/(4*a*c-b^2)/(-4
*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2
)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^4*f-1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1
/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+
b^2)^(1/2)-b)*c)^(1/2))*b^3*e-13/4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*f-c
/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*d+3/4/c^2/(4*a*c-b^2)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*f-1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*b^3*e+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((
-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c
)^(1/2))*a*b*e+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*e+13
/4/c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/
2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*b*f-c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2))*a*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(bc^3d - (b^2c^2 - 2ac^3)e + (b^3c - 3abc^2)f - (b^4 - 4ab^2c + 2a^2c^2)g)x^3 + (2ac^3d - abc^2e + (ab^2c - 2a^2c^2)f - (ab^3 - 3a^2b}{2(ab^2c^3 - 4a^2c^4 + (b^2c^4 - 4ac^5)x^4 + (b^3c^3 - 4abc^4)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*((b*c^3*d - (b^2*c^2 - 2*a*c^3)*e + (b^3*c - 3*a*b*c^2)*f - (b^4 - 4*a*
b^2*c + 2*a^2*c^2)*g)*x^3 + (2*a*c^3*d - a*b*c^2*e + (a*b^2*c - 2*a^2*c^2)*
f - (a*b^3 - 3*a^2*b*c)*g)*x)/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*
x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2) + 1/2*integrate(-(2*a*c^3*d - a*b*c^2*e -
```

```
(b*c^3*d + (b^2*c^2 - 6*a*c^3)*e - (3*b^3*c - 13*a*b*c^2)*f + (5*b^4 - 24*a
*b^2*c + 14*a^2*c^2)*g)*x^2 + (3*a*b^2*c - 10*a^2*c^2)*f - (5*a*b^3 - 19*a^
2*b*c)*g)/(c*x^4 + b*x^2 + a), x)/(b^2*c^3 - 4*a*c^4) + 1/3*(c*g*x^3 + 3*(c
*f - 2*b*g)*x)/c^3
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.127 $\int \frac{x^2\left(d+ex^2+fx^4+gx^6\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=471

$$-\frac{x\left(x^2\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)-ab^2g+bc(af+cd)-2ac(ce-ag)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2c(19\right.}{}$$

```
[Out] (g*x)/c^2 - (x*(b*c*(c*d + a*f) - a*b^2*g - 2*a*c*(c*e - a*g) + (2*c^3*d -
c^2*(b*e + 2*a*f) - b^3*g + b*c*(b*f + 3*a*g))*x^2))/(2*c^2*(b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)) - ((2*c^3*d - c^2*(b*e - 6*a*f) + 3*b^3*g - b*c*(b*f +
13*a*g) + (b^3*c*f - 4*b*c^2*(c*d + 2*a*f) - 3*b^4*g + 4*a*c^2*(c*e - 5*a*g
) + b^2*c*(c*e + 19*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) - ((2*c^3*d - c^2*(b*e - 6*a*f) + 3*b^3*g - b*c*(b*f + 13*a*g
) - (b^3*c*f - 4*b*c^2*(c*d + 2*a*f) - 3*b^4*g + 4*a*c^2*(c*e - 5*a*g) + b^
2*c*(c*e + 19*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4
*a*c]])
```

Rubi [A] time = 6.66183, antiderivative size = 471, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1668, 1676, 1166, 205}

$$-\frac{x\left(x^2\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)-ab^2g+bc(af+cd)-2ac(ce-ag)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2c(19\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] (g*x)/c^2 - (x*(b*c*(c*d + a*f) - a*b^2*g - 2*a*c*(c*e - a*g) + (2*c^3*d -
c^2*(b*e + 2*a*f) - b^3*g + b*c*(b*f + 3*a*g))*x^2))/(2*c^2*(b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)) - ((2*c^3*d - c^2*(b*e - 6*a*f) + 3*b^3*g - b*c*(b*f +
13*a*g) + (b^3*c*f - 4*b*c^2*(c*d + 2*a*f) - 3*b^4*g + 4*a*c^2*(c*e - 5*a*g
) + b^2*c*(c*e + 19*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) - ((2*c^3*d - c^2*(b*e - 6*a*f) + 3*b^3*g - b*c*(b*f + 13*a*g
) - (b^3*c*f - 4*b*c^2*(c*d + 2*a*f) - 3*b^4*g + 4*a*c^2*(c*e - 5*a*g) + b^
2*c*(c*e + 19*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4
*a*c]])
```


Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
```

```
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(d+ex^2+fx^4+gx^6\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{x\left(bc(cd+af)-ab^2g-2ac(ce-ag)+\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag)\right)\right.}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{x\left(bc(cd+af)-ab^2g-2ac(ce-ag)+\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag)\right)\right.}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{gx}{c^2}-\frac{x\left(bc(cd+af)-ab^2g-2ac(ce-ag)+\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+3\right.\right.}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{gx}{c^2}-\frac{x\left(bc(cd+af)-ab^2g-2ac(ce-ag)+\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+3\right.\right.}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{gx}{c^2}-\frac{x\left(bc(cd+af)-ab^2g-2ac(ce-ag)+\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+3\right.\right.}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 2.1155, size = 575, normalized size = 1.22

$$-\frac{2\sqrt{c}x\left(2c\left(a^2g-ac\left(e+fx^2\right)+c^2dx^2\right)+b^2\left(cfx^2-ag\right)+bc\left(a\left(f+3gx^2\right)+c\left(d-ex^2\right)\right)+b^3(-g)x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(2c^2\left(-10a^2g+cd\sqrt{b^2-4ac}+3af\sqrt{b^2-4ac}+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*Sqrt[c]*g*x - (2*Sqrt[c]*x*(-(b^3*g*x^2) + b^2*(-(a*g) + c*f*x^2) + 2*c*
(a^2*g + c^2*d*x^2 - a*c*(e + f*x^2)) + b*c*(c*(d - e*x^2) + a*(f + 3*g*x^2
))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4*g + b^2*c*(c*e
- Sqrt[b^2 - 4*a*c]*f + 19*a*g) + 2*c^2*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e +
3*a*Sqrt[b^2 - 4*a*c]*f - 10*a^2*g) + b^3*(c*f + 3*Sqrt[b^2 - 4*a*c]*g) - b
*c*(4*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 8*a*c*f + 13*a*Sqrt[b^2 - 4*a*c]*g))*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4*g - b^2*c*(c*e + Sqrt[b^2
 - 4*a*c]*f + 19*a*g) + 2*c^2*(c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + 3*a*Sqrt[b
^2 - 4*a*c]*f + 10*a^2*g) + b^3*(-(c*f) + 3*Sqrt[b^2 - 4*a*c]*g) + b*c*(4*c
^2*d - c*Sqrt[b^2 - 4*a*c]*e + 8*a*c*f - 13*a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt
[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(5/2))
```

Maple [B] time = 0.05, size = 2300, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2*g-19/4/c/(4
*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arcta
nh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*b^2*g-3/2/(4*a*c-b^2)*2^
(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2))*a*f+1/4/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1
/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*e-1/2/(4*a*c-b^
2)*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c
+b^2)^(1/2)-b)*c)^(1/2))*d+3/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*f-1/4/(4*a*
c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*b*e+1/2/(4*a*c-b^2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d+3/4/c^
2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*g+3/4/c^2/(4*a*c-b^
2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*
2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^4*g-13/4/c/(4*a*c-b^2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*a*b*g+13/4/c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2
)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*b*g-1/2/c^2/(c*x^
4+b*x^2+a)/(4*a*c-b^2)*x^3*b^3*g+1/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b^2*
f+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*a^2*g-1/4/(4*a*c-b^2)/c*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*b^2*f-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e+c/(c*
x^4+b*x^2+a)/(4*a*c-b^2)*x^3*d-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*f-1/2/(c
*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b*e-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*a*e+1/2/(
c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b*d+1/4/(4*a*c-b^2)/c*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*
f-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/
2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*e+g*x/c^2-1/(4
*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*e-1/4/(4*a*c-b^2)/c/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*f+1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)
```

```
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*b*d+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+
b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
)*a*b*f-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*
c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*e-1/4/(4*a
*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arcta
nh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*f+1/(4*a*c-b^2)*c/(-4*
a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)
/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*a*b*f-3/4/c^2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c
)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*g+3/4/c^2
/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*g-1/2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x
*a*b^2*g+3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*b*g+1/2/c/(c*x^4+b*x^2+a)/
(4*a*c-b^2)*x*a*b*f+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)
^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a^
2*g+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*g
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima"
)

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.128 $\int \frac{d+ex^2+fx^4+gx^6}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=449

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2c(cd-af)-ab^3g+4abc(2ag+ce)-4ac^2(af+3cd)}{c\sqrt{b^2-4ac}}+\frac{ab^2g}{c}+b(af+cd)-2a(3ag+ce)\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\cdots\right.}{\cdots}$$

```
[Out] (x*(c*(b^2*d - 2*a*(c*d - a*f) - (a*b*(c*e + a*g))/c) + (b*c*(c*d + a*f) -
a*b^2*g - 2*a*c*(c*e - a*g))*x^2))/(2*a*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) + ((b*(c*d + a*f) + (a*b^2*g)/c - 2*a*(c*e + 3*a*g) + (b^2*c*(c*d - a*f)
- 4*a*c^2*(3*c*d + a*f) - a*b^3*g + 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2 - 4*
a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*
a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*(c*d + a*f) + (a
*b^2*g)/c - 2*a*(c*e + 3*a*g) - (b^2*c*(c*d - a*f) - 4*a*c^2*(3*c*d + a*f)
- a*b^3*g + 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 2.86695, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1678, 1166, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2c(cd-af)-ab^3g+4abc(2ag+ce)-4ac^2(af+3cd)}{c\sqrt{b^2-4ac}}+\frac{ab^2g}{c}+b(af+cd)-2a(3ag+ce)\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] (x*(c*(b^2*d - 2*a*(c*d - a*f) - (a*b*(c*e + a*g))/c) + (b*c*(c*d + a*f) -
a*b^2*g - 2*a*c*(c*e - a*g))*x^2))/(2*a*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) + ((b*(c*d + a*f) + (a*b^2*g)/c - 2*a*(c*e + 3*a*g) + (b^2*c*(c*d - a*f)
- 4*a*c^2*(3*c*d + a*f) - a*b^3*g + 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2 - 4*
a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*
a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*(c*d + a*f) + (a
*b^2*g)/c - 2*a*(c*e + 3*a*g) - (b^2*c*(c*d - a*f) - 4*a*c^2*(3*c*d + a*f)
- a*b^3*g + 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```


Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
```

```
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4+gx^6}{\left(a+bx^2+cx^4\right)^2}dx = \frac{x\left(c\left(b^2d-2a(cd-af)-\frac{ab(ce+ag)}{c}\right)+\left(bc(cd+af)-ab^2g-2ac(ce-ag)\right)x^2\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\int\frac{-b^2d+2a(3c\cdots}{\cdots}$$

$$=\frac{x\left(c\left(b^2d-2a(cd-af)-\frac{ab(ce+ag)}{c}\right)+\left(bc(cd+af)-ab^2g-2ac(ce-ag)\right)x^2\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b(cd+af)\cdots\right.}{\cdots}$$

$$=\frac{x\left(c\left(b^2d-2a(cd-af)-\frac{ab(ce+ag)}{c}\right)+\left(bc(cd+af)-ab^2g-2ac(ce-ag)\right)x^2\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b(cd+af)\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.83305, size = 512, normalized size = 1.14

$$\frac{\frac{2\sqrt{c}x\left(b\left(a^2(-g)-ace+acfx^2+c^2dx^2\right)+b^2\left(cd-agx^2\right)+2ac\left(a\left(f+gx^2\right)-c\left(d+ex^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(bc\left(8a^2g+cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+4ace\right)-2ac\left(ce\sqrt{b^2-\cdots}\right.\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2\cdots}}}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*Sqrt[c]*x*(b*(-(a*c*e) - a^2*g + c^2*d*x^2 + a*c*f*x^2) + b^2*(c*d - a*
g*x^2) + 2*a*c*(-(c*(d + e*x^2)) + a*(f + g*x^2))))/((b^2 - 4*a*c)*(a + b*x
^2 + c*x^4)) + (Sqrt[2]*(-(a*b^3*g) + b*c*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c*e
+ a*Sqrt[b^2 - 4*a*c]*f + 8*a^2*g) + b^2*(c^2*d - a*c*f + a*Sqrt[b^2 - 4*a*
c]*g) - 2*a*c*(6*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 2*a*c*f + 3*a*Sqrt[b^2 - 4
*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 -
4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(a*b^3*g + b*c*(c*Sqrt
[b^2 - 4*a*c]*d - 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f - 8*a^2*g) + 2*a*c*(6*c^2
*d - c*Sqrt[b^2 - 4*a*c]*e + 2*a*c*f - 3*a*Sqrt[b^2 - 4*a*c]*g) + b^2*(-(c^
2*d) + a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*
a*c^(3/2))
```

Maple [B] time = 0.042, size = 1760, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2/(4*a*c-b^2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+1/4/a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/
2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b
^2)^(1/2)-b)*c)^(1/2))*b^2*d+1/4/a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))*b^2*d+1/4/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*
arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*f-1/2/(4*a*c-b^2)*c
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2))*e-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*f+(-1/2/a*(2*a^
2*c*g-a*b^2*g+a*b*c*f-2*a*c^2*e+b*c^2*d)/(4*a*c-b^2)/c*x^3+1/2*(a^2*b*g-2*a
^2*c*f+a*b*c*e+2*a*c^2*d-b^2*c*d)/a/c/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)-3/2*a/
(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((
(-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*g+3/2*a/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*g+1/
4/(4*a*c-b^2)/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2
)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*g-1/4/(4*a*c-b^2)/c*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*b^2*g-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b
)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*f-3/(4
*a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*a
rctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d-1/4/(4*a*c-b^2)/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*f-3/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*d+1/4/a/(4*a*c-b^2)*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*
c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d-a/(4*a*c
-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh
(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f+1/(4*a*c-b^2)*c/(-4*a*c+b^
2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4
*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*e-1/4/a/(4*a*c-b^2)*c*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d
-a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f+1/(4*a*c-b^2)*c/(-
4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e+2*a/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)
^(1/2)-b)*c)^(1/2))*b*g-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2))*b^3*g+2*a/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*g-1/4/(
4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*g
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(bc^2d - 2\,ac^2e + abcf - \left(ab^2 - 2\,a^2c\right)g\right)x^3 - \left(abce - 2\,a^2cf + a^2bg - \left(b^2c - 2\,ac^2\right)d\right)x}{2\left(a^2b^2c - 4\,a^3c^2 + \left(ab^2c^2 - 4\,a^2c^3\right)x^4 + \left(ab^3c - 4\,a^2bc^2\right)x^2\right)} - \frac{-\int \frac{abce - 2\,a^2cf + a^2bg + \left(bc^2d - 2\,ac^2e + \cdots\right.}{cx^4 + \cdots}}{2\left(ab^2c \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c^2*d - 2*a*c^2*e + a*b*c*f - (a*b^2 - 2*a^2*c)*g)*x^3 - (a*b*c*e -
 2*a^2*c*f + a^2*b*g - (b^2*c - 2*a*c^2)*d)*x)/(a^2*b^2*c - 4*a^3*c^2 + (a*
b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2) - 1/2*integrate(-(a
*b*c*e - 2*a^2*c*f + a^2*b*g + (b*c^2*d - 2*a*c^2*e + a*b*c*f + (a*b^2 - 6*
a^2*c)*g)*x^2 + (b^2*c - 6*a*c^2)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2*c - 4*a
^2*c^2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.129 $\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=460

$$-\frac{x\left(a\left(-2a^2g+\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+x^2\left(-ab(ag+ce)-2ac(cd-af)+b^2cd\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(4a\right.}{}$$

```
[Out] -(d/(a^2*x)) - (x*(a*((b^3*d)/a - b*(3*c*d + b*e) + a*(2*c*e + b*f) - 2*a^2
*g) + (b^2*c*d - 2*a*c*(c*d - a*f) - a*b*(c*e + a*g))*x^2))/(2*a^2*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^2*c*d - 2*a*c*(5*c*d - a*f) - a*b*(c*e +
 a*g) + (3*b^3*c*d - 4*a*b*c*(4*c*d + a*f) - a*b^2*(c*e - a*g) + 4*a^2*c*(3
*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) - ((3*b^2*c*d - 2*a*c*(5*c*d - a*f) - a*b*(c*e + a*g) - (3*b^3*c*d - 4
*a*b*c*(4*c*d + a*f) - a*b^2*(c*e - a*g) + 4*a^2*c*(3*c*e + a*g))/Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[
2]*a^2*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 2.79105, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1669, 1664, 1166, 205}

$$-\frac{x\left(a\left(-2a^2g+\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+x^2\left(-ab(ag+ce)-2ac(cd-af)+b^2cd\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(4a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(d/(a^2*x)) - (x*(a*((b^3*d)/a - b*(3*c*d + b*e) + a*(2*c*e + b*f) - 2*a^2
*g) + (b^2*c*d - 2*a*c*(c*d - a*f) - a*b*(c*e + a*g))*x^2))/(2*a^2*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^2*c*d - 2*a*c*(5*c*d - a*f) - a*b*(c*e +
 a*g) + (3*b^3*c*d - 4*a*b*c*(4*c*d + a*f) - a*b^2*(c*e - a*g) + 4*a^2*c*(3
*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) - ((3*b^2*c*d - 2*a*c*(5*c*d - a*f) - a*b*(c*e + a*g) - (3*b^3*c*d - 4
*a*b*c*(4*c*d + a*f) - a*b^2*(c*e - a*g) + 4*a^2*c*(3*c*e + a*g))/Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[
2]*a^2*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
```

```
NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)-2a^2g\right)+\left(b^2cd-2ac(cd-af)-ab(ce+ag)\right)x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} -$$

$$= -\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)-2a^2g\right)+\left(b^2cd-2ac(cd-af)-ab(ce+ag)\right)x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} -$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)-2a^2g\right)+\left(b^2cd-2ac(cd-af)-ab(ce+ag)\right)\right.}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)-2a^2g\right)+\left(b^2cd-2ac(cd-af)-ab(ce+ag)\right)\right.}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)-2a^2g\right)+\left(b^2cd-2ac(cd-af)-ab(ce+ag)\right)\right.}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 2.63774, size = 529, normalized size = 1.15

$$-\frac{-\frac{2x\left(2a\left(a^2g-ac\left(e+fx^2\right)+c^2dx^2\right)+b^2\left(ae-cdx^2\right)+ab\left(-af+agx^2+3cd+cex^2\right)+b^3(-d)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(2ac\left(2a^2g-5cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+6ace\right)+b^2\right.}{\sqrt{c}\left(b^2-4ac\right)^{3}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -((4*d)/x - (2*x*(-(b^3*d) + b^2*(a*e - c*d*x^2) + a*b*(3*c*d - a*f + c*e*x
^2 + a*g*x^2) + 2*a*(a^2*g + c^2*d*x^2 - a*c*(e + f*x^2))))/((b^2 - 4*a*c)*
```

```
(a + b*x^2 + c*x^4)) + (Sqrt[2]*(3*b^3*c*d + b^2*(3*c*Sqrt[b^2 - 4*a*c]*d -
 a*c*e + a^2*g) + 2*a*c*(-5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e + a*Sqrt[b^2 -
4*a*c]*f + 2*a^2*g) - a*b*(16*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 4*a*c*f + a*S
qrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-3
*b^3*c*d + b^2*(3*c*Sqrt[b^2 - 4*a*c]*d + a*c*e - a^2*g) - 2*a*c*(5*c*Sqrt[
b^2 - 4*a*c]*d + 6*a*c*e - a*Sqrt[b^2 - 4*a*c]*f + 2*a^2*g) + a*b*(16*c^2*d
 - c*Sqrt[b^2 - 4*a*c]*e + 4*a*c*f - a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqr
t[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

Maple [B] time = 0.045, size = 2045, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*e+4/a*c^2/(4
*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arcta
nh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d+1/4/a*c/(4*a*c-b^2)/(-
4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*b*d-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/
2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2
)-b)*c)^(1/2))*b^3*d-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))*b^3*d-a*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)
-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*g-a*c/(4
*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*g+5/2/a*c^2/(4*a*c-b^2)*2^(
1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1
/2)-b)*c)^(1/2))*d-5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d-3*c^2/(4*a*c-
b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*
x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e-3*c^2/(4*a*c-b^2)/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*e-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b*e-3/
2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b*c*d+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^
2)*x^3*b^2*d-1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b*g-a/(c*x^4+b*x^2+a)/(4*a
*c-b^2)*x*g+c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*f+1/2/(c*x^4+b*x^2+a)/(4*a*c-
b^2)*x*b*f+1/4/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh
(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*g-1/4/(4*a*c-b^2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*b*g-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*d-1/2/a/(c*x^4+b*x^2
+a)/(4*a*c-b^2)*x*b^2*e+1/2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^3*d-1/2*c/(
4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((
-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f+1/2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f+1/(
c*x^4+b*x^2+a)/(4*a*c-b^2)*x*c*e-d/a^2/x-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*b^2*g-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2))*b^2*g+c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b
```

```
)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*f+c/(4*a
*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*f-1/4/a*c/(4*a*c-b^2)*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))*b*e-3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^
(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*d+3/4/a^2*c
/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d+1/4/a*c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c
+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2
))*b*e
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**6+f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.130 $\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=542

$$\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-a(bg+2cf)+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2(ce-ag)-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\tan}{}$$

```
[Out] -d/(3*a^2*x^3) + (2*b*d - a*e)/(a^3*x) + (x*(a^2*((b^4*d)/a^2 + 2*c^2*d + 3
*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f - a*(2*c*f + b*g)) + c*(b^3*d - a*b^
2*e - a*b*(3*c*d - a*f) + 2*a^2*(c*e - a*g))*x^2))/(2*a^3*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[c]*(5*b^3*d - 3*a*b^2*e - a*b*(19*c*d - a*f) + 2*
a^2*(5*c*e - a*g) + (5*b^4*d - 3*a*b^3*e + 4*a^2*c*(7*c*d - 3*a*f) - a*b^2*
(29*c*d - a*f) + 4*a^2*b*(4*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(5*b^3*d - 3*a*b^2*e - a*b*(19*c*d - a*f
) + 2*a^2*(5*c*e - a*g) - (5*b^4*d - 3*a*b^3*e + 4*a^2*c*(7*c*d - 3*a*f) -
a*b^2*(29*c*d - a*f) + 4*a^2*b*(4*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)
*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 7.26496, antiderivative size = 542, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1669, 1664, 1166, 205}

$$\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-a(bg+2cf)+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2(ce-ag)-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```


```
[Out] -d/(3*a^2*x^3) + (2*b*d - a*e)/(a^3*x) + (x*(a^2*((b^4*d)/a^2 + 2*c^2*d + 3
*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f - a*(2*c*f + b*g)) + c*(b^3*d - a*b^
2*e - a*b*(3*c*d - a*f) + 2*a^2*(c*e - a*g))*x^2))/(2*a^3*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[c]*(5*b^3*d - 3*a*b^2*e - a*b*(19*c*d - a*f) + 2*
a^2*(5*c*e - a*g) + (5*b^4*d - 3*a*b^3*e + 4*a^2*c*(7*c*d - 3*a*f) - a*b^2*
(29*c*d - a*f) + 4*a^2*b*(4*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(5*b^3*d - 3*a*b^2*e - a*b*(19*c*d - a*f
) + 2*a^2*(5*c*e - a*g) - (5*b^4*d - 3*a*b^3*e + 4*a^2*c*(7*c*d - 3*a*f) -
a*b^2*(29*c*d - a*f) + 4*a^2*b*(4*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)
*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```


Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
```

```
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+bg)\right)+c\left(b^3d-ab^2e-ab(3cd-af)+\right.\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+bg)\right)+c\left(b^3d-ab^2e-ab(3cd-af)+\right.\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+bg)\right)+c\left(b^3d-\right.\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+bg)\right)+c\left(b^3d-\right.\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+bg)\right)+c\left(b^3d-\right.\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 2.33592, size = 612, normalized size = 1.13

$$\frac{\frac{6x\left(ab\left(a^2(-g)+ac\left(3e+fx^2\right)-3c^2dx^2\right)+2a^2c\left(c\left(d+ex^2\right)-a\left(f+gx^2\right)\right)+ab^2\left(af-c\left(4d+ex^2\right)\right)+b^3\left(cdx^2-ae\right)+b^4d\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-2a^2\left(-5ce\sqrt{b^2-4ac}+a\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*a*d)/x^3 + (24*b*d - 12*a*e)/x + (6*x*(b^4*d + b^3*(-(a*e) + c*d*x^2)
+ a*b^2*(a*f - c*(4*d + e*x^2)) + a*b*(-(a^2*g) - 3*c^2*d*x^2 + a*c*(3*e +
f*x^2)) + 2*a^2*c*(c*(d + e*x^2) - a*(f + g*x^2)))))/((b^2 - 4*a*c)*(a + b*x
^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d - 3
*a*e) + a*b^2*(-29*c*d - 3*Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(-19*c*Sqrt[b^2
 - 4*a*c]*d + 16*a*c*e + a*Sqrt[b^2 - 4*a*c]*f + 4*a^2*g) - 2*a^2*(-14*c^2*
d - 5*c*Sqrt[b^2 - 4*a*c]*e + 6*a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(5*b^4*d - b^3*(5*Sqrt[b^2 - 4*a*
c]*d + 3*a*e) + a*b^2*(-29*c*d + 3*Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(19*c*S
qrt[b^2 - 4*a*c]*d + 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f + 4*a^2*g) + 2*a^2*(1
4*c^2*d - 5*c*Sqrt[b^2 - 4*a*c]*e - 6*a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

Maple [B] time = 0.048, size = 2503, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*g-1/2*c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+
b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
)*g-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*e-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-
b^2)*x*b^2*f-1/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*c^2*d+1/2/a^2/(c*x^4+b*x^2+a
)/(4*a*c-b^2)*x*b^3*e-1/2/a^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^4*d-3/4/a^2*c
/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*e-29/4/a^2*c^2/(4*a*
c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c
*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d+5/4/a^3*c/(4*a*c-b^2)/(-
4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/
2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^4*d+5/4/a^3*c/(4*a*c-b^2)/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*d-29/4/a^2*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/
2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b
^2)^(1/2)-b)*c)^(1/2))*b^2*d+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))*b^2*f+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+
b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
)*b^2*f+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)
-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*e-3/4/
a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/
2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*e+4/a*c^2/(4*a
*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e+1/(c*x^4+b*x^2+a)/(4*a*c-
b^2)*x*c*f-1/a^2/x*e+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b*g+1/(c*x^4+b*x^2+a
)*c/(4*a*c-b^2)*x^3*g+2/a^3/x*b*d-1/3*d/a^2/x^3+7/a*c^3/(4*a*c-b^2)/(-4*a*c
+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((
(-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d-1/4/a*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*
f+1/4/a*c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*
2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*f-3/4/a^2*c/(4*a*c-b^2)*2^(1/2)
```

```
/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b)*c)^(1/2))*b^2*e-19/4/a^2*c^2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)
*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d+3/4/a^2
*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e+19/4/a^2*c^2/(4*a*c-b^2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*b*d+7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d+5/
4/a^3*c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^
(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*d-5/4/a^3*c/(4*a*c-b^2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))*b^3*d+c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*g+
c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*g-1/2/a^3/(c*x^4+b*x^
2+a)*c/(4*a*c-b^2)*x^3*b^3*d-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b*f+1/
2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b^2*e+2/a^2/(c*x^4+b*x^2+a)/(4*a*c-
b^2)*x*b^2*c*d+3/2/a^2/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*b*d+5/2/a*c^2/(4
*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-
4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e-5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e-
3*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/
2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f-3*c^2/(4*a*c-b^2
)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f-3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)
*x*b*c*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(a^2bcf-2\,a^3cg+\left(5\,b^3c-19\,abc^2\right)d-\left(3\,ab^2c-10\,a^2c^2\right)e\right)x^6-\left(3\,a^3bg-\left(15\,b^4-62\,ab^2c+14\,a^2c^2\right)d+3\left(3\,ab^3-11\,a^2\right.\right.}{6\left(\left(a^3b^2c-4\,a^4c^2\right)x^7+\left(a^3b^3-4\,a^4bc\right)x^5+\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] 1/6*(3*(a^2*b*c*f - 2*a^3*c*g + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10*
a^2*c^2)*e)*x^6 - (3*a^3*b*g - (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d + 3*(3*
a*b^3 - 11*a^2*b*c)*e - 3*(a^2*b^2 - 2*a^3*c)*f)*x^4 + 2*(5*(a*b^3 - 4*a^2*
b*c)*d - 3*(a^2*b^2 - 4*a^3*c)*e)*x^2 - 2*(a^2*b^2 - 4*a^3*c)*d)/((a^3*b^2*
c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) -
 1/2*integrate(-(a^3*b*g + (a^2*b*c*f - 2*a^3*c*g + (5*b^3*c - 19*a*b*c^2)*
d - (3*a*b^2*c - 10*a^2*c^2)*e)*x^2 + (5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*d -
 (3*a*b^3 - 13*a^2*b*c)*e + (a^2*b^2 - 6*a^3*c)*f)/(c*x^4 + b*x^2 + a), x)/
(a^3*b^2 - 4*a^4*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**6+f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.131 $\int x^2 \left(a+bx^2+cx^4\right)^p \left(3a+b(5+2p)x^2+c(7+4p)x^4\right) dx$

Optimal. Leaf size=20

$$x^3 \left(a+bx^2+cx^4\right)^{p+1}$$

```
[Out] x^3*(a + b*x^2 + c*x^4)^(1 + p)
```

Rubi [A] time = 0.0363838, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {1588}

$$x^3 \left(a+bx^2+cx^4\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^p*(3*a + b*(5 + 2*p)*x^2 + c*(7 + 4*p)*x^4),x]
```

```
[Out] x^3*(a + b*x^2 + c*x^4)^(1 + p)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2 \left(a+bx^2+cx^4\right)^p \left(3a+b(5+2p)x^2+c(7+4p)x^4\right) dx = x^3 \left(a+bx^2+cx^4\right)^{1+p}$$

Mathematica [A] time = 0.147311, size = 20, normalized size = 1.

$$x^3 \left(a+bx^2+cx^4\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4)^p*(3*a + b*(5 + 2*p)*x^2 + c*(7 + 4*p)*x^
4),x]
```

```
[Out] x^3*(a + b*x^2 + c*x^4)^(1 + p)
```

Maple [A] time = 0.014, size = 21, normalized size = 1.1

$$x^3 \left(cx^4+bx^2+a\right)^{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x)
```

```
[Out] x^3*(c*x^4+b*x^2+a)^(1+p)
```

Maxima [A] time = 1.23919, size = 42, normalized size = 2.1

$$\left(cx^7 + bx^5 + ax^3\right)\left(cx^4 + bx^2 + a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algori
thm="maxima")
```

```
[Out] (c*x^7 + b*x^5 + a*x^3)*(c*x^4 + b*x^2 + a)^p
```

Fricas [A] time = 1.85655, size = 63, normalized size = 3.15

$$\left(cx^7 + bx^5 + ax^3\right)\left(cx^4 + bx^2 + a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algori
thm="fricas")
```

```
[Out] (c*x^7 + b*x^5 + a*x^3)*(c*x^4 + b*x^2 + a)^p
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**p*(3*a+b*(5+2*p)*x**2+c*(7+4*p)*x**4),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.1973, size = 78, normalized size = 3.9

$$\left(cx^4 + bx^2 + a\right)^p cx^7 + \left(cx^4 + bx^2 + a\right)^p bx^5 + \left(cx^4 + bx^2 + a\right)^p ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algori
thm="giac")
```

```
[Out] (c*x^4 + b*x^2 + a)^p*c*x^7 + (c*x^4 + b*x^2 + a)^p*b*x^5 + (c*x^4 + b*x^2
+ a)^p*a*x^3
```

3.132 $\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=210

$$-\frac{(d-ex)^{5/2}(d+ex)^{5/2}\left(ae^4+3bd^2e^2+6cd^4\right)}{5e^{10}}+\frac{d^2(d-ex)^{3/2}(d+ex)^{3/2}\left(2ae^4+3bd^2e^2+4cd^4\right)}{3e^{10}}-\frac{d^4\sqrt{d-ex}\sqrt{d+ex}\left(ae^4\right.}{e^{10}}$$

```
[Out] -((d^4*(c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d - e*x]*Sqrt[d + e*x])/e^10) + (d^
2*(4*c*d^4 + 3*b*d^2*e^2 + 2*a*e^4)*(d - e*x)^(3/2)*(d + e*x)^(3/2))/(3*e^1
0) - ((6*c*d^4 + 3*b*d^2*e^2 + a*e^4)*(d - e*x)^(5/2)*(d + e*x)^(5/2))/(5*e
^10) + ((4*c*d^2 + b*e^2)*(d - e*x)^(7/2)*(d + e*x)^(7/2))/(7*e^10) - (c*(d
 - e*x)^(9/2)*(d + e*x)^(9/2))/(9*e^10)
```

Rubi [A] time = 0.314532, antiderivative size = 278, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {520, 1251, 897, 1153}

$$-\frac{\left(d^2-e^2x^2\right)^3\left(ae^4+3bd^2e^2+6cd^4\right)}{5e^{10}\sqrt{d-ex}\sqrt{d+ex}}+\frac{d^2\left(d^2-e^2x^2\right)^2\left(2ae^4+3bd^2e^2+4cd^4\right)}{3e^{10}\sqrt{d-ex}\sqrt{d+ex}}-\frac{d^4\left(d^2-e^2x^2\right)\left(ae^4+bd^2e^2+cd^4\right)}{e^{10}\sqrt{d-ex}\sqrt{d+ex}}+\frac{\left(d^2\right.}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((d^4*(c*d^4 + b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2))/(e^10*Sqrt[d - e*x]*Sqr
t[d + e*x])) + (d^2*(4*c*d^4 + 3*b*d^2*e^2 + 2*a*e^4)*(d^2 - e^2*x^2)^2)/(3
*e^10*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((6*c*d^4 + 3*b*d^2*e^2 + a*e^4)*(d^2
- e^2*x^2)^3)/(5*e^10*Sqrt[d - e*x]*Sqrt[d + e*x]) + ((4*c*d^2 + b*e^2)*(d^
2 - e^2*x^2)^4)/(7*e^10*Sqrt[d - e*x]*Sqrt[d + e*x]) - (c*(d^2 - e^2*x^2)^5
)/(9*e^10*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{x^5\left(a+bx^2+cx^4\right)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{x^5\left(a+bx^2+cx^4\right)}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$
$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{x^2\left(a+bx+cx^2\right)}{\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^2\left(\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{\left(2cd^2+be^2\right)x^2}{e^4}+\frac{cx^4}{e^4}\right)dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \left(\frac{cd^8+bd^6e^2+ad^4e^4}{e^8}-\frac{d^2\left(4cd^4+3bd^2e^2+2ae^4\right)x^2}{e^8}+\frac{\left(6cd^4+3bd^2e^2+ae^4\right)x^4}{e^8}-\frac{\left(4cd^2+be^{2}\right.}{e^8}\right.\right.}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{d^4\left(cd^4+bd^2e^2+ae^4\right)\left(d^2-e^2x^2\right)}{e^{10}\sqrt{d-ex}\sqrt{d+ex}}+\frac{d^2\left(4cd^4+3bd^2e^2+2ae^4\right)\left(d^2-e^2x^2\right)^2}{3e^{10}\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(6cd^4+3b\right.}{5e^{1}}$$

Mathematica [C] time = 1.43718, size = 265, normalized size = 1.26

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(21ae^4\left(4d^2e^2x^2+8d^4+3e^4x^4\right)+9b\left(6d^2e^6x^4+8d^4e^4x^2+16d^6e^2+5e^8x^6\right)+c\left(64d^6e^2x^2+48d^4e^4x\right.\right.}{315e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(Sqrt[d - e*x]*Sqrt[d + e*x]*(21*a*e^4*(8*d^4 + 4*d^2*e^2*x^2 + 3*e^4*x^4)
 + 9*b*(16*d^6*e^2 + 8*d^4*e^4*x^2 + 6*d^2*e^6*x^4 + 5*e^8*x^6) + c*(128*d^
8 + 64*d^6*e^2*x^2 + 48*d^4*e^4*x^4 + 40*d^2*e^6*x^6 + 35*e^8*x^8)) + (630*
d^(9/2)*(c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d + e*x]*ArcSin[Sqrt[d - e*x]/(Sqr
t[2]*Sqrt[d])])/Sqrt[1 + (e*x)/d] - 630*d^5*(c*d^4 + b*d^2*e^2 + a*e^4)*Arc
Tan[Sqrt[d - e*x]/Sqrt[d + e*x]])/(315*e^10)
```

Maple [A] time = 0.006, size = 145, normalized size = 0.7

$$-\frac{35\,cx^8e^8+45\,be^8x^6+40\,cd^2e^6x^6+63\,ae^8x^4+54\,bd^2e^6x^4+48\,cd^4e^4x^4+84\,ad^2e^6x^2+72\,bd^4e^4x^2+64\,cd^6e^2x^2+168}{315\,e^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/315*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(35*c*e^8*x^8+45*b*e^8*x^6+40*c*d^2*e^6
*x^6+63*a*e^8*x^4+54*b*d^2*e^6*x^4+48*c*d^4*e^4*x^4+84*a*d^2*e^6*x^2+72*b*d
^4*e^4*x^2+64*c*d^6*e^2*x^2+168*a*d^4*e^4+144*b*d^6*e^2+128*c*d^8)/e^10
```

Maxima [A] time = 1.64729, size = 398, normalized size = 1.9

$$-\frac{\sqrt{-e^2x^2+d^2}cx^8}{9e^2}-\frac{8\sqrt{-e^2x^2+d^2}cd^2x^6}{63e^4}-\frac{\sqrt{-e^2x^2+d^2}bx^6}{7e^2}-\frac{16\sqrt{-e^2x^2+d^2}cd^4x^4}{105e^6}-\frac{6\sqrt{-e^2x^2+d^2}bd^2x^4}{35e^4}-\frac{\sqrt{-e^2x^2+}}{5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/9*sqrt(-e^2*x^2 + d^2)*c*x^8/e^2 - 8/63*sqrt(-e^2*x^2 + d^2)*c*d^2*x^6/e
^4 - 1/7*sqrt(-e^2*x^2 + d^2)*b*x^6/e^2 - 16/105*sqrt(-e^2*x^2 + d^2)*c*d^4
*x^4/e^6 - 6/35*sqrt(-e^2*x^2 + d^2)*b*d^2*x^4/e^4 - 1/5*sqrt(-e^2*x^2 + d^
2)*a*x^4/e^2 - 64/315*sqrt(-e^2*x^2 + d^2)*c*d^6*x^2/e^8 - 8/35*sqrt(-e^2*x
^2 + d^2)*b*d^4*x^2/e^6 - 4/15*sqrt(-e^2*x^2 + d^2)*a*d^2*x^2/e^4 - 128/315
*sqrt(-e^2*x^2 + d^2)*c*d^8/e^10 - 16/35*sqrt(-e^2*x^2 + d^2)*b*d^6/e^8 - 8
/15*sqrt(-e^2*x^2 + d^2)*a*d^4/e^6
```

Fricas [A] time = 1.77388, size = 317, normalized size = 1.51

$$-\frac{\left(35\,ce^8x^8+128\,cd^8+144\,bd^6e^2+168\,ad^4e^4+5\left(8\,cd^2e^6+9\,be^8\right)x^6+3\left(16\,cd^4e^4+18\,bd^2e^6+21\,ae^8\right)x^4+4\left(16\,cd^6e^2+\right.\right.}{315\,e^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/315*(35*c*e^8*x^8 + 128*c*d^8 + 144*b*d^6*e^2 + 168*a*d^4*e^4 + 5*(8*c*d
^2*e^6 + 9*b*e^8)*x^6 + 3*(16*c*d^4*e^4 + 18*b*d^2*e^6 + 21*a*e^8)*x^4 + 4*
(16*c*d^6*e^2 + 18*b*d^4*e^4 + 21*a*d^2*e^6)*x^2)*sqrt(e*x + d)*sqrt(-e*x +
 d)/e^10
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25458, size = 328, normalized size = 1.56

$$-\frac{1}{2807562240}\left(315\,cd^8e^{81}+315\,bd^6e^{83}+315\,ad^4e^{85}-\left(840\,cd^7e^{81}+630\,bd^5e^{83}+420\,ad^3e^{85}-\left(1932\,cd^6e^{81}+1071\,bd^4e^{8}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/2807562240*(315*c*d^8*e^81 + 315*b*d^6*e^83 + 315*a*d^4*e^85 - (840*c*d^
7*e^81 + 630*b*d^5*e^83 + 420*a*d^3*e^85 - (1932*c*d^6*e^81 + 1071*b*d^4*e^
83 + 462*a*d^2*e^85 - (2952*c*d^5*e^81 + 1116*b*d^3*e^83 + 252*a*d*e^85 - (
3098*c*d^4*e^81 + 729*b*d^2*e^83 - 5*(440*c*d^3*e^81 + 54*b*d*e^83 - (204*c
*d^2*e^81 + 7*((x*e + d)*c*e^81 - 8*c*d*e^81)*(x*e + d) + 9*b*e^83)*(x*e +
d))*(x*e + d) + 63*a*e^85)*(x*e + d))*(x*e + d))*(x*e + d))*(x*e + d))*sqrt
(x*e + d)*sqrt(-x*e + d)*e^(-1)
```

3.133 $\int \frac{x^3\left(a+bx^2+cx^4\right)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=159

$$\frac{(d-ex)^{3/2}(d+ex)^{3/2}\left(ae^4+2bd^2e^2+3cd^4\right)}{3e^8}-\frac{d^2\sqrt{d-ex}\sqrt{d+ex}\left(ae^4+bd^2e^2+cd^4\right)}{e^8}-\frac{(d-ex)^{5/2}(d+ex)^{5/2}\left(be^2+3cd^2\right)}{5e^8}$$

```
[Out] -((d^2*(c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d - e*x]*Sqrt[d + e*x])/e^8) + ((3*
c*d^4 + 2*b*d^2*e^2 + a*e^4)*(d - e*x)^(3/2)*(d + e*x)^(3/2))/(3*e^8) - ((3
*c*d^2 + b*e^2)*(d - e*x)^(5/2)*(d + e*x)^(5/2))/(5*e^8) + (c*(d - e*x)^(7/
2)*(d + e*x)^(7/2))/(7*e^8)
```

Rubi [A] time = 0.189442, antiderivative size = 213, normalized size of antiderivative = 1.34, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {520, 1251, 771}

$$\frac{\left(d^2-e^2x^2\right)^2\left(ae^4+2bd^2e^2+3cd^4\right)}{3e^8\sqrt{d-ex}\sqrt{d+ex}}-\frac{d^2\left(d^2-e^2x^2\right)\left(ae^4+bd^2e^2+cd^4\right)}{e^8\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)^3\left(be^2+3cd^2\right)}{5e^8\sqrt{d-ex}\sqrt{d+ex}}+\frac{c\left(d^2-e^2x^2\right)^4}{7e^8\sqrt{d-ex}\sqrt{d+e}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((d^2*(c*d^4 + b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2))/(e^8*Sqrt[d - e*x]*Sqrt
[d + e*x])) + ((3*c*d^4 + 2*b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2)^2)/(3*e^8*Sq
rt[d - e*x]*Sqrt[d + e*x]) - ((3*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^3)/(5*e^8*S
qrt[d - e*x]*Sqrt[d + e*x]) + (c*(d^2 - e^2*x^2)^4)/(7*e^8*Sqrt[d - e*x]*Sq
rt[d + e*x])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int \frac{x^3\left(a+bx^2+cx^4\right)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{x^3\left(a+bx^2+cx^4\right)}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$
$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{x\left(a+bx+cx^2\right)}{\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \left(\frac{cd^6+bd^4e^2+ad^2e^4}{e^6\sqrt{d^2-e^2x}} + \frac{\left(-3cd^4-2bd^2e^2-ae^4\right)\sqrt{d^2-e^2x}}{e^6} + \frac{\left(3cd^2+be^2\right)\left(d^2-e^2x\right)^{3/2}}{e^6} - \frac{c\left(d^2-e\right.}{e}\right.\right.}{2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{d^2\left(cd^4+bd^2e^2+ae^4\right)\left(d^2-e^2x^2\right)}{e^8\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(3cd^4+2bd^2e^2+ae^4\right)\left(d^2-e^2x^2\right)^2}{3e^8\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(3cd^2+be^2\right)\left(\right.}{5e^8\sqrt{d-ex}}$$

Mathematica [C] time = 1.0886, size = 232, normalized size = 1.46

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(35ae^4\left(2d^2+e^2x^2\right)+7b\left(4d^2e^4x^2+8d^4e^2+3e^6x^4\right)+3c\left(8d^4e^2x^2+6d^2e^4x^4+16d^6+5e^6x^6\right)\right)+\frac{210}{}}{105e^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(Sqrt[d - e*x]*Sqrt[d + e*x]*(35*a*e^4*(2*d^2 + e^2*x^2) + 7*b*(8*d^4*e^2
+ 4*d^2*e^4*x^2 + 3*e^6*x^4) + 3*c*(16*d^6 + 8*d^4*e^2*x^2 + 6*d^2*e^4*x^4
+ 5*e^6*x^6)) + (210*d^(5/2)*(c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d + e*x]*ArcS
in[Sqrt[d - e*x]/(Sqrt[2]*Sqrt[d])])/Sqrt[1 + (e*x)/d] - 210*d^3*(c*d^4 + b
*d^2*e^2 + a*e^4)*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]])/(105*e^8)
```

Maple [A] time = 0.007, size = 109, normalized size = 0.7

$$-\frac{15\,cx^6e^6+21\,be^6x^4+18\,cd^2e^4x^4+35\,ae^6x^2+28\,bd^2e^4x^2+24\,cd^4e^2x^2+70\,ad^2e^4+56\,bd^4e^2+48\,cd^6}{105\,e^8}\sqrt{ex+d}\sqrt{-ex}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/105*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(15*c*e^6*x^6+21*b*e^6*x^4+18*c*d^2*e^4
*x^4+35*a*e^6*x^2+28*b*d^2*e^4*x^2+24*c*d^4*e^2*x^2+70*a*d^2*e^4+56*b*d^4*e
^2+48*c*d^6)/e^8
```

Maxima [A] time = 1.49211, size = 293, normalized size = 1.84

$$-\frac{\sqrt{-e^2x^2+d^2}cx^6}{7\,e^2} - \frac{6\,\sqrt{-e^2x^2+d^2}cd^2x^4}{35\,e^4} - \frac{\sqrt{-e^2x^2+d^2}bx^4}{5\,e^2} - \frac{8\,\sqrt{-e^2x^2+d^2}cd^4x^2}{35\,e^6} - \frac{4\,\sqrt{-e^2x^2+d^2}bd^2x^2}{15\,e^4} - \frac{\sqrt{-e^2x^2}}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/7*sqrt(-e^2*x^2 + d^2)*c*x^6/e^2 - 6/35*sqrt(-e^2*x^2 + d^2)*c*d^2*x^4/e
^4 - 1/5*sqrt(-e^2*x^2 + d^2)*b*x^4/e^2 - 8/35*sqrt(-e^2*x^2 + d^2)*c*d^4*x
^2/e^6 - 4/15*sqrt(-e^2*x^2 + d^2)*b*d^2*x^2/e^4 - 1/3*sqrt(-e^2*x^2 + d^2)
*a*x^2/e^2 - 16/35*sqrt(-e^2*x^2 + d^2)*c*d^6/e^8 - 8/15*sqrt(-e^2*x^2 + d^
2)*b*d^4/e^6 - 2/3*sqrt(-e^2*x^2 + d^2)*a*d^2/e^4
```

Fricas [A] time = 1.91068, size = 238, normalized size = 1.5

$$-\frac{\left(15\,ce^{6}x^{6}+48\,cd^{6}+56\,bd^{4}e^{2}+70\,ad^{2}e^{4}+3\left(6\,cd^{2}e^{4}+7\,be^{6}\right)x^{4}+\left(24\,cd^{4}e^{2}+28\,bd^{2}e^{4}+35\,ae^{6}\right)x^{2}\right)\sqrt{ex+d}\sqrt{-ex+d}}{105\,e^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/105*(15*c*e^6*x^6 + 48*c*d^6 + 56*b*d^4*e^2 + 70*a*d^2*e^4 + 3*(6*c*d^2*
e^4 + 7*b*e^6)*x^4 + (24*c*d^4*e^2 + 28*b*d^2*e^4 + 35*a*e^6)*x^2)*sqrt(e*x
 + d)*sqrt(-e*x + d)/e^8
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14837, size = 239, normalized size = 1.5

$$-\frac{1}{44728320}\left(105\,cd^{6}e^{49}+105\,bd^{4}e^{51}+105\,ad^{2}e^{53}-\left(210\,cd^{5}e^{49}+140\,bd^{3}e^{51}+70\,ade^{53}-\left(357\,cd^{4}e^{49}+154\,bd^{2}e^{51}-3\left(1\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/44728320*(105*c*d^6*e^49 + 105*b*d^4*e^51 + 105*a*d^2*e^53 - (210*c*d^5*
e^49 + 140*b*d^3*e^51 + 70*a*d*e^53 - (357*c*d^4*e^49 + 154*b*d^2*e^51 - 3*
(124*c*d^3*e^49 + 28*b*d*e^51 - (81*c*d^2*e^49 + 5*((x*e + d)*c*e^49 - 6*c*
d*e^49)*(x*e + d) + 7*b*e^51)*(x*e + d))*(x*e + d) + 35*a*e^53)*(x*e + d))*
(x*e + d))*sqrt(x*e + d)*sqrt(-x*e + d)*e^(-1)
```

3.134 $\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=109

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(ae^4+bd^2e^2+cd^4\right)}{e^6}+\frac{(d-ex)^{3/2}(d+ex)^{3/2}\left(be^2+2cd^2\right)}{3e^6}-\frac{c(d-ex)^{5/2}(d+ex)^{5/2}}{5e^6}$$

```
[Out] -(((c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d - e*x]*Sqrt[d + e*x])/e^6) + ((2*c*d^
2 + b*e^2)*(d - e*x)^(3/2)*(d + e*x)^(3/2))/(3*e^6) - (c*(d - e*x)^(5/2)*(d
 + e*x)^(5/2))/(5*e^6)
```

Rubi [A] time = 0.1221, antiderivative size = 149, normalized size of antiderivative = 1.37, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {520, 1247, 698}

$$-\frac{\left(d^2-e^2x^2\right)\left(ae^4+bd^2e^2+cd^4\right)}{e^6\sqrt{d-ex}\sqrt{d+ex}}+\frac{\left(d^2-e^2x^2\right)^2\left(be^2+2cd^2\right)}{3e^6\sqrt{d-ex}\sqrt{d+ex}}-\frac{c\left(d^2-e^2x^2\right)^3}{5e^6\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(((c*d^4 + b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2))/(e^6*Sqrt[d - e*x]*Sqrt[d +
 e*x])) + ((2*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^2)/(3*e^6*Sqrt[d - e*x]*Sqrt[d
 + e*x]) - (c*(d^2 - e^2*x^2)^3)/(5*e^6*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{x(a+bx^2+cx^4)}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$
$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{a+bx+cx^2}{\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \left(\frac{cd^4+bd^2e^2+ae^4}{e^4\sqrt{d^2-e^2x}} + \frac{(-2cd^2-be^2)\sqrt{d^2-e^2x}}{e^4} + \frac{c(d^2-e^2x)^{3/2}}{e^4}\right) dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{(cd^4+bd^2e^2+ae^4)(d^2-e^2x^2)}{e^6\sqrt{d-ex}\sqrt{d+ex}} + \frac{(2cd^2+be^2)(d^2-e^2x^2)^2}{3e^6\sqrt{d-ex}\sqrt{d+ex}} - \frac{c(d^2-e^2x^2)^3}{5e^6\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [C] time = 0.703123, size = 194, normalized size = 1.78

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(5\left(3ae^4+2bd^2e^2+be^4x^2\right)+c\left(4d^2e^2x^2+8d^4+3e^4x^4\right)\right)+\frac{30\sqrt{d}\sqrt{d+ex}\sin^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{2}\sqrt{d}}\right)(ae^4+bd^2e^2+cd^4)}{\sqrt{\frac{ex}{d}+1}}-30d\tan}{15e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(Sqrt[d - e*x]*Sqrt[d + e*x]*(5*(2*b*d^2*e^2 + 3*a*e^4 + b*e^4*x^2) + c*(8
*d^4 + 4*d^2*e^2*x^2 + 3*e^4*x^4)) + (30*Sqrt[d]*(c*d^4 + b*d^2*e^2 + a*e^4
)*Sqrt[d + e*x]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*Sqrt[d])])/Sqrt[1 + (e*x)/d]
- 30*d*(c*d^4 + b*d^2*e^2 + a*e^4)*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]])/(15
*e^6)
```

Maple [A] time = 0.005, size = 73, normalized size = 0.7

$$-\frac{3\,cx^4e^4+5\,be^4x^2+4\,cd^2e^2x^2+15\,ae^4+10\,bd^2e^2+8\,cd^4}{15\,e^6}\sqrt{-ex+d}\sqrt{ex+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/15*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(3*c*e^4*x^4+5*b*e^4*x^2+4*c*d^2*e^2*x^2
+15*a*e^4+10*b*d^2*e^2+8*c*d^4)/e^6
```

Maxima [A] time = 1.63594, size = 188, normalized size = 1.72

$$-\frac{\sqrt{-e^2x^2+d^2}cx^4}{5\,e^2}-\frac{4\,\sqrt{-e^2x^2+d^2}cd^2x^2}{15\,e^4}-\frac{\sqrt{-e^2x^2+d^2}bx^2}{3\,e^2}-\frac{8\,\sqrt{-e^2x^2+d^2}cd^4}{15\,e^6}-\frac{2\,\sqrt{-e^2x^2+d^2}bd^2}{3\,e^4}-\frac{\sqrt{-e^2x^2+d^2}a}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*c*x^4/e^2 - 4/15*sqrt(-e^2*x^2 + d^2)*c*d^2*x^2/e
^4 - 1/3*sqrt(-e^2*x^2 + d^2)*b*x^2/e^2 - 8/15*sqrt(-e^2*x^2 + d^2)*c*d^4/e
^6 - 2/3*sqrt(-e^2*x^2 + d^2)*b*d^2/e^4 - sqrt(-e^2*x^2 + d^2)*a/e^2
```

Fricas [A] time = 1.81681, size = 162, normalized size = 1.49

$$-\frac{\left(3\,ce^{4}x^{4}+8\,cd^{4}+10\,bd^{2}e^{2}+15\,ae^{4}+\left(4\,cd^{2}e^{2}+5\,be^{4}\right)x^{2}\right)\sqrt{ex+d}\sqrt{-ex+d}}{15\,e^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] -1/15*(3*c*e^4*x^4 + 8*c*d^4 + 10*b*d^2*e^2 + 15*a*e^4 + (4*c*d^2*e^2 + 5*b
*e^4)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d)/e^6
```

Sympy [C] time = 77.8217, size = 350, normalized size = 3.21

$$-\frac{iadG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{d^{2}}{e^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}e^{2}}-\frac{adG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{d^{2}e^{-2i\pi}}{e^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}e^{2}}-\frac{ibd^{3}G_{6,6}^{6,2}\left(\begin{matrix} \\ -\frac{3}{2},-\frac{5}{4},\ldots\end{matrix}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -I*a*d*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0)
, ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**2) - a*d*meijerg(((-1, -3/4, -1/2,
 -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), d**2*exp_polar(-2*I
*pi)/(e**2*x**2))/(4*pi**(3/2)*e**2) - I*b*d**3*meijerg(((-5/4, -3/4), (-1,
 -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), d**2/(e**2*x**2))/(4
*pi**(3/2)*e**4) - b*d**3*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7
/4, -5/4), (-2, -3/2, -3/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi
**(3/2)*e**4) - I*c*d**5*meijerg(((-9/4, -7/4), (-2, -2, -3/2, 1)), ((-5/2,
 -9/4, -2, -7/4, -3/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**6) - c*d*
*5*meijerg(((-3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4), (-3, -5/2,
 -5/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**6)
```

Giac [A] time = 1.15936, size = 153, normalized size = 1.4

$$-\frac{1}{276480}\left(15\,cd^{4}e^{25}+15\,bd^{2}e^{27}-\left(20\,cd^{3}e^{25}+10\,bde^{27}-\left(22\,cd^{2}e^{25}+3\left((xe+d)ce^{25}-4\,cde^{25}\right)(xe+d)+5\,be^{27}\right)(xe+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/276480*(15*c*d^4*e^25 + 15*b*d^2*e^27 - (20*c*d^3*e^25 + 10*b*d*e^27 - (
22*c*d^2*e^25 + 3*((x*e + d)*c*e^25 - 4*c*d*e^25)*(x*e + d) + 5*b*e^27)*(x*
e + d))*(x*e + d) + 15*a*e^29)*sqrt(x*e + d)*sqrt(-x*e + d)*e^(-1)
```

3.135 $\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=93

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)}{d}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(be^2+cd^2\right)}{e^4}+\frac{c(d-ex)^{3/2}(d+ex)^{3/2}}{3e^4}$$

```
[Out] -((((c*d^2 + b*e^2)*Sqrt[d - e*x]*Sqrt[d + e*x])/e^4) + (c*(d - e*x)^(3/2)*(
d + e*x)^(3/2))/(3*e^4) - (a*ArcTanh[(Sqrt[d - e*x]*Sqrt[d + e*x])/d])/d
```

Rubi [A] time = 0.164607, antiderivative size = 151, normalized size of antiderivative = 1.62, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {520, 1251, 897, 1153, 208}

$$-\frac{a\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(be^2+cd^2\right)}{e^4\sqrt{d-ex}\sqrt{d+ex}}+\frac{c\left(d^2-e^2x^2\right)^2}{3e^4\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(((c*d^2 + b*e^2)*(d^2 - e^2*x^2))/(e^4*Sqrt[d - e*x]*Sqrt[d + e*x])) + (c
*(d^2 - e^2*x^2)^2)/(3*e^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - (a*Sqrt[d^2 - e^2
*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(d*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{a+bx+cx^2}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{(2cd^2+be^2)x^2}{e^4}+\frac{cx^4}{e^4}}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \left(b+\frac{cd^2}{e^2}-\frac{cx^2}{e^2}+\frac{a}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\right)dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{(cd^2+be^2)(d^2-e^2x^2)}{e^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{c(d^2-e^2x^2)^2}{3e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(a\sqrt{d^2-e^2x^2}\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2\ldots}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{(cd^2+be^2)(d^2-e^2x^2)}{e^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{c(d^2-e^2x^2)^2}{3e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{a\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [B] time = 0.891118, size = 217, normalized size = 2.33

$$\frac{-\frac{3a\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d\sqrt{d-ex}}+\frac{(e^2x^2-d^2)(3be^2+2cd^2+ce^2x^2)}{e^4\sqrt{d-ex}}-\frac{6d^{3/2}\sqrt{\frac{ex}{d}+1}(be^2+cd^2)\sin^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{2}\sqrt{d}}\right)}{e^4}+\frac{6d\sqrt{d+ex}(be^2+cd^2)\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)}{e^4}}{3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (((-d^2 + e^2*x^2)*(2*c*d^2 + 3*b*e^2 + c*e^2*x^2))/(e^4*Sqrt[d - e*x]) - (
6*d^(3/2)*(c*d^2 + b*e^2)*Sqrt[1 + (e*x)/d]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*S
qrt[d])])/e^4 + (6*d*(c*d^2 + b*e^2)*Sqrt[d + e*x]*ArcTan[Sqrt[d - e*x]/Sqr
t[d + e*x]])/e^4 - (3*a*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])
/(d*Sqrt[d - e*x]))/(3*Sqrt[d + e*x])
```

Maple [C] time = 0.044, size = 143, normalized size = 1.5

$$-\frac{\text{csgn}(d)}{3\,de^4}\sqrt{-ex+d}\sqrt{ex+d}\left(\text{csgn}(d)\,x^2cde^2\sqrt{-e^2x^2+d^2}+3\sqrt{-e^2x^2+d^2}\text{csgn}(d)\,bde^2+2\sqrt{-e^2x^2+d^2}\text{csgn}(d)\,cd^3\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/3*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d*(csgn(d)*x^2*c*d*e^2*(-e^2*x^2+d^2)^(1/
2)+3*(-e^2*x^2+d^2)^(1/2)*csgn(d)*b*d*e^2+2*(-e^2*x^2+d^2)^(1/2)*csgn(d)*c*
d^3+3*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*a*e^4)*csgn(d)/(-e^2*x^2+d
^2)^(1/2)/e^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59832, size = 178, normalized size = 1.91

$$\frac{3\,ae^4\log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right)-\left(cde^2x^2+2\,cd^3+3\,bde^2\right)\sqrt{ex+d}\sqrt{-ex+d}}{3\,de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/3*(3*a*e^4*log((sqrt(e*x + d)*sqrt(-e*x + d) - d)/x) - (c*d*e^2*x^2 + 2*c
*d^3 + 3*b*d*e^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d*e^4)
```

Sympy [C] time = 45.0443, size = 304, normalized size = 3.27

$$\frac{iaG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{aG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibdG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
d**2/(e**2*x**2))/(4*pi**(3/2)*d) - a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()
), ((1/4, 3/4), (0, 1/2, 1/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*
pi**(3/2)*d) - I*b*d*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0
, 1/4, 1/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**2) - b*d*meijerg(((-
1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), d**2*
exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**2) - I*c*d**3*meijerg(((-5/
```

```
4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), d**2/(
e**2*x**2))/(4*pi**(3/2)*e**4) - c*d**3*meijerg(((-2, -7/4, -3/2, -5/4, -1,
 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), d**2*exp_polar(-2*I*pi)/(e**
2*x**2))/(4*pi**(3/2)*e**4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac
")
```

```
[Out] sage0*x
```

3.136 $\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=99

$$-\frac{\left(ae^2+2bd^2\right)\tanh^{-1}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)}{2d^3}-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{2d^2x^2}-\frac{c\sqrt{d-ex}\sqrt{d+ex}}{e^2}$$

```
[Out] -((c*Sqrt[d - e*x]*Sqrt[d + e*x])/e^2) - (a*Sqrt[d - e*x]*Sqrt[d + e*x])/(2
*d^2*x^2) - ((2*b*d^2 + a*e^2)*ArcTanh[(Sqrt[d - e*x]*Sqrt[d + e*x])/d])/(2
*d^3)
```

Rubi [A] time = 0.251905, antiderivative size = 155, normalized size of antiderivative = 1.57, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {520, 1251, 897, 1157, 388, 208}

$$-\frac{\sqrt{d^2-e^2x^2}\left(ae^2+2bd^2\right)\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\left(d^2-e^2x^2\right)}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{c\left(d^2-e^2x^2\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^3*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((c*(d^2 - e^2*x^2))/(e^2*Sqrt[d - e*x]*Sqrt[d + e*x])) - (a*(d^2 - e^2*x^
2))/(2*d^2*x^2*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((2*b*d^2 + a*e^2)*Sqrt[d^2 -
 e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^3*Sqrt[d - e*x]*Sqrt[d + e*x
])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx &= \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^3\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}\\
&= \frac{\sqrt{d^2-e^2x^2}\,\mathrm{Subst}\left(\int \frac{a+bx+cx^2}{x^2\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{\sqrt{d^2-e^2x^2}\,\mathrm{Subst}\left(\int \frac{\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{(2cd^2+be^2)x^2}{e^4}+\frac{cx^4}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^2}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{a\left(d^2-e^2x^2\right)}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}}+\frac{\sqrt{d^2-e^2x^2}\,\mathrm{Subst}\left(\int \frac{-a-\frac{2(cd^4+bd^2e^2)}{e^4}+\frac{2cd^2x^2}{e^4}}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{2d^2\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{c\left(d^2-e^2x^2\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\left(d^2-e^2x^2\right)}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}}+\frac{\left(e^2\left(\frac{2cd^4}{e^6}+\frac{-a-\frac{2(cd^4+bd^2e^2)}{e^4}}{e^2}\right)\sqrt{d^2-e^2x^2}\right)\mathrm{Sub}}{2d^2\sqrt{d-ex}\sqrt{d}}\\
&= -\frac{c\left(d^2-e^2x^2\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\left(d^2-e^2x^2\right)}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(2bd^2+ae^2\right)\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3\sqrt{d-ex}\sqrt{d+ex}}
\end{aligned}$$

Mathematica [B] time = 0.216946, size = 233, normalized size = 2.35

$$\frac{-e^2x^2\sqrt{d^2-e^2x^2}\left(ae^2+2bd^2\right)\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-ad^3e^2+ade^4x^2+2cd^3e^2x^4-4cd^{9/2}x^2\sqrt{d-ex}\sqrt{\frac{ex}{d}+1}\sin^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{2}\sqrt{d}}\right)}{2d^3e^2x^2\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^3*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-(a*d^3*e^2) - 2*c*d^5*x^2 + a*d*e^4*x^2 + 2*c*d^3*e^2*x^4 - 4*c*d^(9/2)*x
^2*Sqrt[d - e*x]*Sqrt[1 + (e*x)/d]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*Sqrt[d])]
+ 4*c*d^4*x^2*Sqrt[d - e*x]*Sqrt[d + e*x]*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x
]] - e^2*(2*b*d^2 + a*e^2)*x^2*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x
^2]/d])/(2*d^3*e^2*x^2*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Maple [C] time = 0.025, size = 163, normalized size = 1.7

$$-\frac{\mathrm{csgn}\,(d)}{2\,d^3e^2x^2}\sqrt{-ex+d}\sqrt{ex+d}\left(\ln\left(2\,\frac{d\left(\sqrt{-e^2x^2+d^2}\mathrm{csgn}\,(d)+d\right)}{x}\right)x^2ae^4+2\,\ln\left(2\,\frac{d\left(\sqrt{-e^2x^2+d^2}\mathrm{csgn}\,(d)+d\right)}{x}\right)x^2bd^2e^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/2*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^3*(ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)
+d)/x)*x^2*a*e^4+2*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*x^2*b*d^2*e^2
+2*csgn(d)*x^2*c*d^3*(-e^2*x^2+d^2)^(1/2)+csgn(d)*a*d*e^2*(-e^2*x^2+d^2)^(1
/2))*csgn(d)/(-e^2*x^2+d^2)^(1/2)/e^2/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29746, size = 215, normalized size = 2.17

$$-\frac{2\,cd^4x^2-\left(2\,bd^2e^2+ae^4\right)x^2\log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right)+\left(2\,cd^3x^2+ade^2\right)\sqrt{ex+d}\sqrt{-ex+d}}{2\,d^3e^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*c*d^4*x^2 - (2*b*d^2*e^2 + a*e^4)*x^2*log((sqrt(e*x + d)*sqrt(-e*x
+ d) - d)/x) + (2*c*d^3*x^2 + a*d*e^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^3*e
^2*x^2)
```

Sympy [C] time = 64.1964, size = 270, normalized size = 2.73

$$\frac{iae^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}-\frac{ae^2G_{6,6}^{2,6}\left(\begin{matrix}1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}+\frac{ibG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{bG_{6,}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**3/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] I*a*e**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0
,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**3) - a*e**2*meijerg(((1, 5/4, 3/2, 7
/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), d**2*exp_polar(-2*I*pi)/(e*
*2*x**2))/(4*pi**(3/2)*d**3) + I*b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((
1/2, 3/4, 1, 5/4, 3/2), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d) - b*meijer
g(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), d**2*exp_
polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d) - I*c*d*meijerg(((-1/4, 1/4), (
0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()), d**2/(e**2*x**2))/(4*pi
**(3/2)*e**2) - c*d*meijerg(((-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/
4), (-1, -1/2, -1/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)
*e**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] sage0*x
```

3.137 $\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=126

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)\left(3ae^4+4bd^2e^2+8cd^4\right)}{8d^5}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(3ae^2+4bd^2\right)}{8d^4x^2}-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{4d^2x^4}$$

```
[Out] -(a*Sqrt[d - e*x]*Sqrt[d + e*x])/(4*d^2*x^4) - ((4*b*d^2 + 3*a*e^2)*Sqrt[d
- e*x]*Sqrt[d + e*x])/(8*d^4*x^2) - ((8*c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*ArcT
anh[(Sqrt[d - e*x]*Sqrt[d + e*x])/d])/(8*d^5)
```

Rubi [A] time = 0.276839, antiderivative size = 182, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {520, 1251, 897, 1157, 385, 208}

$$-\frac{\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\left(3ae^4+4bd^2e^2+8cd^4\right)}{8d^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(3ae^2+4bd^2\right)}{8d^4x^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\left(d^2-e^2x^2\right)}{4d^2x^4\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^5*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(a*(d^2 - e^2*x^2))/(4*d^2*x^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((4*b*d^2 +
3*a*e^2)*(d^2 - e^2*x^2))/(8*d^4*x^2*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((8*c*d
^4 + 4*b*d^2*e^2 + 3*a*e^4)*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]
/d])/(8*d^5*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx &= \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^5\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}\\
&= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{a+bx+cx^2}{x^3\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{\left(2cd^2+be^2\right)x^2}{e^4}+\frac{cx^4}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^3}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{a\left(d^2-e^2x^2\right)}{4d^2x^4\sqrt{d-ex}\sqrt{d+ex}}+\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{-3a-\frac{4\left(cd^4+bd^2e^2\right)}{e^4}+\frac{4cd^2x^2}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^2}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{4d^2\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{a\left(d^2-e^2x^2\right)}{4d^2x^4\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(4bd^2+3ae^2\right)\left(d^2-e^2x^2\right)}{8d^4x^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(\left(4b+\frac{8cd^2}{e^2}+\frac{3ae^2}{d^2}\right)\sqrt{d^2-e^2x^2}\right)\text{Su}}{8d^2\sqrt{d-ex}}\\
&= -\frac{a\left(d^2-e^2x^2\right)}{4d^2x^4\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(4bd^2+3ae^2\right)\left(d^2-e^2x^2\right)}{8d^4x^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(8cd^4+4bd^2e^2+3ae^4\right)\sqrt{d^2-e^2x^2}}{8d^5\sqrt{d-ex}\sqrt{d+e}}
\end{aligned}$$

Mathematica [A] time = 0.167105, size = 134, normalized size = 1.06

$$\frac{x^4\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\left(-\left(3ae^4+4bd^2e^2+8cd^4\right)\right)-d\left(d^2-e^2x^2\right)\left(2ad^2+3ae^2x^2+4bd^2x^2\right)}{8d^5x^4\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^5*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-(d*(d^2 - e^2*x^2)*(2*a*d^2 + 4*b*d^2*x^2 + 3*a*e^2*x^2)) - (8*c*d^4 + 4*
b*d^2*e^2 + 3*a*e^4)*x^4*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
)/(8*d^5*x^4*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Maple [C] time = 0.025, size = 222, normalized size = 1.8

$$-\frac{\operatorname{csgn}(d)}{8\,d^5x^4}\sqrt{-ex+d}\sqrt{ex+d}\left(3\,\ln\left(2\,\frac{d\left(\sqrt{-e^2x^2+d^2}\operatorname{csgn}(d)+d\right)}{x}\right)x^4ae^4+4\,\ln\left(2\,\frac{d\left(\sqrt{-e^2x^2+d^2}\operatorname{csgn}(d)+d\right)}{x}\right)x^4bd^2e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/8*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^5*(3*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(
d)+d)/x)*x^4*a*e^4+4*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*x^4*b*d^2*e
^2+8*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*x^4*c*d^4+3*csgn(d)*x^2*a*d
*e^2*(-e^2*x^2+d^2)^(1/2)+4*csgn(d)*x^2*b*d^3*(-e^2*x^2+d^2)^(1/2)+2*csgn(d
)*a*d^3*(-e^2*x^2+d^2)^(1/2))*csgn(d)/(-e^2*x^2+d^2)^(1/2)/x^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36739, size = 224, normalized size = 1.78

$$\frac{\left(8\,cd^4+4\,bd^2e^2+3\,ae^4\right)x^4\log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right)-\left(2\,ad^3+\left(4\,bd^3+3\,ade^2\right)x^2\right)\sqrt{ex+d}\sqrt{-ex+d}}{8\,d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/8*((8*c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*x^4*log((sqrt(e*x + d)*sqrt(-e*x + d
) - d)/x) - (2*a*d^3 + (4*b*d^3 + 3*a*d*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x +
 d))/(d^5*x^4)
```

Sympy [C] time = 89.724, size = 253, normalized size = 2.01

$$\frac{iae^4G_{6,6}^{5,3}\left(\begin{matrix}\frac{11}{4},\frac{13}{4},1 & 3,3,\frac{7}{2}\\ \frac{5}{2},\frac{11}{4},3,\frac{13}{4},\frac{7}{2} & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^5}-\frac{ae^4G_{6,6}^{2,6}\left(\begin{matrix}2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3,1 & \\ \frac{9}{4},\frac{11}{4} & 2,\frac{5}{2},\frac{5}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^5}+\frac{ibe^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**5/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] I*a*e**4*meijerg(((11/4, 13/4, 1), (3, 3, 7/2)), ((5/2, 11/4, 3, 13/4, 7/2)
, (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**5) - a*e**4*meijerg(((2, 9/4, 5/
2, 11/4, 3, 1), ()), ((9/4, 11/4), (2, 5/2, 5/2, 0)), d**2*exp_polar(-2*I*p
i)/(e**2*x**2))/(4*pi**(3/2)*d**5) + I*b*e**2*meijerg(((7/4, 9/4, 1), (2, 2
, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d*
*3) - b*e**2*meijerg(((1, 5/4, 3/2, 7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2,
3/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d**3) + I*c*mei
jerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), d**2/(e
**2*x**2))/(4*pi**(3/2)*d) - c*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/
4, 3/4), (0, 1/2, 1/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/
2)*d)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] sage0*x
```

3.138 $\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=212

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(5ae^4+6bd^2e^2+8cd^4\right)}{16d^6x^2}-\frac{e^2\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\left(5ae^4+6bd^2e^2+8cd^4\right)}{16d^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{\sqrt{d-ex}\sqrt{d+ex}\,(5}{24d^4x^4}$$

```
[Out] -(a*Sqrt[d - e*x]*Sqrt[d + e*x])/(6*d^2*x^6) - ((6*b*d^2 + 5*a*e^2)*Sqrt[d
- e*x]*Sqrt[d + e*x])/(24*d^4*x^4) - ((8*c*d^4 + 6*b*d^2*e^2 + 5*a*e^4)*Sqr
t[d - e*x]*Sqrt[d + e*x])/(16*d^6*x^2) - (e^2*(8*c*d^4 + 6*b*d^2*e^2 + 5*a*
e^4)*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(16*d^7*Sqrt[d - e
*x]*Sqrt[d + e*x])
```

Rubi [A] time = 0.372665, antiderivative size = 248, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {520, 1251, 897, 1157, 385, 199, 208}

$$-\frac{\left(d^2-e^2x^2\right)\left(5ae^4+6bd^2e^2+8cd^4\right)}{16d^6x^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{e^2\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\left(5ae^4+6bd^2e^2+8cd^4\right)}{16d^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(5ae^2+6bd\right.}{24d^4x^4\sqrt{d-ex}\sqrt{d+e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^7*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(a*(d^2 - e^2*x^2))/(6*d^2*x^6*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((6*b*d^2 +
5*a*e^2)*(d^2 - e^2*x^2))/(24*d^4*x^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((8*c*
d^4 + 6*b*d^2*e^2 + 5*a*e^4)*(d^2 - e^2*x^2))/(16*d^6*x^2*Sqrt[d - e*x]*Sqr
t[d + e*x]) - (e^2*(8*c*d^4 + 6*b*d^2*e^2 + 5*a*e^4)*Sqrt[d^2 - e^2*x^2]*Ar
cTanh[Sqrt[d^2 - e^2*x^2]/d])/(16*d^7*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^7\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{a+bx+cx^2}{x^4\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{(2cd^2+be^2)x^2}{e^4}+\frac{cx^4}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^4}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{6d^2x^6\sqrt{d-ex}\sqrt{d+ex}} + \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{-5a-\frac{6(cd^4+bd^2e^2)}{e^4}+\frac{6cd^2x^2}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^3}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{6d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{6d^2x^6\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(6bd^2+5ae^2\right)\left(d^2-e^2x^2\right)}{24d^4x^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(\left(6b+\frac{8cd^2}{e^2}+\frac{5ae^2}{d^2}\right)\sqrt{d^2-e^2x^2}\right)\text{Subst}}{8d^2\sqrt{d-ex}\sqrt{}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{6d^2x^6\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(6bd^2+5ae^2\right)\left(d^2-e^2x^2\right)}{24d^4x^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(8cd^4+6bd^2e^2+5ae^4\right)\left(d^2-e^2x^2\right)}{16d^6x^2\sqrt{d-ex}\sqrt{d+ex}} -$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{6d^2x^6\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(6bd^2+5ae^2\right)\left(d^2-e^2x^2\right)}{24d^4x^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(8cd^4+6bd^2e^2+5ae^4\right)\left(d^2-e^2x^2\right)}{16d^6x^2\sqrt{d-ex}\sqrt{d+ex}} -$$

Mathematica [A] time = 0.20004, size = 173, normalized size = 0.82

$$\frac{-d\left(d^2-e^2x^2\right)\left(a\left(10d^2e^2x^2+8d^4+15e^4x^4\right)+6\left(3bd^2e^2x^4+2bd^4x^2+4cd^4x^4\right)\right)-3e^2x^6\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)(5a}{48d^7x^6\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^7*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-(d*(d^2 - e^2*x^2)*(6*(2*b*d^4*x^2 + 4*c*d^4*x^4 + 3*b*d^2*e^2*x^4) + a*(
8*d^4 + 10*d^2*e^2*x^2 + 15*e^4*x^4))) - 3*e^2*(8*c*d^4 + 6*b*d^2*e^2 + 5*a
*e^4)*x^6*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(48*d^7*x^6*S
qrt[d - e*x]*Sqrt[d + e*x])
```

Maple [C] time = 0.033, size = 306, normalized size = 1.4

$$-\frac{\text{csgn}(d)}{48\,d^7x^6}\sqrt{-ex+d}\sqrt{ex+d}\left(15\ln\left(2\frac{d\left(\sqrt{-e^2x^2+d^2}\text{csgn}(d)+d\right)}{x}\right)x^6ae^6+18\ln\left(2\frac{d\left(\sqrt{-e^2x^2+d^2}\text{csgn}(d)+d\right)}{x}\right)x^6bd\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/48*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^7*(15*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csg
n(d)+d)/x)*x^6*a*e^6+18*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*x^6*b*d^
2*e^4+24*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*x^6*c*d^4*e^2+15*csgn(d
)*d*(-e^2*x^2+d^2)^(1/2)*x^4*a*e^4+18*csgn(d)*d^3*(-e^2*x^2+d^2)^(1/2)*x^4*
b*e^2+24*csgn(d)*d^5*(-e^2*x^2+d^2)^(1/2)*x^4*c+10*csgn(d)*x^2*a*d^3*e^2*(-
e^2*x^2+d^2)^(1/2)+12*csgn(d)*x^2*b*d^5*(-e^2*x^2+d^2)^(1/2)+8*csgn(d)*a*d^
5*(-e^2*x^2+d^2)^(1/2))*csgn(d)/(-e^2*x^2+d^2)^(1/2)/x^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44854, size = 298, normalized size = 1.41

$$\frac{3\left(8\,cd^4e^2+6\,bd^2e^4+5\,ae^6\right)x^6\log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right)-\left(8\,ad^5+3\left(8\,cd^5+6\,bd^3e^2+5\,ade^4\right)x^4+2\left(6\,bd^5+5\,ad^3e^2\right)x^2\right)}{48\,d^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(8*c*d^4*e^2 + 6*b*d^2*e^4 + 5*a*e^6)*x^6*log((sqrt(e*x + d)*sqrt(-
e*x + d) - d)/x) - (8*a*d^5 + 3*(8*c*d^5 + 6*b*d^3*e^2 + 5*a*d*e^4)*x^4 + 2
*(6*b*d^5 + 5*a*d^3*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^7*x^6)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: MellinTransformStripError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**7/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Exception raised: MellinTransformStripError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] sage0*x
```

3.139 $\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=216

$$\frac{d^2\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(8ae^4+6bd^2e^2+5cd^4\right)}{16e^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{x\sqrt{d-ex}\sqrt{d+ex}\left(8ae^4+6bd^2e^2+5cd^4\right)}{16e^6}-\frac{x^3\sqrt{d-ex}\sqrt{d+\cdots}}{24\cdots}$$

```
[Out] -((5*c*d^4 + 6*b*d^2*e^2 + 8*a*e^4)*x*Sqrt[d - e*x]*Sqrt[d + e*x])/(16*e^6)
 - ((5*c*d^2 + 6*b*e^2)*x^3*Sqrt[d - e*x]*Sqrt[d + e*x])/(24*e^4) + (c*x^5*
(-d + e*x)*Sqrt[d + e*x])/(6*e^2*Sqrt[d - e*x]) + (d^2*(5*c*d^4 + 6*b*d^2*e
^2 + 8*a*e^4)*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^
7*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rubi [A] time = 0.205081, antiderivative size = 245, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {520, 1267, 459, 321, 217, 203}

$$-\frac{x\left(d^2-e^2x^2\right)\left(8ae^4+6bd^2e^2+5cd^4\right)}{16e^6\sqrt{d-ex}\sqrt{d+ex}}+\frac{d^2\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(8ae^4+6bd^2e^2+5cd^4\right)}{16e^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{x^3\left(d^2-e^2x^2\right)\left(6be^2\cdots\right.}{24e^4\sqrt{d-ex}\sqrt{d\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((5*c*d^4 + 6*b*d^2*e^2 + 8*a*e^4)*x*(d^2 - e^2*x^2))/(16*e^6*Sqrt[d - e*x
]*Sqrt[d + e*x]) - ((5*c*d^2 + 6*b*e^2)*x^3*(d^2 - e^2*x^2))/(24*e^4*Sqrt[d
 - e*x]*Sqrt[d + e*x]) - (c*x^5*(d^2 - e^2*x^2))/(6*e^2*Sqrt[d - e*x]*Sqrt[
d + e*x]) + (d^2*(5*c*d^4 + 6*b*d^2*e^2 + 8*a*e^4)*Sqrt[d^2 - e^2*x^2]*ArcT
an[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^7*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
```

```
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(a+bx^2+cx^4\right)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx &= \frac{\sqrt{d^2-e^2x^2}\int \frac{x^2\left(a+bx^2+cx^4\right)}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{cx^5\left(d^2-e^2x^2\right)}{6e^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{\sqrt{d^2-e^2x^2}\int \frac{x^2\left(-6ae^2-\left(5cd^2+6be^2\right)x^2\right)}{\sqrt{d^2-e^2x^2}}\,dx}{6e^2\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{\left(5cd^2+6be^2\right)x^3\left(d^2-e^2x^2\right)}{24e^4\sqrt{d-ex}\sqrt{d+ex}}-\frac{cx^5\left(d^2-e^2x^2\right)}{6e^2\sqrt{d-ex}\sqrt{d+ex}}+\frac{\left(\left(5cd^4+6bd^2e^2+8ae^4\right)\sqrt{d^2-e^2x^2}\right)\int}{8e^4\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{\left(5cd^4+6bd^2e^2+8ae^4\right)x\left(d^2-e^2x^2\right)}{16e^6\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(5cd^2+6be^2\right)x^3\left(d^2-e^2x^2\right)}{24e^4\sqrt{d-ex}\sqrt{d+ex}}-\frac{cx^5\left(d^2-e^2x^2\right)}{6e^2\sqrt{d-ex}\sqrt{d+ex}}+\\
&= -\frac{\left(5cd^4+6bd^2e^2+8ae^4\right)x\left(d^2-e^2x^2\right)}{16e^6\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(5cd^2+6be^2\right)x^3\left(d^2-e^2x^2\right)}{24e^4\sqrt{d-ex}\sqrt{d+ex}}-\frac{cx^5\left(d^2-e^2x^2\right)}{6e^2\sqrt{d-ex}\sqrt{d+ex}}+\\
&= -\frac{\left(5cd^4+6bd^2e^2+8ae^4\right)x\left(d^2-e^2x^2\right)}{16e^6\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(5cd^2+6be^2\right)x^3\left(d^2-e^2x^2\right)}{24e^4\sqrt{d-ex}\sqrt{d+ex}}-\frac{cx^5\left(d^2-e^2x^2\right)}{6e^2\sqrt{d-ex}\sqrt{d+ex}}+
\end{aligned}
$$

Mathematica [A] time = 0.808645, size = 202, normalized size = 0.94

$$
-\frac{ex\sqrt{d-ex}\sqrt{d+ex}\left(6\left(4ae^4+3bd^2e^2+2be^4x^2\right)+c\left(10d^2e^2x^2+15d^4+8e^4x^4\right)\right)-\frac{6d^{3/2}\sqrt{d+ex}\sin^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{2}\sqrt{d}}\right)\left(8ae^4+10bd^2e^2+11cd^4\right)}{\sqrt{\frac{ex}{d}+1}}}{48e^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(e*x*Sqrt[d - e*x]*Sqrt[d + e*x]*(6*(3*b*d^2*e^2 + 4*a*e^4 + 2*b*e^4*x^2)
+ c*(15*d^4 + 10*d^2*e^2*x^2 + 8*e^4*x^4)) - (6*d^(3/2)*(11*c*d^4 + 10*b*d^
```

```
2*e^2 + 8*a*e^4)*Sqrt[d + e*x]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*Sqrt[d])])/Sqr
t[1 + (e*x)/d] + 96*d^2*(c*d^4 + b*d^2*e^2 + a*e^4)*ArcTan[Sqrt[d - e*x]/Sq
rt[d + e*x]])/(48*e^7)
```

Maple [C] time = 0.035, size = 273, normalized size = 1.3

$$-\frac{\operatorname{csgn}(e)}{48\,e^7}\sqrt{-ex+d}\sqrt{ex+d}\left(8\operatorname{csgn}(e)\,x^5ce^5\sqrt{-e^2x^2+d^2}+12\operatorname{csgn}(e)\,x^3be^5\sqrt{-e^2x^2+d^2}+10\operatorname{csgn}(e)\,x^3cd^2e^3\sqrt{-e^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/48*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(8*csgn(e)*x^5*c*e^5*(-e^2*x^2+d^2)^(1/2
)+12*csgn(e)*x^3*b*e^5*(-e^2*x^2+d^2)^(1/2)+10*csgn(e)*x^3*c*d^2*e^3*(-e^2*
x^2+d^2)^(1/2)+24*csgn(e)*e^5*(-e^2*x^2+d^2)^(1/2)*x*a+18*csgn(e)*e^3*(-e^2
*x^2+d^2)^(1/2)*x*b*d^2+15*csgn(e)*e*(-e^2*x^2+d^2)^(1/2)*x*c*d^4-24*arctan
(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/2))*a*d^2*e^4-18*arctan(csgn(e)*e*x/(-e^2*x^
2+d^2)^(1/2))*b*d^4*e^2-15*arctan(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/2))*c*d^6)*
csgn(e)/e^7/(-e^2*x^2+d^2)^(1/2)
```

Maxima [A] time = 1.56008, size = 309, normalized size = 1.43

$$-\frac{\sqrt{-e^2x^2+d^2}cx^5}{6\,e^2}-\frac{5\sqrt{-e^2x^2+d^2}cd^2x^3}{24\,e^4}-\frac{\sqrt{-e^2x^2+d^2}bx^3}{4\,e^2}+\frac{5\,cd^6\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{16\sqrt{e^2}e^6}+\frac{3\,bd^4\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{8\sqrt{e^2}e^4}+\frac{ad^2\operatorname{arcs}}{2\,\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/6*sqrt(-e^2*x^2 + d^2)*c*x^5/e^2 - 5/24*sqrt(-e^2*x^2 + d^2)*c*d^2*x^3/e
^4 - 1/4*sqrt(-e^2*x^2 + d^2)*b*x^3/e^2 + 5/16*c*d^6*arcsin(e^2*x/sqrt(d^2*
e^2))/(sqrt(e^2)*e^6) + 3/8*b*d^4*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^
4) + 1/2*a*d^2*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^2) - 5/16*sqrt(-e^2
*x^2 + d^2)*c*d^4*x/e^6 - 3/8*sqrt(-e^2*x^2 + d^2)*b*d^2*x/e^4 - 1/2*sqrt(-
e^2*x^2 + d^2)*a*x/e^2
```

Fricas [A] time = 1.43201, size = 298, normalized size = 1.38

$$-\frac{\left(8\,ce^5x^5+2\left(5\,cd^2e^3+6\,be^5\right)x^3+3\left(5\,cd^4e+6\,bd^2e^3+8\,ae^5\right)x\right)\sqrt{ex+d}\sqrt{-ex+d}+6\left(5\,cd^6+6\,bd^4e^2+8\,ad^2e^4\right)\operatorname{arc}}{48\,e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/48*((8*c*e^5*x^5 + 2*(5*c*d^2*e^3 + 6*b*e^5)*x^3 + 3*(5*c*d^4*e + 6*b*d^
2*e^3 + 8*a*e^5)*x)*sqrt(e*x + d)*sqrt(-e*x + d) + 6*(5*c*d^6 + 6*b*d^4*e^2
```

```
+ 8*a*d^2*e^4)*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(e*x)))/e^7
```

Sympy [C] time = 125.927, size = 362, normalized size = 1.68

$$-\frac{iad^2G_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1 \\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^3}+\frac{ad^2G_{6,6}^{2,6}\left(\begin{matrix} -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^3}-\frac{ibd^4G_{6,6}^{6,2}\left(\begin{matrix} \\ -2 \end{matrix}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -I*a*d**2*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/
4, 0, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**3) + a*d**2*meijerg(((-3/2
, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), d**2*ex
p_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**3) - I*b*d**4*meijerg(((-7/4,
 -5/4), (-3/2, -3/2, -1, 1)), ((-2, -7/4, -3/2, -5/4, -1, 0), ()), d**2/(e*
*2*x**2))/(4*pi**(3/2)*e**5) + b*d**4*meijerg(((-5/2, -9/4, -2, -7/4, -3/2,
 1), ()), ((-9/4, -7/4), (-5/2, -2, -2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*
x**2))/(4*pi**(3/2)*e**5) - I*c*d**6*meijerg(((-11/4, -9/4), (-5/2, -5/2, -
2, 1)), ((-3, -11/4, -5/2, -9/4, -2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2
)*e**7) + c*d**6*meijerg(((-7/2, -13/4, -3, -11/4, -5/2, 1), ()), ((-13/4,
-11/4), (-7/2, -3, -3, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/
2)*e**7)
```

Giac [A] time = 1.1521, size = 257, normalized size = 1.19

$$\frac{1}{34603008}\left(\left(33\,cd^5e^{36}+30\,bd^3e^{38}+24\,ade^{40}-\left(85\,cd^4e^{36}+54\,bd^2e^{38}-2\left(55\,cd^3e^{36}+18\,bde^{38}-\left(45\,cd^2e^{36}+4\left((xe+d)\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/34603008*((33*c*d^5*e^36 + 30*b*d^3*e^38 + 24*a*d*e^40 - (85*c*d^4*e^36 +
 54*b*d^2*e^38 - 2*(55*c*d^3*e^36 + 18*b*d*e^38 - (45*c*d^2*e^36 + 4*((x*e
+ d)*c*e^36 - 5*c*d*e^36)*(x*e + d) + 6*b*e^38)*(x*e + d))*(x*e + d) + 24*a
*e^40)*(x*e + d))*sqrt(x*e + d)*sqrt(-x*e + d) + 6*(5*c*d^6*e^36 + 6*b*d^4*
e^38 + 8*a*d^2*e^40)*arcsin(1/2*sqrt(2)*sqrt(x*e + d)/sqrt(d)))*e^(-1)
```

3.140 $\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=128

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)\left(8ae^4+4bd^2e^2+3cd^4\right)}{4e^5}-\frac{x\sqrt{d-ex}\sqrt{d+ex}\left(4be^2+3cd^2\right)}{8e^4}+\frac{cx^3(ex-d)\sqrt{d+ex}}{4e^2\sqrt{d-ex}}$$

```
[Out] -((3*c*d^2 + 4*b*e^2)*x*Sqrt[d - e*x]*Sqrt[d + e*x])/(8*e^4) + (c*x^3*(-d +
 e*x)*Sqrt[d + e*x])/(4*e^2*Sqrt[d - e*x]) - ((3*c*d^4 + 4*b*d^2*e^2 + 8*a*
e^4)*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]])/(4*e^5)
```

Rubi [A] time = 0.0911319, antiderivative size = 179, normalized size of antiderivative = 1.4, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {520, 1159, 388, 217, 203}

$$\frac{\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(8ae^4+4bd^2e^2+3cd^4\right)}{8e^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{x\left(d^2-e^2x^2\right)\left(4be^2+3cd^2\right)}{8e^4\sqrt{d-ex}\sqrt{d+ex}}-\frac{cx^3\left(d^2-e^2x^2\right)}{4e^2\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((3*c*d^2 + 4*b*e^2)*x*(d^2 - e^2*x^2))/(8*e^4*Sqrt[d - e*x]*Sqrt[d + e*x]
) - (c*x^3*(d^2 - e^2*x^2))/(4*e^2*Sqrt[d - e*x]*Sqrt[d + e*x]) + ((3*c*d^4
 + 4*b*d^2*e^2 + 8*a*e^4)*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x
^2]])/(8*e^5*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{cx^3\left(d^2-e^2x^2\right)}{4e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-4ae^2-\left(3cd^2+4be^2\right)x^2}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{\left(3cd^2+4be^2\right)x\left(d^2-e^2x^2\right)}{8e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx^3\left(d^2-e^2x^2\right)}{4e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(\left(-8ae^4+d^2\left(-3cd^2-4be^2\right)\right)\sqrt{d^2-e^2x^2}\right.}{8e^4\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{\left(3cd^2+4be^2\right)x\left(d^2-e^2x^2\right)}{8e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx^3\left(d^2-e^2x^2\right)}{4e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(\left(-8ae^4+d^2\left(-3cd^2-4be^2\right)\right)\sqrt{d^2-e^2x^2}\right.}{8e^4\sqrt{d-ex}\sqrt{d}}$$

$$= -\frac{\left(3cd^2+4be^2\right)x\left(d^2-e^2x^2\right)}{8e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx^3\left(d^2-e^2x^2\right)}{4e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(3cd^4+4bd^2e^2+8ae^4\right)\sqrt{d^2-e^2x^2}\tan^{-1}}{8e^5\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] time = 0.581851, size = 157, normalized size = 1.23

$$-\frac{16\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)\left(ae^4+bd^2e^2+cd^4\right)+ex\sqrt{d-ex}\sqrt{d+ex}\left(4be^2+3cd^2+2ce^2x^2\right)-\frac{2d^{5/2}\sqrt{\frac{ex}{d}+1}\left(4be^2+5cd^2\right)\sin^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{2}\sqrt{d}}\right)}{\sqrt{d+ex}}}{8e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(e*x*Sqrt[d - e*x]*Sqrt[d + e*x]*(3*c*d^2 + 4*b*e^2 + 2*c*e^2*x^2) - (2*d^
(5/2)*(5*c*d^2 + 4*b*e^2)*Sqrt[1 + (e*x)/d]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*S
qrt[d])])/Sqrt[d + e*x] + 16*(c*d^4 + b*d^2*e^2 + a*e^4)*ArcTan[Sqrt[d - e*
x]/Sqrt[d + e*x]])/(8*e^5)
```

Maple [C] time = 0.017, size = 191, normalized size = 1.5

$$-\frac{\mathrm{csgn}(e)}{8e^5}\sqrt{-ex+d}\sqrt{ex+d}\left(2\,\mathrm{csgn}(e)\,x^3ce^3\sqrt{-e^2x^2+d^2}+4\,\mathrm{csgn}(e)\,e^3\sqrt{-e^2x^2+d^2}xb+3\,\mathrm{csgn}(e)\,e\sqrt{-e^2x^2+d^2}xcd^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/8*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(2*csgn(e)*x^3*c*e^3*(-e^2*x^2+d^2)^(1/2)
+4*csgn(e)*e^3*(-e^2*x^2+d^2)^(1/2)*x*b+3*csgn(e)*e*(-e^2*x^2+d^2)^(1/2)*x*
```

```
c*d^2-8*arctan(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/2))*a*e^4-4*arctan(csgn(e)*e*x
/(-e^2*x^2+d^2)^(1/2))*b*d^2*e^2-3*arctan(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/2))
*c*d^4)*csgn(e)/e^5/(-e^2*x^2+d^2)^(1/2)
```

Maxima [A] time = 1.50604, size = 201, normalized size = 1.57

$$-\frac{\sqrt{-e^2x^2+d^2}cx^3}{4\,e^2}+\frac{a\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{\sqrt{e^2}}+\frac{3\,cd^4\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{8\,\sqrt{e^2}e^4}+\frac{bd^2\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{2\,\sqrt{e^2}e^2}-\frac{3\,\sqrt{-e^2x^2+d^2}cd^2x}{8\,e^4}-\frac{\sqrt{-e^2x^2+d^2}\ldots}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/4*sqrt(-e^2*x^2 + d^2)*c*x^3/e^2 + a*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^
2) + 3/8*c*d^4*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^4) + 1/2*b*d^2*arcs
in(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^2) - 3/8*sqrt(-e^2*x^2 + d^2)*c*d^2*x/
e^4 - 1/2*sqrt(-e^2*x^2 + d^2)*b*x/e^2
```

Fricas [A] time = 1.58024, size = 227, normalized size = 1.77

$$-\frac{\left(2\,ce^3x^3+\left(3\,cd^2e+4\,be^3\right)x\right)\sqrt{ex+d}\sqrt{-ex+d}+2\left(3\,cd^4+4\,bd^2e^2+8\,ae^4\right)\arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{ex}\right)}{8\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/8*((2*c*e^3*x^3 + (3*c*d^2*e + 4*b*e^3)*x)*sqrt(e*x + d)*sqrt(-e*x + d)
+ 2*(3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4)*arctan((sqrt(e*x + d)*sqrt(-e*x + d)
- d)/(e*x)))/e^5
```

Sympy [C] time = 37.9723, size = 325, normalized size = 2.54

$$-\frac{iaG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e}+\frac{aG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e}-\frac{ibd^2G_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & \ldots\\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\ldots\right)}{4\pi^{\frac{3}{2}}e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -I*a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, d**2/(e**2*x**2))/(4*pi**(3/2)*e) + a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2,
1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2
))/(4*pi**(3/2)*e) - I*b*d**2*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), (
(-1, -3/4, -1/2, -1/4, 0, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**3) + b
*d**2*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2,
-1, -1, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**3) - I*c*
```

```
d**4*meijerg(((-7/4, -5/4), (-3/2, -3/2, -1, 1)), ((-2, -7/4, -3/2, -5/4, -
1, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**5) + c*d**4*meijerg(((-5/2, -
9/4, -2, -7/4, -3/2, 1), ()), ((-9/4, -7/4), (-5/2, -2, -2, 0)), d**2*exp_p
olar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**5)
```

Giac [A] time = 1.13785, size = 170, normalized size = 1.33

$$\frac{1}{114688}\left(\left(5\,cd^3e^{16}+4\,bde^{18}-\left(9\,cd^2e^{16}+2\left((xe+d)ce^{16}-3\,cde^{16}\right)(xe+d)+4\,be^{18}\right)(xe+d)\right)\sqrt{xe+d}\sqrt{-xe+d}+2\left(3\,cd^4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/114688*((5*c*d^3*e^16 + 4*b*d*e^18 - (9*c*d^2*e^16 + 2*((x*e + d)*c*e^16
- 3*c*d*e^16)*(x*e + d) + 4*b*e^18)*(x*e + d))*sqrt(x*e + d)*sqrt(-x*e + d)
 + 2*(3*c*d^4*e^16 + 4*b*d^2*e^18 + 8*a*e^20)*arcsin(1/2*sqrt(2)*sqrt(x*e +
 d)/sqrt(d)))*e^(-1)
```

3.141 $\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=102

$$-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{d^2x}-\frac{\left(2be^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)}{e^3}+\frac{cx(ex-d)\sqrt{d+ex}}{2e^2\sqrt{d-ex}}$$

```
[Out] -((a*Sqrt[d - e*x]*Sqrt[d + e*x])/(d^2*x)) + (c*x*(-d + e*x)*Sqrt[d + e*x])
/(2*e^2*Sqrt[d - e*x]) - ((c*d^2 + 2*b*e^2)*ArcTan[Sqrt[d - e*x]/Sqrt[d + e
*x]])/e^3
```

Rubi [A] time = 0.121831, antiderivative size = 155, normalized size of antiderivative = 1.52, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {520, 1265, 388, 217, 203}

$$-\frac{a\left(d^2-e^2x^2\right)}{d^2x\sqrt{d-ex}\sqrt{d+ex}}+\frac{\sqrt{d^2-e^2x^2}\left(2be^2+cd^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{cx\left(d^2-e^2x^2\right)}{2e^2\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((a*(d^2 - e^2*x^2))/(d^2*x*Sqrt[d - e*x]*Sqrt[d + e*x])) - (c*x*(d^2 - e^
2*x^2))/(2*e^2*Sqrt[d - e*x]*Sqrt[d + e*x]) + ((c*d^2 + 2*b*e^2)*Sqrt[d^2 -
 e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3*Sqrt[d - e*x]*Sqrt[d +
e*x])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^2\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{a\left(d^2-e^2x^2\right)}{d^2x\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-bd^2-cd^2x^2}{\sqrt{d^2-e^2x^2}}\,dx}{d^2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{a\left(d^2-e^2x^2\right)}{d^2x\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx\left(d^2-e^2x^2\right)}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(\left(2b+\frac{cd^2}{e^2}\right)\sqrt{d^2-e^2x^2}\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{a\left(d^2-e^2x^2\right)}{d^2x\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx\left(d^2-e^2x^2\right)}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(\left(2b+\frac{cd^2}{e^2}\right)\sqrt{d^2-e^2x^2}\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x,\right.}{2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{a\left(d^2-e^2x^2\right)}{d^2x\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx\left(d^2-e^2x^2\right)}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(cd^2+2be^2\right)\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] time = 0.564506, size = 135, normalized size = 1.32

$$-\frac{\frac{e\sqrt{d-ex}\sqrt{d+ex}\left(2ae^2+cd^2x^2\right)}{d^2x} + 4\left(be^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right) - \frac{2cd^{5/2}\sqrt{\frac{ex}{d}+1}\sin^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{2}\sqrt{d}}\right)}{\sqrt{d+ex}}}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((e*Sqrt[d - e*x]*Sqrt[d + e*x]*(2*a*e^2 + c*d^2*x^2))/(d^2*x) - (2*c*d^(5
/2)*Sqrt[1 + (e*x)/d]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*Sqrt[d])])/Sqrt[d + e*x
] + 4*(c*d^2 + b*e^2)*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]])/(2*e^3)
```

Maple [C] time = 0.022, size = 148, normalized size = 1.5

$$-\frac{\text{csgn}(e)}{2d^2e^3x}\sqrt{-ex+d}\sqrt{ex+d}\left(\text{csgn}(e)x^2cd^2e\sqrt{-e^2x^2+d^2} - 2\arctan\left(\frac{\text{csgn}(e)ex}{\sqrt{-e^2x^2+d^2}}\right)xbd^2e^2 - \arctan\left(\text{csgn}(e)ex\frac{1}{\sqrt{-e^2x}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/2*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2*(csgn(e)*x^2*c*d^2*e*(-e^2*x^2+d^2)^(
1/2)-2*arctan(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/2))*x*b*d^2*e^2-arctan(csgn(e)*
e*x/(-e^2*x^2+d^2)^(1/2))*x*c*d^4+2*csgn(e)*e^3*(-e^2*x^2+d^2)^(1/2)*a)*csg
```

```
n(e)/e^3/(-e^2*x^2+d^2)^(1/2)/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52415, size = 203, normalized size = 1.99

$$-\frac{2\left(cd^4+2\,bd^2e^2\right)x\arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{ex}\right)+\left(cd^2ex^2+2\,ae^3\right)\sqrt{ex+d}\sqrt{-ex+d}}{2\,d^2e^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*(c*d^4 + 2*b*d^2*e^2)*x*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(
e*x)) + (c*d^2*e*x^2 + 2*a*e^3)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^2*e^3*x)
```

Sympy [C] time = 54.2156, size = 287, normalized size = 2.81

$$\frac{iaeG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{aeG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**2/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] I*a*e*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,))
, d**2/(e**2*x**2))/(4*pi**(3/2)*d**2) + a*e*meijerg(((1/2, 3/4, 1, 5/4, 3/
2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**
2))/(4*pi**(3/2)*d**2) - I*b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1
/4, 1/2, 3/4, 1, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e) + b*meijerg(((-
1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), d**2*exp_p
olar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e) - I*c*d**2*meijerg(((-3/4, -1/4)
, (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), d**2/(e**2*x**2)
)/(4*pi**(3/2)*e**3) + c*d**2*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ())
, ((-5/4, -3/4), (-3/2, -1, -1, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(
4*pi**(3/2)*e**3)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError
```

3.142 $\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=157

$$-\frac{(d^2-e^2x^2)(2ae^2+3bd^2)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}}-\frac{a(d^2-e^2x^2)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}}+\frac{c\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] -(a*(d^2 - e^2*x^2))/(3*d^2*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((3*b*d^2 +
2*a*e^2)*(d^2 - e^2*x^2))/(3*d^4*x*Sqrt[d - e*x]*Sqrt[d + e*x]) + (c*Sqrt[d
^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(e*Sqrt[d - e*x]*Sqrt[d +
e*x])
```

Rubi [A] time = 0.124514, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {520, 1265, 451, 217, 203}

$$-\frac{(d^2-e^2x^2)(2ae^2+3bd^2)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}}-\frac{a(d^2-e^2x^2)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}}+\frac{c\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(a*(d^2 - e^2*x^2))/(3*d^2*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((3*b*d^2 +
2*a*e^2)*(d^2 - e^2*x^2))/(3*d^4*x*Sqrt[d - e*x]*Sqrt[d + e*x]) + (c*Sqrt[d
^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(e*Sqrt[d - e*x]*Sqrt[d +
e*x])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
```

```
Q[m + n, -1]))
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx &= \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^4\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{a\left(d^2-e^2x^2\right)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-3bd^2-2ae^2-3cd^2x^2}{x^2\sqrt{d^2-e^2x^2}}\,dx}{3d^2\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{a\left(d^2-e^2x^2\right)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(3bd^2+2ae^2\right)\left(d^2-e^2x^2\right)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(c\sqrt{d^2-e^2x^2}\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{a\left(d^2-e^2x^2\right)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(3bd^2+2ae^2\right)\left(d^2-e^2x^2\right)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(c\sqrt{d^2-e^2x^2}\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{a\left(d^2-e^2x^2\right)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(3bd^2+2ae^2\right)\left(d^2-e^2x^2\right)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} + \frac{c\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e\sqrt{d-ex}\sqrt{d+ex}}
\end{aligned}$$

Mathematica [A] time = 0.128342, size = 81, normalized size = 0.52

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(a\left(d^2+2e^2x^2\right)+3bd^2x^2\right)}{3d^4x^3} - \frac{2c\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(Sqrt[d - e*x]*Sqrt[d + e*x]*(3*b*d^2*x^2 + a*(d^2 + 2*e^2*x^2)))/(3*d^4*x
^3) - (2*c*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]])/e
```

Maple [C] time = 0.022, size = 146, normalized size = 0.9

$$-\frac{\text{csgn}(e)}{3\,d^4x^3e}\sqrt{-ex+d}\sqrt{ex+d}\left(-3\,\arctan\left(\frac{\text{csgn}(e)\,ex}{\sqrt{-e^2x^2+d^2}}\right)x^3cd^4+2\,\text{csgn}(e)\,e^3\sqrt{-e^2x^2+d^2}x^2a+3\,\text{csgn}(e)\,e\sqrt{-e^2x^2+d^2}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/3*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^4*(-3*arctan(csgn(e)*e*x/(-e^2*x^2+d^2)
^(1/2))*x^3*c*d^4+2*csgn(e)*e^3*(-e^2*x^2+d^2)^(1/2)*x^2*a+3*csgn(e)*e*(-e^
```

```
2*x^2+d^2)^(1/2)*x^2*b*d^2+a*(-e^2*x^2+d^2)^(1/2)*d^2*csgn(e)*e)*csgn(e)/(-
e^2*x^2+d^2)^(1/2)/x^3/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38233, size = 203, normalized size = 1.29

$$-\frac{6\,cd^4x^3\arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{ex}\right)+\left(ad^2e+\left(3\,bd^2e+2\,ae^3\right)x^2\right)\sqrt{ex+d}\sqrt{-ex+d}}{3\,d^4ex^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/3*(6*c*d^4*x^3*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(e*x)) + (a*d^2
*e + (3*b*d^2*e + 2*a*e^3)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^4*e*x^3)
```

Sympy [C] time = 77.4416, size = 257, normalized size = 1.64

$$\frac{iae^3G_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3\\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}+\frac{ae^3G_{6,6}^{2,6}\left(\begin{matrix}\frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}+\frac{ibeG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**4/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] I*a*e**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3),
(0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**4) + a*e**3*meijerg(((3/2, 7/4, 2,
 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), d**2*exp_polar(-2*I*pi)/(
e**2*x**2))/(4*pi**(3/2)*d**4) + I*b*e*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2
)), ((1, 5/4, 3/2, 7/4, 2), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**2) + b
*e*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)),
d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d**2) - I*c*meijerg(((1/4
, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), d**2/(e**2*x**2)
)/(4*pi**(3/2)*e) + c*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1
/4), (-1/2, 0, 0, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError
```

3.143 $\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=160

$$-\frac{\left(d^2-e^2x^2\right)\left(8ae^4+10bd^2e^2+15cd^4\right)}{15d^6x\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(4ae^2+5bd^2\right)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\left(d^2-e^2x^2\right)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] -(a*(d^2 - e^2*x^2))/(5*d^2*x^5*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((5*b*d^2 +
4*a*e^2)*(d^2 - e^2*x^2))/(15*d^4*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((15*c
*d^4 + 10*b*d^2*e^2 + 8*a*e^4)*(d^2 - e^2*x^2))/(15*d^6*x*Sqrt[d - e*x]*Sqr
t[d + e*x])
```

Rubi [A] time = 0.145075, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {520, 1265, 453, 264}

$$-\frac{\left(d^2-e^2x^2\right)\left(8ae^4+10bd^2e^2+15cd^4\right)}{15d^6x\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(4ae^2+5bd^2\right)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\left(d^2-e^2x^2\right)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^6*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(a*(d^2 - e^2*x^2))/(5*d^2*x^5*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((5*b*d^2 +
4*a*e^2)*(d^2 - e^2*x^2))/(15*d^4*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((15*c
*d^4 + 10*b*d^2*e^2 + 8*a*e^4)*(d^2 - e^2*x^2))/(15*d^6*x*Sqrt[d - e*x]*Sqr
t[d + e*x])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx &= \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^6\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}\\ &= -\frac{a\left(d^2-e^2x^2\right)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{\sqrt{d^2-e^2x^2}\int \frac{-5bd^2-4ae^2-5cd^2x^2}{x^4\sqrt{d^2-e^2x^2}}\,dx}{5d^2\sqrt{d-ex}\sqrt{d+ex}}\\ &= -\frac{a\left(d^2-e^2x^2\right)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(5bd^2+4ae^2\right)\left(d^2-e^2x^2\right)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}}+\frac{\left(\left(15cd^4-2e^2\left(-5bd^2-4ae^2\right)\right)\sqrt{d^2-\ldots}\right.}{15d^4\sqrt{d-ex}\sqrt{d+\ldots}}\\ &= -\frac{a\left(d^2-e^2x^2\right)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(5bd^2+4ae^2\right)\left(d^2-e^2x^2\right)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(15cd^4+10bd^2e^2+8ae^4\right)\left(d^2-e^2x^2\right)}{15d^6x\sqrt{d-ex}\sqrt{d+ex}}\end{aligned}$$

Mathematica [A] time = 0.12539, size = 87, normalized size = 0.54

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(a\left(4d^2e^2x^2+3d^4+8e^4x^4\right)+5bd^2x^2\left(d^2+2e^2x^2\right)+15cd^4x^4\right)}{15d^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^6*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(Sqrt[d - e*x]*Sqrt[d + e*x]*(15*c*d^4*x^4 + 5*b*d^2*x^2*(d^2 + 2*e^2*x^2)
 + a*(3*d^4 + 4*d^2*e^2*x^2 + 8*e^4*x^4)))/(15*d^6*x^5)
```

Maple [A] time = 0.005, size = 82, normalized size = 0.5

$$-\frac{8\,ae^4x^4+10\,bd^2e^2x^4+15\,cd^4x^4+4\,ad^2e^2x^2+5\,bd^4x^2+3\,ad^4}{15\,x^5d^6}\sqrt{ex+d}\sqrt{-ex+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/15*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(8*a*e^4*x^4+10*b*d^2*e^2*x^4+15*c*d^4*x
^4+4*a*d^2*e^2*x^2+5*b*d^4*x^2+3*a*d^4)/x^5/d^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.41799, size = 173, normalized size = 1.08

$$-\frac{\left(3\,ad^4+\left(15\,cd^4+10\,bd^2e^2+8\,ae^4\right)x^4+\left(5\,bd^4+4\,ad^2e^2\right)x^2\right)\sqrt{ex+d}\sqrt{-ex+d}}{15\,d^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/15*(3*a*d^4 + (15*c*d^4 + 10*b*d^2*e^2 + 8*a*e^4)*x^4 + (5*b*d^4 + 4*a*d
^2*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d)/(d^6*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**6/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.83967, size = 1489, normalized size = 9.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -4/15*(15*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*
e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^2 + 15*b*d^2*((sqrt(2)*sqrt(
d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(
-x*e + d)))^9*e^4 - 240*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e
+ d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^2 + 15*a*((sqr
t(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(
d) - sqrt(-x*e + d)))^9*e^6 - 160*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))
/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^4 +
1440*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d
)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^2 - 80*a*((sqrt(2)*sqrt(d) - sqrt
(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)
))^7*e^6 + 800*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sq
rt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^4 - 3840*c*d^4*((sqrt(2
)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d)
- sqrt(-x*e + d)))^3*e^2 + 928*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x
*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^6 - 2560*b*
d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt
(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^4 + 3840*c*d^4*((sqrt(2)*sqrt(d) - sqrt(
```

```
-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d))
)*e^2 - 1280*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e
 + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^6 + 3840*b*d^2*((sqrt(2)*sqrt
(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt
(-x*e + d)))*e^4 + 3840*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d)
 - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))*e^6)*e^(-1)/((((sqrt(2
)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d)
- sqrt(-x*e + d)))^2 - 4)^5*d^6)
```

3.144 $\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=226

$$-\frac{2e^2\left(d^2-e^2x^2\right)\left(24ae^4+28bd^2e^2+35cd^4\right)}{105d^8x\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(24ae^4+28bd^2e^2+35cd^4\right)}{105d^6x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(6ae^2+7bd^2\right)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}}-$$

```
[Out] -(a*(d^2 - e^2*x^2))/(7*d^2*x^7*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((7*b*d^2 +
6*a*e^2)*(d^2 - e^2*x^2))/(35*d^4*x^5*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((35*c
*d^4 + 28*b*d^2*e^2 + 24*a*e^4)*(d^2 - e^2*x^2))/(105*d^6*x^3*Sqrt[d - e*x]
*Sqrt[d + e*x]) - (2*e^2*(35*c*d^4 + 28*b*d^2*e^2 + 24*a*e^4)*(d^2 - e^2*x^
2))/(105*d^8*x*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rubi [A] time = 0.178386, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {520, 1265, 453, 271, 264}

$$-\frac{2e^2\left(d^2-e^2x^2\right)\left(24ae^4+28bd^2e^2+35cd^4\right)}{105d^8x\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(24ae^4+28bd^2e^2+35cd^4\right)}{105d^6x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(6ae^2+7bd^2\right)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^8*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(a*(d^2 - e^2*x^2))/(7*d^2*x^7*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((7*b*d^2 +
6*a*e^2)*(d^2 - e^2*x^2))/(35*d^4*x^5*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((35*c
*d^4 + 28*b*d^2*e^2 + 24*a*e^4)*(d^2 - e^2*x^2))/(105*d^6*x^3*Sqrt[d - e*x]
*Sqrt[d + e*x]) - (2*e^2*(35*c*d^4 + 28*b*d^2*e^2 + 24*a*e^4)*(d^2 - e^2*x^
2))/(105*d^8*x*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^8\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-7bd^2-6ae^2-7cd^2x^2}{x^6\sqrt{d^2-e^2x^2}}\,dx}{7d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(7bd^2+6ae^2\right)\left(d^2-e^2x^2\right)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(\left(35cd^4-4e^2\left(-7bd^2-6ae^2\right)\right)\sqrt{d^2-\right.}{35d^4\sqrt{d-ex}\sqrt{d+e}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(7bd^2+6ae^2\right)\left(d^2-e^2x^2\right)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(35cd^4+28bd^2e^2+24ae^4\right)\left(d^2-e^2x^2\right)}{105d^6x^3\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(7bd^2+6ae^2\right)\left(d^2-e^2x^2\right)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(35cd^4+28bd^2e^2+24ae^4\right)\left(d^2-e^2x^2\right)}{105d^6x^3\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] time = 0.149583, size = 124, normalized size = 0.55

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(3a\left(6d^4e^2x^2+8d^2e^4x^4+5d^6+16e^6x^6\right)+7b\left(4d^4e^2x^4+8d^2e^4x^6+3d^6x^2\right)+35cd^4x^4\left(d^2+2e^2x^2\right)\right)}{105d^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^8*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(Sqrt[d - e*x]*Sqrt[d + e*x]*(35*c*d^4*x^4*(d^2 + 2*e^2*x^2) + 7*b*(3*d^6*
x^2 + 4*d^4*e^2*x^4 + 8*d^2*e^4*x^6) + 3*a*(5*d^6 + 6*d^4*e^2*x^2 + 8*d^2*e
^4*x^4 + 16*e^6*x^6)))/(105*d^8*x^7)
```

Maple [A] time = 0.005, size = 118, normalized size = 0.5

$$-\frac{48\,ae^6x^6+56\,bd^2e^4x^6+70\,cd^4e^2x^6+24\,ad^2e^4x^4+28\,bd^4e^2x^4+35\,cd^6x^4+18\,ad^4e^2x^2+21\,bd^6x^2+15\,ad^6}{105\,x^7d^8}\sqrt{ex+d}\sqrt{-e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/105*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(48*a*e^6*x^6+56*b*d^2*e^4*x^6+70*c*d^4
*e^2*x^6+24*a*d^2*e^4*x^4+28*b*d^4*e^2*x^4+35*c*d^6*x^4+18*a*d^4*e^2*x^2+21
*b*d^6*x^2+15*a*d^6)/x^7/d^8
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54043, size = 251, normalized size = 1.11

$$-\frac{\left(15\,ad^{6}+2\left(35\,cd^{4}e^{2}+28\,bd^{2}e^{4}+24\,ae^{6}\right)x^{6}+\left(35\,cd^{6}+28\,bd^{4}e^{2}+24\,ad^{2}e^{4}\right)x^{4}+3\left(7\,bd^{6}+6\,ad^{4}e^{2}\right)x^{2}\right)\sqrt{ex+d}\sqrt{-}}{105\,d^{8}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/105*(15*a*d^6 + 2*(35*c*d^4*e^2 + 28*b*d^2*e^4 + 24*a*e^6)*x^6 + (35*c*d
^6 + 28*b*d^4*e^2 + 24*a*d^2*e^4)*x^4 + 3*(7*b*d^6 + 6*a*d^4*e^2)*x^2)*sqrt
(e*x + d)*sqrt(-e*x + d)/(d^8*x^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**8/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.50619, size = 2048, normalized size = 9.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -4/105*(105*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(
x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^13*e^4 + 105*b*d^2*((sqrt(2)*s
```

```
qrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - s
qrt(-x*e + d)))^13*e^6 - 1960*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqr
t(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^11*e^4 + 105
*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(
2)*sqrt(d) - sqrt(-x*e + d)))^13*e^8 - 1400*b*d^2*((sqrt(2)*sqrt(d) - sqrt(
-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d))
)^11*e^6 + 16240*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) -
sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^4 - 840*a*((sqrt(2)*s
qrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - s
qrt(-x*e + d)))^11*e^8 + 12656*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sq
rt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^6 - 806
40*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/
(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^4 + 14448*a*((sqrt(2)*sqrt(d) - sqr
t(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d
)))^9*e^8 - 69888*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) -
 sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^6 + 259840*c*d^4*((s
qrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqr
t(d) - sqrt(-x*e + d)))^5*e^4 - 40704*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))
/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^8 +
202496*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e +
 d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^6 - 501760*c*d^4*((sqrt(2)*sqrt
(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt
(-x*e + d)))^3*e^4 + 231168*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e
+ d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^8 - 358400*b*d
^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(
2)*sqrt(d) - sqrt(-x*e + d)))^3*e^6 + 430080*c*d^4*((sqrt(2)*sqrt(d) - sqrt
(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)
))*e^4 - 215040*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(
x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^8 + 430080*b*d^2*((sqrt(2)
*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) -
 sqrt(-x*e + d)))*e^6 + 430080*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x
*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))*e^8)*e^(-1)/(((
(sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*s
qrt(d) - sqrt(-x*e + d)))^2 - 4)^7*d^8)
```

3.145 $\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=292

$$-\frac{8e^4\left(d^2-e^2x^2\right)\left(16ae^4+18bd^2e^2+21cd^4\right)}{315d^{10}x\sqrt{d-ex}\sqrt{d+ex}}-\frac{4e^2\left(d^2-e^2x^2\right)\left(16ae^4+18bd^2e^2+21cd^4\right)}{315d^8x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(16ae^4+18bd^2\right.}{105d^6x^5\sqrt{d-ex}\sqrt{d}}$$

```
[Out] -(a*(d^2 - e^2*x^2))/(9*d^2*x^9*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((9*b*d^2 +
8*a*e^2)*(d^2 - e^2*x^2))/(63*d^4*x^7*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((21*c
*d^4 + 18*b*d^2*e^2 + 16*a*e^4)*(d^2 - e^2*x^2))/(105*d^6*x^5*Sqrt[d - e*x]
*Sqrt[d + e*x]) - (4*e^2*(21*c*d^4 + 18*b*d^2*e^2 + 16*a*e^4)*(d^2 - e^2*x^
2))/(315*d^8*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - (8*e^4*(21*c*d^4 + 18*b*d^2
*e^2 + 16*a*e^4)*(d^2 - e^2*x^2))/(315*d^10*x*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rubi [A] time = 0.241533, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {520, 1265, 453, 271, 264}

$$-\frac{8e^4\left(d^2-e^2x^2\right)\left(16ae^4+18bd^2e^2+21cd^4\right)}{315d^{10}x\sqrt{d-ex}\sqrt{d+ex}}-\frac{4e^2\left(d^2-e^2x^2\right)\left(16ae^4+18bd^2e^2+21cd^4\right)}{315d^8x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(16ae^4+18bd^2\right.}{105d^6x^5\sqrt{d-ex}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^10*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```


```
[Out] -(a*(d^2 - e^2*x^2))/(9*d^2*x^9*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((9*b*d^2 +
8*a*e^2)*(d^2 - e^2*x^2))/(63*d^4*x^7*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((21*c
*d^4 + 18*b*d^2*e^2 + 16*a*e^4)*(d^2 - e^2*x^2))/(105*d^6*x^5*Sqrt[d - e*x]
*Sqrt[d + e*x]) - (4*e^2*(21*c*d^4 + 18*b*d^2*e^2 + 16*a*e^4)*(d^2 - e^2*x^
2))/(315*d^8*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - (8*e^4*(21*c*d^4 + 18*b*d^2
*e^2 + 16*a*e^4)*(d^2 - e^2*x^2))/(315*d^10*x*Sqrt[d - e*x]*Sqrt[d + e*x])
```


Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-9bd^2-8ae^2-9cd^2x^2}{x^8\sqrt{d^2-e^2x^2}}\,dx}{9d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(9bd^2+8ae^2\right)\left(d^2-e^2x^2\right)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(\left(63cd^4-6e^2\left(-9bd^2-8ae^2\right)\right)\sqrt{d^2-\right.}{63d^4\sqrt{d-ex}\sqrt{d+}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(9bd^2+8ae^2\right)\left(d^2-e^2x^2\right)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(21cd^4+18bd^2e^2+16ae^4\right)\left(d^2-e^2x^2\right.}{105d^6x^5\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(9bd^2+8ae^2\right)\left(d^2-e^2x^2\right)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(21cd^4+18bd^2e^2+16ae^4\right)\left(d^2-e^2x^2\right.}{105d^6x^5\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(9bd^2+8ae^2\right)\left(d^2-e^2x^2\right)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(21cd^4+18bd^2e^2+16ae^4\right)\left(d^2-e^2x^2\right.}{105d^6x^5\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] time = 0.180761, size = 158, normalized size = 0.54

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(a\left(40d^6e^2x^2+48d^4e^4x^4+64d^2e^6x^6+35d^8+128e^8x^8\right)+9b\left(6d^6e^2x^4+8d^4e^4x^6+16d^2e^6x^8+5d^8x^2\right)\right.}{315d^{10}x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^10*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(Sqrt[d - e*x]*Sqrt[d + e*x]*(21*c*d^4*x^4*(3*d^4 + 4*d^2*e^2*x^2 + 8*e^4*
x^4) + 9*b*(5*d^8*x^2 + 6*d^6*e^2*x^4 + 8*d^4*e^4*x^6 + 16*d^2*e^6*x^8) + a
*(35*d^8 + 40*d^6*e^2*x^2 + 48*d^4*e^4*x^4 + 64*d^2*e^6*x^6 + 128*e^8*x^8))
)/(315*d^10*x^9)
```

Maple [A] time = 0.007, size = 154, normalized size = 0.5

$$-\frac{128\,ae^8x^8+144\,bd^2e^6x^8+168\,cd^4e^4x^8+64\,ad^2e^6x^6+72\,bd^4e^4x^6+84\,cd^6e^2x^6+48\,ad^4e^4x^4+54\,bd^6e^2x^4+63\,cd^8x^4+}{315\,x^9d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/315*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(128*a*e^8*x^8+144*b*d^2*e^6*x^8+168*c*
d^4*e^4*x^8+64*a*d^2*e^6*x^6+72*b*d^4*e^4*x^6+84*c*d^6*e^2*x^6+48*a*d^4*e^4
*x^4+54*b*d^6*e^2*x^4+63*c*d^8*x^4+40*a*d^6*e^2*x^2+45*b*d^8*x^2+35*a*d^8)/
x^9/d^10
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86087, size = 327, normalized size = 1.12

$$-\frac{\left(35\,ad^8 + 8\left(21\,cd^4e^4 + 18\,bd^2e^6 + 16\,ae^8\right)x^8 + 4\left(21\,cd^6e^2 + 18\,bd^4e^4 + 16\,ad^2e^6\right)x^6 + 3\left(21\,cd^8 + 18\,bd^6e^2 + 16\,ad^4e^4\right.}{315\,d^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] -1/315*(35*a*d^8 + 8*(21*c*d^4*e^4 + 18*b*d^2*e^6 + 16*a*e^8)*x^8 + 4*(21*c
*d^6*e^2 + 18*b*d^4*e^4 + 16*a*d^2*e^6)*x^6 + 3*(21*c*d^8 + 18*b*d^6*e^2 +
16*a*d^4*e^4)*x^4 + 5*(9*b*d^8 + 8*a*d^6*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x
+ d)/(d^10*x^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**10/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.4791, size = 2607, normalized size = 8.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] -4/315*(315*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(
x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^17*e^6 + 315*b*d^2*((sqrt(2)*s
qrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - s
qrt(-x*e + d)))^17*e^8 - 6720*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqr
t(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^15*e^6 + 315
*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(
2)*sqrt(d) - sqrt(-x*e + d)))^17*e^10 - 5040*b*d^2*((sqrt(2)*sqrt(d) - sqrt
(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)
))^15*e^8 + 76608*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) -
 sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^13*e^6 - 3360*a*((sqrt(2
)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d)
- sqrt(-x*e + d)))^15*e^10 + 68544*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d)
)/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^13*e^8
- 580608*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e
 + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^11*e^6 + 76608*a*((sqrt(2)*sqrt(d
) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-
x*e + d)))^13*e^10 - 509184*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(
x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^11*e^8 + 28922
88*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/
(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^6 - 327168*a*((sqrt(2)*sqrt(d) - sq
rt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e +
d)))^11*e^10 + 2363904*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e +
 d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^8 - 9289728*c*d
^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(
2)*sqrt(d) - sqrt(-x*e + d)))^7*e^6 + 2728448*a*((sqrt(2)*sqrt(d) - sqrt(-x
*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^
9*e^10 - 8146944*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) -
sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^8 + 19611648*c*d^4*((
sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sq
rt(d) - sqrt(-x*e + d)))^5*e^6 - 5234688*a*((sqrt(2)*sqrt(d) - sqrt(-x*e +
d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^1
0 + 17547264*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt
(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^8 - 27525120*c*d^4*((sqrt
(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d
) - sqrt(-x*e + d)))^3*e^6 + 19611648*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))
/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^10 -
 20643840*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*
e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^8 + 20643840*c*d^4*((sqrt(2)
*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) -
 sqrt(-x*e + d)))*e^6 - 13762560*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt
(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^10 + 2064
3840*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d
)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))*e^8 + 20643840*a*((sqrt(2)*sqrt(d) -
sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e
+ d)))*e^10)*e^(-1)/((((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - s
qrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^2 - 4)^9*d^10)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
   (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```